BASIC NONLINEAR CONTROL SYSTEMS

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Contents

Summary

This section is written to introduce the reader to nonlinearity and its effect in control systems, and also to 'set the stage' for the subsequent articles. Nonlinearity is first defined and then some of the physical effects which cause nonlinear behavior are discussed. This is followed by some comments on the stability of nonlinear systems, the existence of limit cycles, and the unique behavioral aspects which nonlinear feedback systems may possess. Finally a few brief comments are made on designing controllers for nonlinear systems.

1. Introduction

Linear systems have the important property that they satisfy the superposition principle. This leads to many important advantages in methods for their analysis. For example, in circuit theory when an RLC circuit has both d.c and a.c input voltages, the voltage or current elsewhere in the circuit can be found by summing the results of separate analyses for the d.c. and a.c. inputs taken individually, also if the magnitude of the a.c. voltage is doubled then the a.c. voltages or currents elsewhere in the circuit will be doubled.

Thus, mathematically a linear system with input $x(t)$ and output $y(t)$ satisfies the property that the output for an input $ax_1(t)+bx_2(t)$ is $ay_1(t)+by_2(t)$, if $y_1(t)$ and $y_2(t)$ are the outputs in response to the inputs $x_1(t)$ and $x_2(t)$, respectively, and a and b are constants. A nonlinear system is defined as one which does not satisfy the superposition property. The simplest form of nonlinear system is the static nonlinearity where the output depends only on the current value of input but in a nonlinear manner, for example

$$y(t) = ax(t) + bx^3(t) \quad (1)$$

More commonly a nonlinear differential equation, for example

$$d^2y(t)/dt^2 + a(dy(t)/dt)^3 + by(t) = x(t) \quad (2)$$

will describe the behavior. From an engineering viewpoint it may be desirable to think of this equation in terms of a block diagram consisting of linear dynamic elements and a static nonlinearity, in this case a cubic, which with input $dy(t)/dt$ gives an output $a(dy(t)/dt)^3$. A nonlinear system can possess many different forms of behavior, which are dependent on its structure, and are unique to nonlinear systems as discussed later. The major point about nonlinear systems, however is that their response is amplitude dependent so that if a particular form of response, or some measure of it, occurs for one input magnitude it may not result for some other input magnitude.

This means that in a feedback control system if the controller designed does not linearize the system then to adequately describe the system behavior one needs to cover the whole state space. In practice, of course, this means the whole space covered by the allowable range, due to saturation, of the system states. For a linear system one can claim that a system has an optimum response, assuming optimum is precisely defined, for example minimization of the integral squared error, using results for just one input amplitude, but all amplitudes, over the allowable range have to be checked for a nonlinear system. Perhaps the most interesting aspect of nonlinear systems is that they exhibit forms of behavior not possible in linear systems and more will be said on this later.

2. Forms of nonlinearity

In control engineering nonlinearity may occur in the dynamics of the plant to be controlled or in the components used to implement the control, for example a dead zone in a valve, which may be referred to as inherent nonlinearity, because it exists although one would probably prefer this not to be the case. Alternatively one may have intentional nonlinearities which have been purposely designed into the system to improve the system specifications, either from a technical or economic viewpoint. Identifying the precise form of a nonlinearity may not be easy and like all modeling exercises the golden rule is to be aware of the approximations in a nonlinear model and the conditions for its validity.

Some people would argue that linear systems theory is not applicable to practical control engineering problems because they are always nonlinear. This is an overstatement, of course, but a valid reminder. All systems have actuator saturation and in some cases it might occur for relatively low error signals, for example in rotary position control it is not unusual for a step input of say 10°, or even less, to produce maximum motor drive torque. It simply is the result of good economical design. Valves used to control fluid or gas flow, apart from having a saturation value also possess a dead zone due to friction and have slightly different behavior when opening compared with closing due to the unidirectional pressure of the fluid.

Friction always occurs in mechanical systems and is very difficult to model, with many quite sophisticated models having been presented in the literature. The simplest is to assume the three components of stiction, an abbreviation for static friction, Coulomb friction and viscous friction. As its name implies stiction is assumed to exist only at zero differential speed between the two contact surfaces.

Coulomb friction with a value less than stiction is assumed to be constant at all speeds, and viscous friction is a linear effect being directly proportional to speed. In practice there is often a term proportional to a higher power of speed, and this is also the situation for many shaft loads, for example a fan. Many mechanical loads are driven through gearing rather than directly. Although geared drives, like all areas of technology, have improved through the years they always have some small backlash.

This may be avoided by using anti-backlash gears, which are only available for low torques. Backlash which is a very complicated phenomenon involving impacts between surfaces is often modeled in a very simplistic manner. For example, the simple approach used in some digital simulation languages, such as Simulink the simulation component of the well-known software package Matlab, consists of an input- output position characteristic of two parallel straight lines with possible horizontal movement between them. This makes two major assumptions, first that the load shaft friction is high enough for contact to be maintained with the drive side of the backlash when the drive slows down to rest. Secondly when the drive reverses the backlash is crossed and the new drive side of the gear 'picks up' the load instantaneously with no loss of energy in the impact and both then move at the drive shaft speed. Clearly both these assumptions are never true in practice but no checks exist in Simulink to determine how good they are or, indeed, when they are completely invalid.

The most widely used intentional nonlinearity is the relay. The on-off type, which can be described mathematically by the signum function, that is switches on if its input exceeds a given value and off if it goes below the value, is widely used normally with some hysteresis between the switching levels. Use of this approach provides a control strategy where the controlled variable oscillates about the desired level. The switching mechanism varies significantly according to the application from electromechanical relays at low speed to fast electronic switches employing transistors or thyristors.

A common usage of the relay is in temperature control of buildings, where typically the switching is provided by a pool of mercury on a metal expansion coil. As the temperature drops the coil contracts and this causes a change in angle of the mercury capsule so that eventually the mercury moves and closes a contact. Electronic switching controllers are being used in many modern electric motor drive systems, for example, to regulate phase currents in stepping motors and switched reluctance motors and to control currents in vector control drives for induction motors. Relays with a dead zone, that is, three position relays giving positive, negative and a zero output are also used. The zero output allows for a steady state position within the dead zone but this affects the resulting steady state control accuracy.

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Biographical Sketch

Derek Atherton was born in Bradford, England on 21 April 1934. He has a B.Eng from Sheffield University and Ph.D and D.Sc from Manchester University. He taught at Manchester University, and McMaster University and the University of New Brunswick in Canada before taking up the appointment of Professor of Control Engineering at the University of Sussex in 1980, where he currently has a part-time appointment. He has served on committees of the Science and Engineering Research Council, as President of the Institute of Measurement and Control in 1990 and President of the Control Systems Society of the Institute of Electrical and Electronic Engineers, USA in 1995, and also served for six years on the International Federation of Automatic Control (IFAC) Council. His major research interests are in nonlinear control theory, computer aided control system design, simulation and target tracking. He has written three books, one of which is jointly authored and published over 300 papers.