

## ANTI WINDUP AND OVERRIDE CONTROL

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### Summary

Linear controllers are very successful in industrial applications. They imply linear time invariant plant models. This typically holds for small deviations from steady state operation. But for large deviations from steady state most plants exhibit ‘input constraints’ introduced by actuator saturation, and also ‘output constraints’ on secondary plant outputs. Typical examples are operational limits on current in DC-drives with speed control, or on temperature differences in thermal power plants, etc.

Such design problems may be solved analytically by using the Maximum Principle from Optimal Control Theory. They may also be solved by numerical trajectory optimization in a receding time horizon frame (such as in Model Predictive Control). A third approach has been established much earlier by control engineers on an intuitive basis. It is a two step design procedure. A linear control system is designed for the small deviations operation. Then large deviations are applied. Any performance degradation is countered by nonlinear add-on's to the linear control algorithm. This standard technique is known as 'anti windup' and 'override' design. This text is an introduction to the third approach. Three aspects of design and analysis are considered, that is structure, transient response, and nonlinear stability properties.

## 1. Introduction

### 1.1. Control Systems with Input Constraints

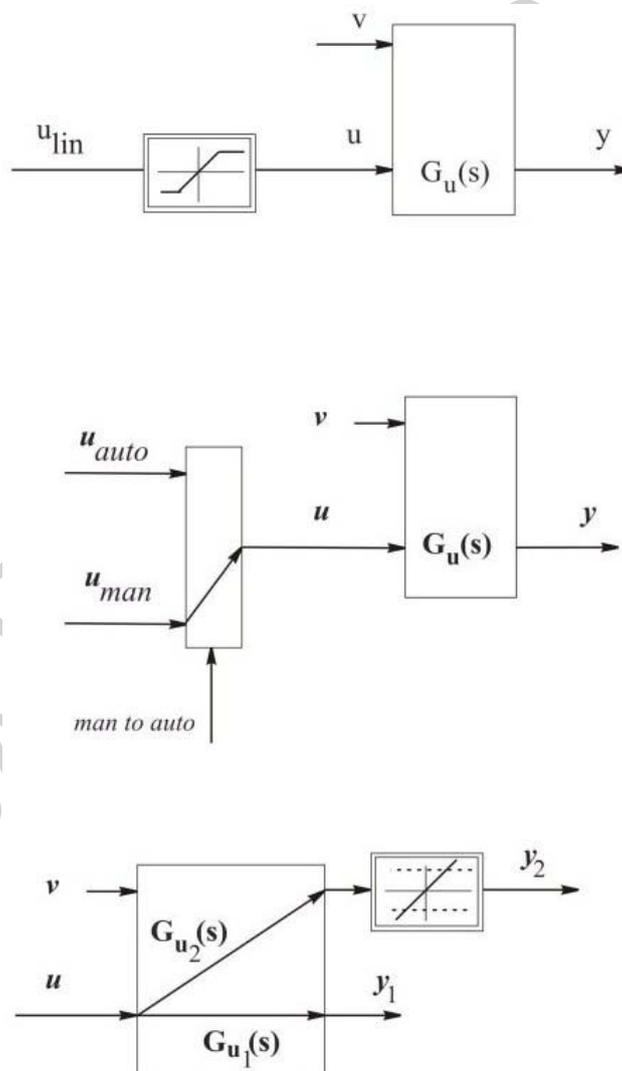


Figure1. Control system with 'input' saturation (top), with 'manual to automatic' transfer switch (center), and with constraint on the secondary output  $y_2$  (bottom)

Consider the system to be controlled, or ‘plant’ for short, in Figure 1. There are two (scalar) inputs, the manipulated or control variable  $u$  and the disturbance input  $v$ , and one (scalar) output, the controlled variable  $y$ . A typical example would be a tank with level  $y$  to be controlled to its reference value  $r$ , with persistent outflow  $\bar{v}$  and manipulated inflow  $u$ . Thus  $\bar{v}$  determines the operating point  $\bar{u}$ .

The  *saturations*  on the control variable  $u(t)$  are due to the working range of the actuator, which is always bounded by physical reasons to a low and a high limit. An often used model is

$$\begin{aligned}
 u = SAT(u_{lin}) \quad & u = u_{low} \quad \text{if } u_{lin} < u_{low} \\
 & u = u_{lin} \quad \text{if } u_{low} \leq u_{lin} \leq u_{high} \\
 & u = u_{high} \quad \text{if } u_{lin} > u_{high}
 \end{aligned} \tag{1}$$

This is the ‘input constraint’ situation, Figure 1 top. Here ‘input’ refers to the ‘plant’, and not to the closed loop system.

In the range of operating conditions  $\bar{v}$  for which the feedback loop was originally designed, the plant shall finally stabilize at some equilibrium  $\bar{u}$ :

$$u_{low} + \Delta u_{low} < \bar{u} < u_{high} - \Delta u_{high} \tag{2}$$

situated at a finite distance ( $\Delta u$ ) inside the working range limits of  $u$ . This allows regulation as specified for sufficiently small sized loop input changes (to the setpoint  $r$  and/or to the disturbance  $v$ ), such that  $u(t)$  does not touch the limit values. This is the typical situation in control design. Here the model of the dynamic response may be linearized around the equilibrium, leading to a linear time invariant plant model and then to a linear controller, such as standard PID, state feedback, etc.

For larger loop input changes however, the control variable  $u$  will saturate, either transiently or permanently. In the second case the control function will be permanently interrupted, and control objectives clearly cannot be met any longer. In other words this must be remedied by re-design of the actuator subsystem, such as expanded working range or additional actuators. This case shall not be considered further here.

The focus will be on the transiently saturating case, where a new equilibrium can be established with  $\bar{u}$  inside limits, see Eq. (1).

## 1.2. Control Systems with Mode Switch

Another basic nonlinearity in almost all control loops is the ‘Manual-to-Automatic’ control mode switch on  $u(t)$ , Figure 1 center.

‘Bumpless transfer’ on  $u(t)$  is specified for Manual-to-Auto-mode switch-over and vice versa, independent of all past  $u(t)$ . This requires some mutual output tracking feature

on  $u_{man}$  and  $u_{auto}$ . Note that the second control  $u_{man}$  may also be the output of a ‘Programmable Logic Controller’ or another regulator.

The most interesting phase is mainly the switching to the automatic mode, if state variables and the controlled plant output  $y(t)$  have not attained equilibrium values for the current setpoint. The subsequent transient may move  $u$  to its limits, or output constraints may be met.

### 1.3. Control Systems with Output Constraints

A third major class of basic nonlinearities is that of ‘output constraints’, Figure 1 bottom. The ‘constrained output’  $y_c(t)$  is proportional to a single state variable of the plant or to a linear combination of those.

Typical examples are speed and torque in electromechanical positioning loops, winding temperature in high performance DC-Servomotors, pressure and flow in hydroelectric plants, temperature and temperature differences (that is differential expansion, thermoshock, etc.) in thermal power plants, and many similar cases in chemical plants. In most situations, more than one constrained output is present, but only cases of one  $y_c$  shall be considered here.

Such output constraints are operational limits rather than ‘hard’ physical limits. Transgressing them is possible, but will add to low cycle fatigue of process equipment. Very large transgressions will cause spontaneous equipment failure, but this is to be avoided by separate safety functions and shall not be considered here.

Output constraints are typically ‘soft’ constraints, that is there exists a continuous model of the response along the constraint values  $y_{c_{low}}$  or  $y_{c_{high}}$  respectively, for small deviations within the span indicated by the  $\Delta$ 's.

$$y_{c_{low}} - \Delta_{low} < y_c(t) < y_{c_{high}} + \Delta_{high} \quad (3)$$

Thus feedback control is feasible along the constraints.

In other words the main assumption is that  $y_c(t)$  is controllable by  $u(t)$  along its constraint values in a sufficient working range. If not, no active control is feasible, and again the actuator subsystem will have to be re-designed.

### 1.4. Design Approaches

Three general design approaches are available for such nonlinear control problems:

- The *analytic approach* considers the problem of optimizing the system trajectory to the new equilibrium subject to both input and output constraints by applying the ‘Maximum Principle’ and produces an optimal control function  $u^*(t)$ . The

solution is primarily feedforward and thus requires an exact model of the system and all input signals. The optimization problem is termed ‘regular’ for input constraints only, and ‘singular’ if additional output constraints are present.

- The *numerical approach* optimizes a discrete time control sequence  $u^*(kT_s)$  over a given time interval by iteration. Then the first control move is implemented. At the end of the sampling interval  $T_s$ , the optimization is redone, using the actual state variables and/or outputs, and using the same time look-ahead intervals (receding horizon). This amounts to feedback. Such Model Predictive Control (MPC) is very successful in chemical industry on complex processes with many interacting inputs, outputs and relatively large time constants.
- The *intuitive approach* has been developed by practitioners for handling such problems with available control equipment before theory and adequate computers were available. It is also called the *two-step design* procedure.

As a first design step a linear control system is designed for small enough deviations around the operating point to meet the performance specifications. Any of the classical linear methods may be used.

If this control system is also used for situations where input or output constraints are encountered, clearly then performance specifications can no longer be met. This is countered in the second design step by intuitive nonlinear add-on's to the linear regulator. Some of them have turned out to be effective in most routine applications, and are now standard practice in industrial control design.

This text will focus on the third approach. As such solutions have been developed on an empirical basis, a more theoretical analysis is needed to clearly establish properties and show the limits for their ‘good usage’.

Three views will be used in the following, namely ‘Structure’, ‘Transient response’ and ‘Stability properties’, because from experience it is crucial to have a good balance of these three views to obtain a good design result. Note that many discussions of alternatives have been about ‘Structure’ only, while the other views were not considered.

Here the ‘input saturation’ and ‘output constraint’ situations are addressed. The ‘bumpless transfer’ situation shall not be investigated here beyond mentioning that standard anti windup structures on the controller delivering  $u_{auto}(t)$  are being used.

The most common case of PI control systems with input saturation shown in Figure 2 top shall be studied first, demonstrating the ‘windup effect’ and different ‘anti windup’ features. Then plants of dominant higher order are considered, where ‘plant windup’ may appear, and suitable anti windup schemes are investigated. Finally ‘output constraint’ control systems of different forms are considered, such as with the ‘override’ structure shown in Figure 2 bottom.

Concerning *methodology*, the investigation begins with a detailed specification of the problem and of a benchmark case. Then the linear controller is designed for the small

linear region around steady state. Next the nonlinear extensions are considered. Several current structure alternatives are introduced and their transient responses are discussed.

But then any conclusions about the performance of the different alternatives drawn from the simulations are valid only for this benchmark case. Therefore extended simulations would be required to cover the spread of possible cases.

This need can be satisfied more effectively by *nonlinear stability analysis*. It turns out to be a powerful tool to obtain a deeper insight beyond mere simulations. In this text the sector criteria shall be used. Their theoretical background is not covered here, and the reader is referred to the references.

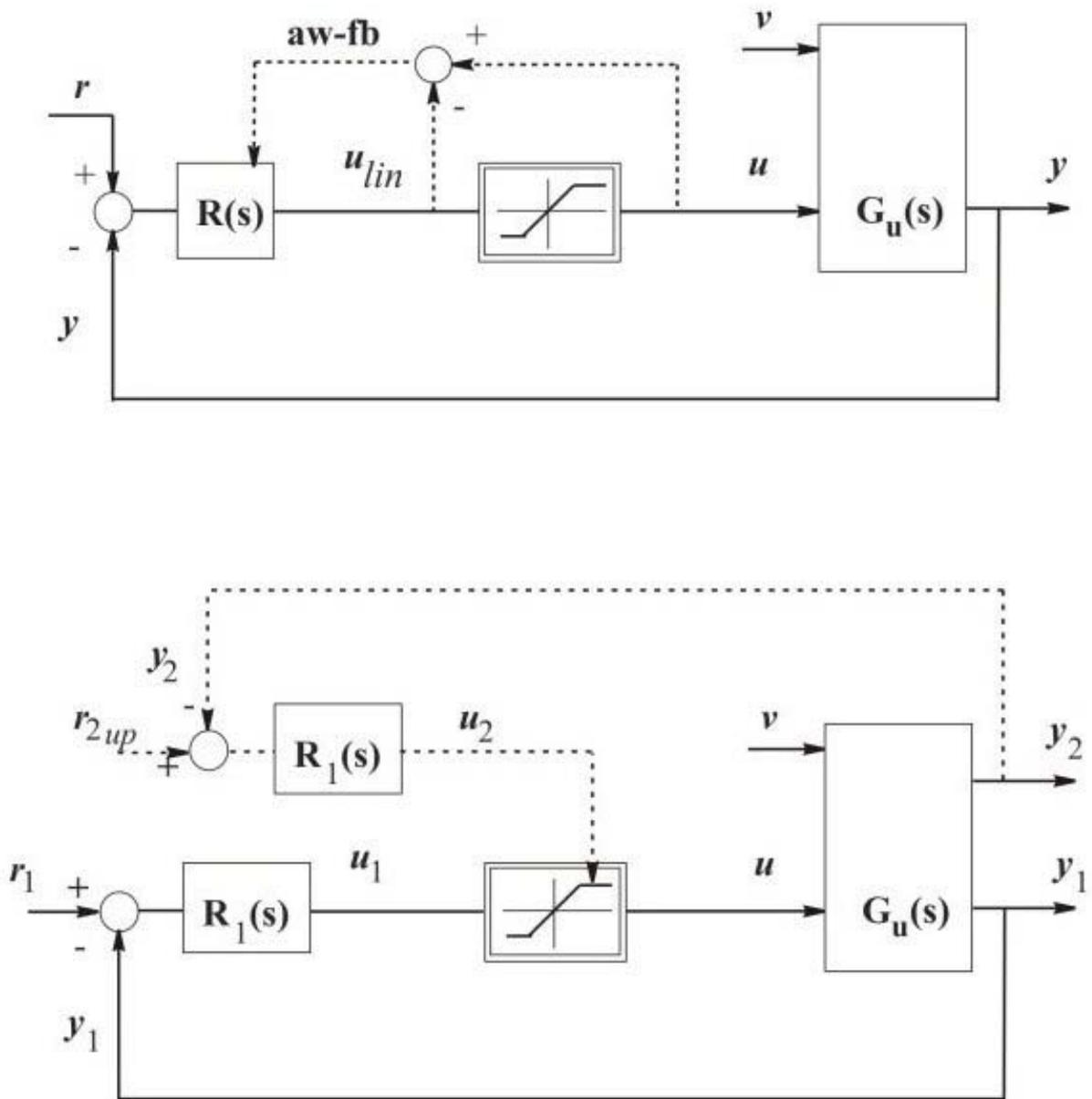


Figure 2. ‘Anti windup control’ for input saturation (top) and ‘Override control’ for an upward constraint on  $y_2$  at  $r_{2up}$

Concerning *references*, there are several authoritative books on the underlying nonlinear stability analysis. The intuitive design approach to such control systems is covered in many Conference and Journal papers. No comprehensive overview in book form exists so far.

## 2. PI-Control with Input Saturations

### 2.1. Problem Statement and Test Cases

The following set of specifications is used for the control system under study. It defines also a benchmark for testing controllers with alternate anti windup features.

(a) The **plant** is given by its transfer function

$$G(s) = \frac{y(s)}{u(s)} = e^{-sD} \frac{b}{sT + a} \quad (4)$$

This corresponds to the standardized plant model used by the Ziegler-Nichols and Chien-Hrones-Reswick design methods, (e.g. Aström, Hägglund, 1995). It covers a large percentage of industrial control cases, such as speed, level, pressure, concentration and temperature control. The small delay  $D$  shall represent the fast non-modeled dynamics of actuator, process and sensor, and will limit the attainable closed loop bandwidth to a realistic value.

Neglecting the small delay yields the 'dominant first order' plant model

$$G_d(s) = \frac{y(s)}{u(s)} = \frac{b}{sT + a} \quad (5)$$

to be used for controller design. For the load input  $v(t)$ :

$$G_v(s) = \frac{y(s)}{v(s)} = -G_d \quad (6)$$

*Numerical values* to be used in the test cases are:

$$b := 1.0; \quad T := 1.0; \quad D := 0.020; \quad \text{and} \quad a := 1.0; \quad \text{or} \quad := 0.0; \quad (7)$$

which covers both standard cases 'unity gain first order lag' and 'open integrator'.

(b) The **PI-controller** shall be derived from the standard continuous form used in the Ziegler-Nichols rules, by replacing the continuous integration by its discrete Euler equivalent (e.g. Aström, Hägglund, 1995)

$$R(s) = k_p \left( 1 + \frac{1}{sT_i} \right) \rightarrow R(z) = k_p \left( 1 + \frac{1}{T_i} T_s \frac{z^{-1}}{1 - z^{-1}} \right) \quad (8)$$

For the test cases, the sampling time  $T_s$  is set to  $T_s = 0.020$  [s].

(c) The **PI-controller settings**  $k_p$  and  $T_i$  are obtained by pole placement using the ‘dominant first order’ plant dynamics  $G_d(s)$  and the continuous time form of the controller  $R(s)$ . This implies  $T_s$  and  $D$  to be sufficiently short. From the closed loop characteristic equation

$$0 = 1 + R(s)G_d(s) = s^2 + s \frac{a + k_p b}{T} + \frac{k_p b}{T_i T} = (s + \Omega)^2 \quad (9)$$

then

$$k_p b = 2\Omega T - a; \quad \frac{1}{T_i} = \frac{\Omega^2 T}{k_p b} \quad \text{and for } a = 0; \quad k_p b = 2\Omega T; \quad \frac{1}{T_i} = \frac{\Omega}{2} \quad (10)$$

Setting the closed loop bandwidth to  $\Omega = 5$  [rad/s] provides a sufficient margin with respect to both the sampling frequency from  $T_s$  and the ‘ultimate frequency’ from  $D$ . And it produces realistic values for the controller settings (with  $a = 0$ :  $k_p = 10$ ; and  $T_i = 0.40$ ).

From experience the linear control loop must be designed for good but realistic performance. Not respecting this (obvious)  $T_s$  rule may lead to very misleading conclusions about the performance of anti windup schemes.

(d) The following **test sequence** shall be applied to the closed loop

- Initially (index<sub>0</sub>) the loop is to be at ‘standstill’ conditions:  
 $r_0 = 0$ ; and  $v_0 = 0$ ; that is  $\bar{y}_0 = 0$ ; and  $\bar{u}_0 = 0$ ;
- At time  $T_1$ , a setpoint step to  $r_1 = 0.95$  is applied while there is still no load, that is  $v_1 = 0$ .
- Then at time  $T_2$ , a setpoint step to  $r_2 = 1.0$  is applied while there is still no load, that is  $v_2 = 0$ . This will show the small signal (linear) closed loop response. Then at equilibrium, a plant input  $\bar{u}_1 = a \cdot r_1$  results.
- At time  $T_3$  a load step  $v_3 = 0.90$  is applied, while the setpoint is constant,  $r_3 = r_2 = 1$ ,
- and finally at time  $T_4$ , a full load reversal is applied  $v_4 = -v_3$ , again with constant setpoint  $r_4 = r_3 = 1$ .

No high frequency measurement disturbance shall be considered here, but is a problem in some applications.

(e) The **actuator saturation** is specified as follows:

$$\text{for } a = 0: \quad u_{low} = -1.0; \quad u_{high} = +1.0; \quad (11)$$

$$\text{for } a = 1: \quad u_{low} = -0.0; \quad u_{high} = +2.0; \quad (12)$$

This provides the same 'margin to maneuver' for setpoint steps around the ' $r = 1$  and no load' condition for both values of  $a$  introduced above. Also  $\bar{v}_3 = +0.90$  yields a realistic steady state control margin (see Eq. (2)) of  $\Delta u_{high} = \Delta u_{low} = 0.10$  for the load swings.

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### Biographical Sketches

**Adolf Hermann Glattfelder** studied mechanical engineering at ETH Zürich and obtained his diploma degree there in 1964. He started his professional career working with Brown Boveri Cie, Baden, Switzerland. He obtained his Ph.D degree in 1969 at ETH Zürich, and his "Habilitation" in 1973. He has been teaching in this field at ETH Zürich since then as a part time lecturer, and is a titular Professor at ETH Zürich since 1989. In 1976 he continued his professional career in industry, with Sulzer Escher Wyss, Zürich, as R&D manager and then as manager for automatic control systems of hydroelectric plants (Ober-Ingenieur, 1977), and then with Sulzer Corporate Research (Vize-Direktor, 1985).

He has been at the Automatic Control Laboratory of ETH Zürich since 1991. His research and teaching activities are in the field of application oriented control system design, and especially in "modeling for control" and in "control systems with saturations and overrides". He is leading a research group of Ph.D. students in "Automatic Control in Anesthesia", in a research cooperation with the Research group in anesthesia at the University Hospital Bern (Switzerland) and has frequent contacts with industry for design of control systems for mechanical, hydraulic and thermal systems.

**Walter Schaufelberger** studied electrical engineering at the ETH Zürich during the early sixties. He earned a doctorate at ETH Zürich in 1969. Following an assignment as guest lecturer at Queen's University in Kingston, Canada, he was appointed assistant professor 1972 at ETH Zürich; 1977 as associate professor and in 1983 he was named Professor Ordinarius. He is currently Dean of International Relations of ETH. He is especially interested in the many different aspects of Control Engineering covering both theory and practical applications and using computers for the design and implementation of control systems. He was President of the European Society for Engineering Education (SEFI) from 1997 to 1999 and Treasurer of the International Federation of Automatic Control (IFAC) from 1993 to 2002.