GAIN-SCHEDULING

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Contents

Summary

The gain-scheduling approach is perhaps one of the most popular nonlinear control design approaches which has been widely and successfully applied in fields ranging from aerospace to process control. While much of the classical gain-scheduling theory originates from the 1960s, there has recently been a considerable increase in interest in

gain-scheduling in the literature with many new results obtained. This chapter discusses the main theoretical results and design procedures relating to continuous gain-scheduling (in the sense of decomposition of nonlinear design into linear sub-problems) control.

1. Introduction

Gain-scheduling is perhaps one of the most popular approaches to nonlinear control design and has been widely and successfully applied in fields ranging from aerospace to process control. In recent years, there has been a marked resurgence of interest in gain-scheduling methods within the research community with many fundamental developments in gain-scheduling theory. Although a wide variety of control methods are often described as "gain-scheduling" approaches, these are usually linked by a divide-and-conquer type of design procedure whereby the nonlinear control design task is decomposed into a number of linear sub-problems.

This divide-and-conquer approach is the source of much of the popularity of gain-scheduling methods since it enables well established linear design methods to be applied to nonlinear problems. (While the analysis and design of nonlinear systems remains relatively difficult, techniques for the analysis and design of linear time-invariant systems are rather better developed *Stability Concepts*, *Stability Theory* , *Control of Linear Multivariable Systems*, *Robust Control*). However, it is also emphasized that the benefits of continuity with linear methods often extend beyond purely technical considerations; for example, safety certification requirements are often based on linear methods and the development of new certification procedures using nonlinear approaches may well be prohibitive.

Of course, the question must be asked as to whether the basic premise of such design approaches is in fact reasonable; that is, whether a wide class of nonlinear design tasks can genuinely be decomposed into linear sub-problems. While few results are available which relate directly to this fundamental issue, and it is well known that certain classes of problem present greater difficulty than others for gain-scheduling methods, the general usefulness of such methods is nevertheless well established both in practice and from a theoretical viewpoint.

The chapter is organized as follows. The theoretical results relating the dynamic characteristics of a nonlinear system to those of a family of linear systems are reviewed in Section 2. The classical gain-scheduling design procedure is discussed in Section 3 followed by a number of recent divide-and-conquer approaches which attempt to address a number of deficiencies of classical methods. LPV gain-scheduling approaches, which have recently been the subject of considerable research activity but are less strongly based on divide-and-conquer ideas, are reviewed in Section 4 and the outlook is briefly discussed in Section 5. The notation used is standard.

2. Linearization theory

Gain-scheduling design typically employs a divide-and-conquer approach whereby the nonlinear design task is decomposed into a number of linear sub-tasks. Such a decomposition depends on establishing a relationship between a nonlinear system and a family of linear systems. The main theoretical results which, for a broad class of nonlinear systems, relate the dynamic characteristics of a member of the class to those of an associated family of linear systems are reviewed in this section.

These results fall into two main sub-classes: first, stability results which establish a relationship between the stability of a nonlinear system and the stability of an associated linear system and second, approximation results which establish a direct relationship between the solution to a nonlinear system and the solution to associated linear systems. It is important to distinguish between these classes of result.

The former are typically much more limited than the latter, being confined to specifying conditions under which boundedness of the solution to a particular linear system implies boundedness of the solution to the nonlinear system for an appropriate class of inputs and initial conditions. Notice that under such conditions the solutions are bounded but may otherwise be quite dissimilar. Reflecting this distinction, the discussion in the following sections often separately considers results relating both to stability and approximation.

The section is organized as follows. Perhaps the most widespread approach for associating a linear system with a nonlinear one, namely series expansion linearization theory, is first reviewed The series expansion linearization is only valid in the vicinity of a specific trajectory or equilibrium point, and so there is considerable incentive to develop techniques which relax this restriction. Approaches which aim to increase the allowable operating envelope by utilizing a family of linearizations (rather than just a single linearization) are reviewed in Sections 2.2-2.3.

2.1. Series Expansion Linearization about a Single Trajectory or Equilibrium Point

Consider the nonlinear system,

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{r}), \quad \mathbf{y} = \mathbf{G}(\mathbf{x},\mathbf{r}) \tag{1}$$

where $r \in R^m$ $y \in R^p$ $x \in R^n$. Let $(\tilde{\mathbf{x}}(t),\ \tilde{\mathbf{r}}(t),\ \tilde{\mathbf{y}}(t))$ denote a specific trajectory of the nonlinear system (the trajectory could simply be an equilibrium operating point in which case $\tilde{\mathbf{x}}$ is constant). Neglecting higher-order terms, it follows from series expansion theory that the nonlinear system, (1), may be approximated, locally to the trajectory, $(\tilde{\mathbf{x}}(t),\ \tilde{\mathbf{r}}(t),\ \tilde{\mathbf{y}}(t))$, by the linear time-varying system

$$\delta\dot{\hat{\mathbf{x}}} = \nabla_{\mathbf{x}}\mathbf{F}(\tilde{\mathbf{x}},\tilde{\mathbf{r}})\delta\hat{\mathbf{x}} + \nabla_{\mathbf{r}}\mathbf{F}(\tilde{\mathbf{x}},\tilde{\mathbf{r}})\delta\mathbf{r} \tag{2}$$

$$\delta\hat{\mathbf{y}} = \nabla_{\mathbf{x}}\mathbf{G}(\tilde{\mathbf{x}},\tilde{\mathbf{r}})\delta\hat{\mathbf{x}} + \nabla_{\mathbf{r}}\mathbf{G}(\tilde{\mathbf{x}},\tilde{\mathbf{r}})\delta\mathbf{r} \tag{3}$$

where

$$\delta\mathbf{r} = \mathbf{r} - \tilde{\mathbf{r}}, \mathbf{y} = \delta\mathbf{y} + \tilde{\mathbf{y}}, \delta\mathbf{x} = \mathbf{x} - \tilde{\mathbf{x}} \quad (4)$$

The nonlinear system, (1), is stable relative to the trajectory ($\tilde{\mathbf{x}}(t)$, $\tilde{\mathbf{r}}(t)$, $\tilde{\mathbf{y}}(t)$) provided the linear time-varying dynamics (2)-(3) are robustly stable with respect to the approximation error involved in truncating the series expansion. In fact, is turns out that nonlinear system, (1), is locally BIBO stable if and only if the linear system (2)-(3) is exponentially stable when $\delta\mathbf{r}$ is zero (*i.e.* the unforced case) - see *Lyapunov Stability*.

In the special case when the system, (2)-(3), is linear time-invariant, simple necessary and sufficient conditions for its stability are well-known (see *Stability Concepts, Stability Theory* ,). However, in the time-varying case (see also *Design Techniques for Time-Varying Systems*), the stability analysis is, in general, not so straightforward. In the context of gain-scheduling, frozen-time theory is widely employed to establish stability conditions for linear time-varying systems.

Specifically, it can be shown that the stability of the linear time-varying system, (2)-(3), is guaranteed provided that the time variation of $\nabla_{\mathbf{x}}\mathbf{F}(\tilde{\mathbf{x}},\tilde{\mathbf{r}})$ is sufficiently slow in some appropriate sense (for example, that $\sup\limits_{t\geq 0} \left| d/dt\left(\nabla_{\mathbf{x}}\mathbf{F}(\tilde{\mathbf{x}},\tilde{\mathbf{r}})\right)\right|$ is sufficiently small). Although classical frozen-time results mainly relate to nominal stability, it can also be shown that, provided the rate of variation is sufficiently slow, the linear time-varying system (2)-(3) inherits the worst-case stability robustness of the family of frozen-time linear time-invariant systems $\delta\dot{\hat{\mathbf{x}}} = \mathbf{A}_\tau\delta\hat{\mathbf{x}}$ where $\mathbf{A}_\tau$ denotes the value of $\nabla_{\mathbf{x}}\mathbf{F}(\tilde{\mathbf{x}},\tilde{\mathbf{r}})$ at time τ .

Frozen-time theory is generally conservative in that it only establishes sufficient conditions for stability. In addition, it is important to note that in all of the frozen-time robustness results an increase in robustness requires a decrease in the allowable rate of variation and that the linear time-varying system fully inherits the robustness of the frozen-time family only as the allowable rate of variation becomes arbitrarily small.

The foregoing results relate to stability properties only. Were the linear dynamics, (2)-(3), an accurate approximation to the nonlinear dynamics, (1), then it might be expected that, when starting from the same initial conditions, the solutions of (1) and (2)-(3) remain correlated for some time.

However, the solution to (2)-(3) is, in fact, only a *zeroeth order* approximation to the solution to (1). This poor approximation property is inevitably reflected in the weakness of any approximation result. Available results are essentially confined to a restatement of bounded-input bounded-output (BIBO) stability; that is, the solutions of (1) and (2)-(3) both remain within a bounded region enclosing the origin provided the input and the initial conditions are sufficiently small.

2.2. Series expansion linearization families

The foregoing results are confined to the dynamic behavior locally to a *single* trajectory or equilibrium operating point. This is a significant limitation of the series expansion linearization theory particularly since the local neighborhood within which the analysis is valid may, in general, be very small. Within a gain-scheduling context, it is almost always required to consider the behavior of a system relative to a *family* of operating points, which spans the envelope of operation, rather than relative to a single operating point. In order to increase the size of the operating region within which a series expansion linearization is valid, it is therefore natural to consider combining, in some sense, the series expansion linearizations associated with a number of equilibrium points.

At this point it is perhaps worth emphasizing the clear distinction which exists between a single dynamic system and a family of dynamic systems, regardless of any superficial similarity between the two. The linear time-varying system (2)-(3),for example, is a quite different object (being a distinct dynamic system) from the associated family of frozen linear time-invariant systems (being a collection of dynamic systems). The importance of this distinction becomes particularly great when the state, input and/or output of the members of the family differ from one another as is the case when considering the family of series expansion linearizations of (1) relative to the equilibrium points.

The state, input and output of each series expansion linearization are perturbation quantities which depend on the equilibrium point considered. The relationship between the solution to a nonlinear system and the solutions to its series expansion linearizations is thus not straightforward when the system is not confined to the vicinity of a single equilibrium point. Nevertheless, it is possible to establish a weaker relationship. Namely, a relationship between the local stability of a nonlinear system and the stability of the associated series expansion equilibrium linearizations.

The relevant theory stems primarily from an early lemma by Hoppensteadt from 1966, originally derived in the context of singular perturbation theory. Using this so-called frozen-input theory, it can be shown that the nonlinear system, (1), is locally BIBO stable in the vicinity of equilibrium operation provided that the members of its family of equilibrium linearizations are uniformly stable and the rate of variation is sufficiently slow. In addition, a trivial extension of this result is that, provided that the rate of variation is sufficiently slow, the nonlinear system also inherits the stability robustness of the equilibrium linearizations to smooth, finite dimensional, nonlinear perturbations (although there is the usual trade-off between robustness and the restrictiveness of the slow variation condition required).

The slow variation condition in these results generally takes the form of a restriction both on the initial conditions of the system and on the rate of variation of the forcing input. This slow variation condition plays two roles: firstly, it ensures that the system stays sufficiently near to equilibrium operation (necessary owing to the use of equilibrium linearizations) and secondly, it ensures that the system evolves sufficiently slowly from the vicinity of one equilibrium point to the vicinity of another. It is emphasized that the analysis is *inherently* confined to a small neighborhood enclosing the equilibrium operating points and consequently may be extremely conservative. Such a restriction is, of course, to be expected since the analysis is based on the properties of the series expansion linearizations of the nonlinear plant relative to the equilibrium points and so only utilizes information regarding the dynamics at equilibrium. It is also worth noting that it is often difficult to test whether the stability conditions obtained are satisfied since they typically involve quantities which are difficult to evaluate.

Indeed, perhaps owing to this difficulty, although frozen-input results are widely invoked in the literature to justify control designs it is quite rare for the theoretical slow variation conditions applying in a particular application to be actually determined. An additional technical requirement in frozen-input stability analysis is that the equilibrium operating points are smoothly parameterized by the system input, **r**. This requirement is not unduly restrictive in an analysis context but may be undesirable in the gain-scheduling design context, where it is more natural to parameterize the equilibrium operating points by the scheduling variable.

2.3. Off-equilibrium linearizations

Classical series expansion theory associates a linear time-invariant system only with equilibrium operating points. Consequently, any analysis/design based on this theory is generally only valid during near equilibrium operation. This limitation arises due to the characteristics of classical series expansions but may be resolved by, instead, considering an alternative linearization framework.

Before proceeding, in order to streamline the later discussion it is useful to explicitly highlight the linear and nonlinear dependencies of the dynamics by reformulating the nonlinear system (1) as

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Br} + \mathbf{f}(\boldsymbol{\rho}), \quad \mathbf{y} = \mathbf{Cx} + \mathbf{Dr} + \mathbf{g}(\boldsymbol{\rho}) \tag{5}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are appropriately dimensioned constant matrices, $\mathbf{f}(\bullet)$ and $\mathbf{g}(\bullet)$ are nonlinear functions and $\boldsymbol{\rho}(\mathbf{x},\mathbf{r})$ embodies the nonlinear dependence of the dynamics on the state and input with $\nabla_x\boldsymbol{\rho}$, $\nabla_r\boldsymbol{\rho}$ constant. Trivially, this reformulation can always be achieved by letting $\boldsymbol{\rho} = [\mathbf{x}^T \ \mathbf{r}^T]^T$. However, the nonlinearity of the system is frequently dependent on only a subset of the states and inputs, in which case the dimension of $\boldsymbol{\rho}$ is reduced. The formulation, (5), defines a scheduling variable $\boldsymbol{\rho}$ which explicitly embodies the nonlinear dependence of the dynamics.

The solution to the velocity-based linearization

$$\dot{\hat{\mathbf{x}}} = \hat{\mathbf{w}} \tag{6}$$

$$\dot{\hat{\mathbf{w}}} = (\mathbf{A}\nabla_{\mathbf{f}}(\boldsymbol{\rho}_1))\hat{\mathbf{w}} + (\mathbf{B} + \nabla_{\mathbf{f}}(\boldsymbol{\rho}_1))\dot{\mathbf{r}} \tag{7}$$

$$\dot{\hat{\mathbf{y}}} = (\mathbf{C} + \nabla_{\mathbf{g}}(\boldsymbol{\rho}_1))\hat{\mathbf{w}} + (\mathbf{D} + \nabla_{\mathbf{g}}(\boldsymbol{\rho}_1))\dot{\mathbf{r}} \tag{8}$$

approximates the solution to the nonlinear system, (5) (and so (1)), to second-order locally to an operating point $(\mathbf{x}_1, \mathbf{r}_1)$ at which $\boldsymbol{\rho}_1 = \boldsymbol{\rho}(\mathbf{x}_1, \mathbf{r}_1)$. It is emphasized that $(\mathbf{x}_1, \mathbf{r}_1)$ may be a general operating point (it need not be an equilibrium point and, indeed, may lie far from any equilibria). While the solution to the velocity-based linearization is only a local approximation, there is a velocity-based linearization associated with every operating point of a nonlinear system and the solutions to these linearizations may be pieced together to globally approximate the solution to the nonlinear system, (5), to an arbitrary degree of accuracy.

Hence, the velocity-based linearization family embodies the entire dynamics of a nonlinear system, with no loss of information, and is, in fact, an alternative representation of the nonlinear system. The velocity-based linearization family is parameterized by the scheduling variable, $\boldsymbol{\rho}$, and in this sense $\boldsymbol{\rho}$ captures the nonlinear structure of a system. The relationship between the nonlinear system, (5), and its velocity-based linearization, (6)-(8), is direct. Differentiating (5), an alternative representation of the nonlinear system is

$$\dot{\mathbf{x}} = \mathbf{w} \tag{9}$$

$$\mathbf{w} = (\mathbf{A} + \nabla_{\mathbf{f}}(\boldsymbol{\rho}))\dot{\mathbf{w}} + (\mathbf{B} + \nabla_{\mathbf{f}}(\boldsymbol{\rho}))\dot{\mathbf{r}} \tag{10}$$

$$\dot{\mathbf{y}} = (\mathbf{C} + \nabla_{\mathbf{g}}(\boldsymbol{\rho}))\dot{\mathbf{w}} + (\mathbf{D} + \nabla_{\mathbf{g}}(\boldsymbol{\rho}))\dot{\mathbf{r}} \tag{11}$$

Evidently, the velocity-based linearization, (6)-(8), is simply the frozen form of (9)-(11) at the operating point, $(\mathbf{x}_1, \mathbf{r}_1)$.

The relationship between the solution to a nonlinear system and the solutions to the members of the associated velocity-based linearization family can be used to derive conditions relating the stability of a nonlinear system to the stability of its velocity-based linearizations.

General stability analysis methods such as small gain theory (see Input-Output Stability) and Lyapunov theory (see Lyapunov Stability) can be applied to derive velocity-based stability conditions (including the methods in Section 4 below). In addition, by adopting the velocity-based framework, it is possible to extend and strengthen the classical frozen-input stability results discussed in Section 2.2. Specifically, BIBO stability of the nonlinear system (1) is guaranteed provided the members of its velocity-based linearization family are uniformly stable, unboundedness of the state $\mathbf{x}$ implies that $\mathbf{w}$ is unbounded (assuming the input $\mathbf{r}$ is bounded) and the class of inputs and initial conditions is restricted to limit the rate of evolution of the nonlinear system to be sufficiently slow.

In addition, provided that the rate of evolution is sufficiently slow, the nonlinear system inherits the stability robustness of the members of the velocity-based linearization family (with the usual trade-off between robustness and the restrictiveness of the slow variation condition required).. This velocity-based result involves no restriction to near equilibrium operation other than that implicit in the slow variation requirement; for example, for some systems where the slow variation condition is automatically satisfied the class of allowable inputs and initial conditions is unrestricted and the stability analysis is global.

3. Divide & Conquer Gain-Scheduling Design

Gain-scheduling design approaches conventionally construct a nonlinear controller, with certain required dynamic properties, by combining, in some sense, the members of an appropriate family of linear time-invariant controllers. Design approaches may be broadly classified according to the linear family utilized. Classical gain-scheduling design approaches, based on the series expansion linearization of a system about its equilibrium points, are discussed in Section 3.1. Recent, and closely related, approaches based on neural/fuzzy modeling and off-equilibrium linearizations are considered, respectively, in Sections 3.2 and 3.3.

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Bibliography

[Section 2. Key publications establishing the theoretical basis of classical gain-scheduling methods. Desoer's paper underlies the slow variation theory for linear time-varying systems discussed in Section 2.1. Hoppensteadt's seminal work, extended by Khalil & Kokotovic, establishes frozen input theory discussed in Section 2.2].

[Section 3.1. The classical gain-scheduling approach discussed is often followed implicitly rather than explicitly. A clear exposition of the approach is given by Astrom and illustrated by Shamma and Athans, Hyde and Glover]

[Section 3.1. The main theoretically-based approaches to the choice of realization of classical gain-scheduling controllers are discussed in these papers. The local linear equivalence formalism was initially proposed by Kaminer and co-workers and generalized by Rugh and co-workers. The deficiencies of the approach were subsequently highlighted by Leith and Leithead and their extended local linear equivalent approach is discussed in detail in the 1998 Int. J. Control article].

[Section 3.2. The Local Model Network approach to gain-scheduling was originated by Johansen and co-workers. The paper by Hunt and Johansen presents details of an example illustrating the application of these ideas. Takagi-Sugeno methods employed LMI design techniques similar to those used in LPV gain-scheduling are described by Tanaka and co-workers. The negative impact of blending and the use of affine rather than linear local models in Local Model Networks and Takagi-Sugeno models was highlighted by Shorten and co-workers and is analysed in detail by Leith and Leithead.]

[Section 3.3. Off-equilibrium/velocity-based linearization theory was introduced by Leith and Leithead. The velocity-based gain-scheduling design approach was introduced in the companion articles. The article by Leith, Tsoudos *et al*. presents a detailed application of these ideas.]

[Section 4. Small-gain LFT based LPV gain-scheduling methods appear to have been independently proposed by Packard and by Apkarian and co-workers. Quadratic Lyapunov function LPV methods were introduced by Becker. Parameter dependent Lyapunov function methods were introduced by Wu and co-workers and are discussed in detail by Apkarian and Adams.]

[The following books provide good references to underlying theoretical tools]

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Biographical Sketches

Dr. Douglas Leith graduated from the University of Glasgow in 1986 with a first class BSc (Eng) degree in Electronics and Electrical Engineering. During this course of study he was awarded prizes including the *Alexander J. Younger Memorial*, the *John Oliphant Bursary*, the *Sir John Pender Bursary*, the *ICI Prize in Control Engineering* and the *Howe Prize*. He was awarded his PhD, also from the University of Glasgow, in 1989. Following a one year post-doctoral Fellowship and a short spell lecturing at the Victoria University of Technology in Melbourne, Dr. Leith joined the University of Strathclyde in 1992. He was subsequently awarded a prestigious personal *Royal Society Fellowship* to pursue his interests in the mathematical modeling, analysis and design of nonlinear dynamic systems which has included the development of *VB linearization theory*. Prof. Leith is currently a Research Professor at the National University of Ireland Maynooth and is Director of the Hamilton Institute (www.hamilton.may.ie).

Dr. William E. Leithead is Professor of Systems and Control within the Department of Electronic and Electrical Engineering at the University of Strathclyde and a Senior Researcher at the Hamilton Institute. In addition to control design for nonlinear stochastic systems, Dr. Leithead's research interests include wind turbine dynamics and control, multivariable control and implementation aspects of control design. With regard to wind turbine modeling and regulation, Dr. Leithead has led numerous projects funded by the EPRSC, DTI and directly by industry and is a founding member of the recently inaugurated Centre for Economic Renewable Power Delivery based at the Universities of Glasgow and Strathclyde. Dr. Leithead also has strong links with the U.K. aerospace industry and is actively involved in the development of design methodologies for multivariable control applications.