ESTIMATION WITH UNKNOWN NOISE MODEL

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Summary

The consistent identification schemes presented in *Estimation with known Noise Model* and *Frequency Domain Subspace Algorithms*, assumed explicitly that the covariance matrix of the disturbing noise is known *a priori*. In practice this information should also be extracted from the experimental data. In this chapter, it is shown that a utilizable non-parametric frequency domain noise model can be obtained from a very small number of repeated experiments. Under these conditions the consistency of the estimates is maintained, while the loss in efficiency is small. A consistent estimator, which does not use (an estimate of) the noise covariance matrix is also presented. Since this chapter relies strongly on the results of *Estimation with known Noise Model* and *Frequency Domain Subspace Algorithms*, it can not be read independently of these chapters.

1. Introduction

1.1. Problem Statement

In *Estimation with known Noise Model* and *Frequency Domain Subspace Algorithms* a large variety of estimators were discussed, ranging from linear least squares methods to maximum likelihood estimators. The more advanced estimators like GTLS, BTLS and ML estimators require knowledge of the covariance matrix with the disturbing noise as a function of the frequency. For example, the ML estimator of transfer function model

$$G(\Omega_k,\theta)=\frac{\sum_{r=0}^{n_b} b_r\Omega^r}{\sum_{r=0}^{n_a} a_r\Omega^r} \tag{1}$$

minimizes:

$$V_{\mathrm{ML}}(\theta,Z)=\sum_{k=1}^{F}\frac{\left|e(\Omega_k,\theta,Z(k))\right|^2}{\sigma_e^2(\Omega_k,\theta)}=\sum_{k=1}^{F}\left|\varepsilon(\Omega_k,\theta,Z(k))\right|^2 \tag{2}$$

with $e(\Omega_k,\theta,Z(k))=A(\Omega_k,\theta)Y(k)-B(\Omega_k,\theta)U(k)$ the equation error, $\sigma_e^2(\Omega_k,\theta)=\mathrm{var}(e(\Omega_k,\ \theta,N_Z(k)))$ the variance of the equation error,

$$\sigma_e^2(\Omega_k,\theta)=\sigma_Y^2(k)\left|A(\Omega_k,\theta)\right|^2+\sigma_U^2(k)\left|B(\Omega_k,\theta)\right|^2-2\mathrm{Re}(\sigma_{YU}^2(k)A(\Omega_k,\theta)\bar{B}(\Omega_k,\theta)) \tag{3}$$

and $\varepsilon(\Omega_k,\theta,Z(k))=e(\Omega_k,\theta,Z(k))/\sigma_e(\Omega_k,\theta)$ the normalized equation error. The noise (co-) variances $\sigma_U^2(k)$, $\sigma_Y^2(k)$ and $\sigma_{YU}^2(k)$ were assumed to be known exactly, and under these conditions the properties of the estimators were studied. In practice, this information is not available, but should be extracted from the experimental data. In this chapter we will replace the exact noise (co-)variances by their sample values. This is only possible if independent, repeated experiments are available. A practical solution consists of applying periodic excitations to the plant and observing M consecutive periods of the steady state response. These M experiments result in a set of M input/output DFT spectra

$$U^{[l]}(k),Y^{[l]}(k),l=1,\ldots,M \text{ and } k=1,\ldots,F\,. \tag{4}$$

The sample means and sample (co-)variances of this set of measurements is calculated

$$\hat{U}(k)=\frac{1}{M}\sum_{l=1}^{M}U^{[l]}(k) \qquad \hat{Y}(k)=\frac{1}{M}\sum_{l=1}^{M}Y^{[l]}(k) \tag{5}$$

$$\begin{gathered}\hat{\sigma}_U^2(k)=\frac{1}{M-1}\sum_{l=1}^{M}\left|U^{[l]}(k)-\hat{U}(k)\right|^2 \quad \hat{\sigma}_Y^2(k)=\frac{1}{M-1}\sum_{l=1}^{M}\left|Y^{[l]}(k)-\hat{Y}(k)\right|^2\\ \hat{\sigma}_{YU}^2(k)=\frac{1}{M-1}\sum_{l=1}^{M}(Y^{[l]}(k)-\hat{Y}(k))\overline{(U^{[l]}(k)-\hat{U}(k))}.\end{gathered} \tag{6}$$

The sample means $\hat{U}(k)$, $\hat{Y}(k)$ and the sample noise (co-)variances $\hat{\sigma}_U^2(k)$, $\hat{\sigma}_Y^2(k)$ and $\hat{\sigma}_{YU}^2(k)$ are used in (2) and (3) instead of $U(k)$, $Y(k)$ and the true noise (co-)variances. To compare this approach with the classical framework that deals with arbitrary excitations (Ljung, 1999), we have to simplify the errors-in-variables framework to a weighted output error problem. This means that only process noise is considered, the measurement noise on the input and the output is assumed to be zero ($\sigma_U^2(k)=0$ and also $\sigma_{YU}^2(k)=0$) so that the cost function (2) reduces to

$$V_{\mathrm{ML}}(\theta,Z)=\sum_{k=1}^{F}\frac{\left|Y(k)-\dfrac{B(\Omega_k,\theta)}{A(\Omega_k,\theta)}U(k)\right|^2}{\sigma_Y^2(k)}=\sum_{k=1}^{F}\frac{\left|Y(k)-G(\Omega_k,\theta)U(k)\right|^2}{\sigma_Y^2(k)}. \qquad (7)$$

Because in this classical framework no repeated measurements are imposed, the sample variance $\hat{\sigma}_Y^2(k)$ cannot be calculated. Instead a parametric noise model $\sigma_Y^2(k)=\sigma^2\left|H(z_k^{-1},\theta)\right|^2$ is used and the additional noise model parameters are estimated together with the plant model parameters (Ljung, 1999). This poses to the question as to what approach should be preferred: the parametric or the non-parametric (sample (co-)variances) noise modeling approach?

The major advantage of the parametric modeling approach is its applicability to arbitrary excitations. Its major disadvantages are the need for a double model selection problem (plant model and noise model), the more complex optimization problem, and the fact that the quality of the estimated noise model strongly depends on the quality of plant model. The reader is referred to Ljung (1999) for a comprehensive discussion of these techniques.

The major disadvantages of the non-parametric approach are the restriction to periodic excitations and the loss in frequency resolution of a factor M w.r.t. the parametric approach (see also Section 3.1). However, whenever periodic excitations can be applied, significant advantages appear: the non-parametric model is generated automatically, without any user interaction; the errors-in-variables problem can be solved straightforwardly (no equivalent solution is available in the classical approach); the cost function is absolutely interpretable, which simplifies the validation process significantly (see Section 6). For these reasons, we prefer to use the non-parametric noise models whenever it is possible to apply periodic excitations, independent of the fact that a time or frequency domain method will be used later on.

1.2. Noise Model

The estimators are studied under the following idealized noise assumptions. First, we require that M independent repeated experiments are available. Next, we make an assumption about the disturbing errors of the *l*th experiment.

1. The measured input/output DFT spectra , $U^{[l]}(k), Y^{[l]}(k), k = 1, 2, \ldots, F$ and $l = 1, 2, \ldots, M$, satisfy

$$\begin{aligned} Y^{[l]}(k) &= Y_0(k) + N_Y^{[l]}(k), \\ U^{[l]}(k) &= U_0(k) + N_U^{[l]}(k), \end{aligned} \tag{8}$$

where the true unknown deterministic values $U_0(k)$, $Y_0(k)$ are independent of l, and where the disturbing input/output errors $N_U^{[l]}(k)$, $N_Y^{[l]}(k)$ are independent over l.

2. The disturbing noise $N_Z^{[l]}(k) = [N_Y^{[l]}(k)\ N_U^{[l]}(k)]^T$ is independent over the frequency k, has zero mean, and is circular complex normally distributed with covariance matrix

$$C_{N_Z}(k) = E\{N_Z^{[l]}(k)(N_Z^{[l]}(k))^H\} = \begin{bmatrix} \sigma_Y^2(k) & \sigma_{YU}^2(k) \\ \overline{\sigma}_{YU}^2(k) & \sigma_U^2(k) \end{bmatrix} \tag{9}$$

and with $E\{N_Z^{[l]}(k)(N_Z^{[l]}(k))^T\} = 0$ (Picinbono, 1993).

The noise behavior is characterized using the sample mean and sample variance, obtained from a set of repeated measurements. Hence the two conditions are met in practice for a frequency domain experiment (see Section 1.1 of *Estimation with known Noise Model* for the definition of a frequency and time domain experiment). For a time domain experiment we often obtain these repeated measurements by measuring M successive periods in one record. For each period, we calculate the Fourier coefficients and consider them as independent experiments from one period to the other as formalized in (8). This is only approximately met in practice since some correlation exists between neighboring periods. Because the correlation of filtered white noise (time domain experiment assumption) decays exponentially, the correlation between two neighboring periods reduces as the length of the period is increased. In practice it can be neglected if the period length is large compared with the correlation length of the noise.

2. Estimation Algorithms

2.1. Maximum Likelihood

A new cost function is defined putting $\hat{U}(k)$, $\hat{Y}(k)$ and $\hat{\sigma}_U^2(k)$, $\hat{\sigma}_Y^2(k)$, $\hat{\sigma}_{YU}^2(k)$ as the measurements and the variances, respectively, into the cost function (2):

$$V_{\text{SML}}(\theta, Z) = \sum_{k=1}^{F} \frac{\left|\hat{e}(\Omega_k, \theta, \hat{Z}(k))\right|^2}{\hat{\sigma}_{\hat{e}}^2(\Omega_k, \theta)} = \sum_{k=1}^{F} \hat{\varepsilon}(\Omega_k, \theta, \hat{Z}(k)) \tag{10}$$

with $\hat{\varepsilon}(\Omega_k, \theta, \hat{Z}(k)) = \hat{e}(\Omega_k, \theta, \hat{Z}(k)) / \hat{\sigma}_{\hat{e}}(\Omega_k, \theta)$ and

$$\begin{aligned}
&\hat{e}(\Omega_k, \theta, \hat{Z}(k)) = A(\Omega_k, \theta)\hat{Y}(k) - B(\Omega_k, \theta)\hat{U}(k) \\
&\hat{\sigma}_{\hat{e}}^2(\Omega_k, \theta) = \hat{\sigma}_e^2(\Omega_k, \theta) / M \\
&\hat{\sigma}_e^2(\Omega_k, \theta) = \hat{\sigma}_Y^2(k)\left|A(\Omega_k, \theta)\right|^2 + \hat{\sigma}_U^2(k)\left|B(\Omega_k, \theta)\right|^2 \\
&-2\,\mathrm{Re}(\hat{\sigma}_{YU}^2(k)A(\Omega_k, \theta)\bar{B}(\Omega_k, \theta)).
\end{aligned} \qquad (11)$$

Note that $\hat{\sigma}_e^2(\Omega_k, \theta) = \mathrm{var}(\hat{e}(\Omega_k, \theta, N_Z^{[l]}(k)))$ stands for the variance of the equation error of one experiment, while $\hat{\sigma}_{\hat{e}}^2(\Omega_k, \theta) = \mathrm{var}(\hat{e}(\Omega_k, \theta, \hat{N}_Z(k)))$ is the variance of the sample mean of the equation error.

The most important concern, when replacing the exact noise (co-)variances by their sample values, is the loss in quality of the new estimator $\hat{\theta}_{\mathrm{SML}}(Z)$ (minimizer of (10)) with respect to the original estimate $\hat{\theta}_{\mathrm{ML}}(Z)$ (minimizer of (2)) due to this change. Taking the expected value of (10) taking into account that the sample mean and sample variance of Gaussian random variables are independently distributed, gives

$$E\{V_{\mathrm{SML}}(\theta, Z)\} = \frac{M-1}{M-2} E\{V_{\mathrm{ML}}(\theta, Z)\}\,. \qquad (12)$$

(Schoukens *et al.*, 1997 and Pintelon and Schoukens, 2001). Applying quick analysis by tools 2 and 3 of Section 2.3 of *Estimation with known Noise Model* shows that (i) in the absence of model errors $\hat{\theta}_{\mathrm{SML}}(Z)$ converges (in stochastic sense for $F \to \infty$) to the true value θ_0, and (ii) in the presence of model errors (unmodeled dynamics or nonlinear distortions) $\hat{\theta}_{\mathrm{SML}}(Z)$ converges (in stochastic sense for $F \to \infty$) to $\tilde{\theta}_{\mathrm{SML}}(Z_0) = \tilde{\theta}_{\mathrm{ML}}(Z_0)$, the minimizer of the expected value of the cost function. It also follows that the minimizers of the limit cost functions ($F \to \infty$ in (2) and (10)) are equal $\theta_{*\mathrm{SML}} = \theta_{*\mathrm{ML}}$. If no model errors are present it can also be shown that the covariance matrix of $\hat{\theta}_{\mathrm{SML}}(Z)$ is a factor $(M-2)/(M-3)$ larger than the covariance matrix of $\hat{\theta}_{\mathrm{ML}}(Z)$

$$\mathrm{Cov}(\hat{\theta}_{\mathrm{SML}}(Z)) = \frac{M-2}{M-3} \mathrm{Cov}(\hat{\theta}_{\mathrm{ML}}(Z)) \qquad (13)$$

(Schoukens *et al.*, 1997 and Pintelon and Schoukens, 2001). For example, for $M = 4, 5, 6$ and 10 the standard deviation increases by 41%, 22%, 15% and 7%, respectively. We conclude that $\hat{\theta}_{\mathrm{SML}}(Z)$ has the same asymptotic properties as $\hat{\theta}_{\mathrm{ML}}(Z)$ described in Sections 2.4 and 3.4 of *Estimation with known Noise Model*. Each property requires, however, a minimal number of independent repeated experiments: $M = 4$ for the convergence, $M = 6$ for the convergence rate and $M = 7$ for the asymptotic normality (Pintelon and Schoukens, 2001).

Equation (13) quantifies the loss in efficiency due to the use of the sample variances. However, it does not give an answer how to calculate $\mathrm{Cov}(\hat{\theta}_{\mathrm{SML}}(Z))$ from the available information. $\mathrm{Cov}(\hat{\theta}_{\mathrm{ML}}(Z))$ is approximated by

$$\mathrm{Cov}(\hat{\theta}_{\mathrm{ML}}(Z)) \approx \left[2\,\mathrm{Re}\left(\left(\frac{\partial \varepsilon(\theta, Z)}{\partial \hat{\theta}_{\mathrm{ML}}(Z)} \right)^H \left(\frac{\partial \varepsilon(\theta, Z)}{\partial \hat{\theta}_{\mathrm{ML}}(Z)} \right) \right) \right]^{-1},$$

(see Section 3.4 of *Estimation with known Noise Model*) and in practice, during the calculations of the covariance matrix, the exact variances in $\varepsilon(\theta, Z)$ are again replaced by the sample variances, and only $\hat{\varepsilon}(\theta, Z)$ is available. Using similar calculations as in (12), it turns out that

$$\begin{aligned} & 2\,\mathrm{Re}\left(\left(\frac{\partial \hat{\varepsilon}(\theta, Z)}{\partial \hat{\theta}_{\mathrm{SML}}(Z)} \right)^H \left(\frac{\partial \hat{\varepsilon}(\theta, Z)}{\partial \hat{\theta}_{\mathrm{SML}}(Z)} \right) \right) \approx \\ & \frac{M-1}{M-2}\, 2\,\mathrm{Re}\left(\left(\frac{\partial \varepsilon(\theta, Z)}{\partial \hat{\theta}_{\mathrm{ML}}(Z)} \right)^H \left(\frac{\partial \varepsilon(\theta, Z)}{\partial \hat{\theta}_{\mathrm{ML}}(Z)} \right) \right) \end{aligned} \tag{14}$$

so that (13) is replaced by

$$\mathrm{Cov}(\hat{\theta}_{\mathrm{SML}}(Z)) = \left[2\frac{M-3}{M-1}\mathrm{Re}\left(\left(\frac{\partial \hat{\varepsilon}(\theta, Z)}{\partial \hat{\theta}_{\mathrm{SML}}(Z)} \right)^H \left(\frac{\partial \hat{\varepsilon}(\theta, Z)}{\partial \hat{\theta}_{\mathrm{SML}}(Z)} \right) \right) \right]^{-1}. \tag{15}$$

Note that the expression between brackets in (15) equals, within a factor of $2(M-3)/(M-1)$, the matrix of the normal equation in the last Newton-Gauss step of the minimization of (10) (see Eq. (16) of *Estimation with known Noise Model*).

2.2. Generalized Total Least Squares

The general form of the cost function of the GTLS estimator is given by Eq. (38) of *Estimation with known Noise Model*:

$$V_{\mathrm{GTLS}}(\theta, Z) = \frac{\sum_{k=1}^{F} \left| e(\Omega_k, \theta, Z(k)) \right|^2}{\sum_{k=1}^{F} \sigma_e^2(\Omega_k, \theta)}. \tag{16}$$

Replacing in this expression $Z(k)$ by the sample mean $\hat{Z}(k)$ and the exact noise (co-)variances by the sample noise (co-)variances gives the sample GTLS (SGTLS) cost function

$$V_{\mathrm{SGTLS}}(\theta, Z) = \frac{\sum_{k=1}^{F} \left| \hat{e}(\Omega_k, \theta, \hat{Z}(k)) \right|^2}{\sum_{k=1}^{F} \hat{\sigma}_{\hat{e}}^2(\Omega_k, \theta)}, \tag{17}$$

where $\hat{e}(\Omega_k, \theta, \hat{Z}(k))$ and $\hat{\sigma}_{\hat{e}}^2(\Omega_k, \theta)$ are defined in (11). The minimizer $\hat{\theta}_{\mathrm{SGTLS}}(Z)$ of (17) is not calculated using the iterative Newton-Gauss scheme (see Eq. (15) or (17) of *Estimation with known Noise Model*), but via the generalized singular value decomposition of the matrix pair $(\hat{J}_{re}(Z), \hat{C})$ with $\hat{J}(Z) = \partial\hat{e}(\theta, \hat{Z}) / \partial\theta$ and $\hat{C}$ a square root of the column covariance matrix of $\hat{J}_{re}(N_Z)$, calculated using the sample noise (co-)variances (see Section 3.3 of *Estimation with known Noise Model*). Just like the GTLS estimate, $\hat{\theta}_{\mathrm{SGTLS}}(Z)$ suffers from the amplification of the high frequency errors (see Section 3.3.3 of *Estimation with known Noise Model*). To cope with this problem weighted SGTLS versions can be constructed as in Sections 3.3.3 and 3.5.3 of *Estimation with known Noise Model*.

The properties of $\hat{\theta}_{\mathrm{SGTLS}}(Z)$ can be analyzed by using the quick analysis tools of *Estimation with known Noise Model* where $V_F(\theta, Z) = V_{\mathrm{SGTLS}}(\theta, Z)$, and $f_F(\theta, \eta(Z), Z) = V_{\mathrm{SGTLS}}(\theta, Z)$ with $\eta(Z) = F^{-1} \sum_{k=1}^{F} \hat{\sigma}_{\hat{e}}^2(\Omega_k, \theta)$. Since $\eta(Z)$ converges for $M \geq 2$ to

$$\eta_* = E\{F^{-1} \sum_{k=1}^{F} \hat{\sigma}_{\hat{e}}^2(\Omega_k, \theta)\} = \sum_{k=1}^{F} \sigma_e^2(\Omega_k, \theta) / (FM) \tag{18}$$

(strong law of large numbers), it can easily be found that

$$V_F(\theta) = E\{f_F(\theta, \eta_*, Z)\} = \frac{\sum_{k=1}^{F} \left| e(\Omega_k, \theta, Z_0(k)) \right|^2}{\frac{1}{M} \sum_{k=1}^{F} \sigma_e^2(\Omega_k, \theta)} + 1. \tag{19}$$

Hence, in the absence of model errors $\hat{\theta}_{\mathrm{SGTLS}}(Z)$ converges to the true value θ_0 (quick tool 2 of *Estimation with known Noise Model*), and in case of model errors $\hat{\theta}_{\mathrm{SGTLS}}(Z)$ converges to $\tilde{\theta}_{\mathrm{SGTLS}}(Z_0) = \tilde{\theta}_{\mathrm{GTLS}}(Z_0)$ (quick tool 3 of *Estimation with known Noise Model*). It also follows that the minimizers of the limit cost functions ($F \to \infty$ in (16)and (17)) are equal $\theta_{*\mathrm{SGTLS}} = \theta_{*\mathrm{GTLS}}$. If no model errors are present it can also be shown that

$$\mathrm{Cov}(\hat{\theta}_{\mathrm{SGTLS}}(Z)) = \mathrm{Cov}(\hat{\theta}_{\mathrm{GTLS}}(Z)) \tag{20}$$

(Pintelon and Schoukens, 2001). This is no longer true if model errors are present. The basic reason for the similar asymptotic behavior of $\hat{\theta}_{\mathrm{SGTLS}}(Z)$ and $\hat{\theta}_{\mathrm{GTLS}}(Z)$ is that the "poor quality" sample (co-)variances are averaged over the frequency in the cost function (17), resulting in a "high quality" estimate of the denominator of the cost function. We conclude that $\hat{\theta}_{\mathrm{SGTLS}}(Z)$, with $M \geq 2$, has exactly the same asymptotic properties as $\hat{\theta}_{\mathrm{GTLS}}(Z)$ described in Sections 2.4 and 3.3.3 of *Estimation with known Noise Model*.

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Biographical Sketches

Rik Pintelon was born in Ghent, Belgium, on December 4, 1959. He received the degree of electrical engineer (burgerlijk ingenieur) in July 1982, the degree of doctor in applied sciences in January 1988, and the qualification to teach at university level (geaggregeerde voor het hoger onderwijs) in April 1994, all from the Vrije Universiteit Brussel (VUB), Brussels, Belgium. From October 1982 till September 2000 he was a researcher of the Fund for Scientific Research – Flanders at the VUB. Since October 2000 he is professor at the VUB in the Electrical Measurement Department (ELEC). His main research interests are in the field of parameter estimation / system identification, and signal processing.

Johan Schoukens was born in Belgium in 1957. He received the degree of engineer in 1980 and the degree of doctor in applied sciences in 1985, both from the Vrije Universiteit Brussel. The prime factors of his interest are in the filed of system identification for linear and nonlinear systems and growing tomatoes in his green house.