MODAL ANALYSIS

Patrick Guillaume
Department of Mechanical Engineering, Acoustics & Vibration Research Group, Vrije Universiteit Brussel, Belgium.

Keywords: Vibration, Estimation, Frequency domain, Modal analysis, Modal parameters, Natural frequency, Damping, Mode shapes, Transfer function, SISO, MIMO, Mechanical systems, SDOF, MDOF.

Contents

Summary

In this chapter the applicability of frequency-domain estimators in the field of modal analysis will be illustrated. The basics of vibration and modal analysis are briefly summarized. In modal analysis, mechanical systems with a few inputs and hundreds of outputs have to be identified. This requires dedicated frequency-domain estimators designed to handle large amount of data in a reasonable amount of time.

1. Introduction

It is well known that (mechanical) structures can resonate, i.e. that small forces can result in significant deformation, and possibly, damage can be induced in the structure.

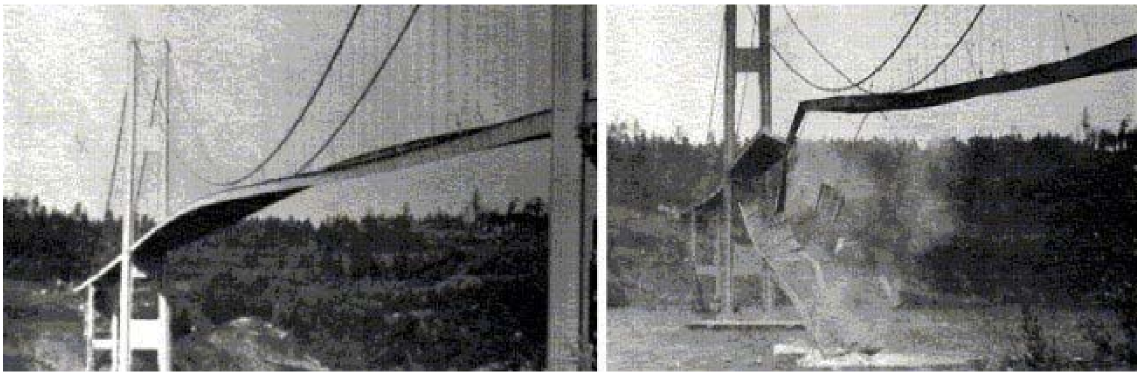

Figure 1: Tacoma Narrows Bridge Disaster.

The Tacoma Narrows bridge disaster (Figure 1) is a typical example of this. On November 7, 1940, the Tacoma Narrows suspension bridge collapsed due to wind-induced vibration (i.e. flutter). Situated on the Tacoma Narrows in Puget Sound, near the city of Tacoma, Washington, the bridge had only been open for traffic a few months.

Wings of airplanes can be subjected to similar flutter phenomena during flight. Before an airplane is released, flight flutter tests have to be performed to detect possible onset of flutter. The classical flight flutter testing approach is to expand the flight envelope of a airplane by performing a vibration test at constant flight conditions, curve-fit the data to estimate the resonance frequencies and damping ratios, and then to plot these frequencies and damping estimates against flight speed or Mach number. The damping values are then extrapolated in order to determine whether it is safe to proceed to the next flight test point. Flutter will occur when one of the damping values tends to become negative. Before starting the flight tests, ground vibration tests as well as numerical simulations and wind tunnel tests (see Figure 2) are used to get some prior insight into the problem.

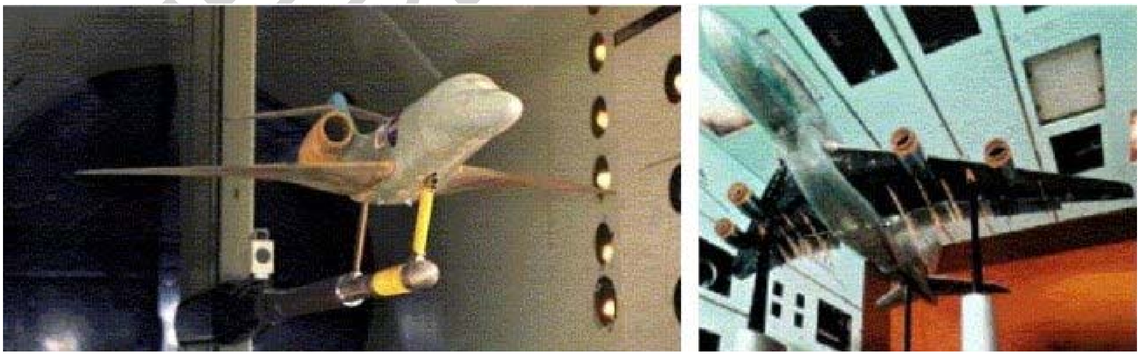

Figure 2: Wind tunnel tests on a scaled model of (a) a Cessna and (b) an Airbus A380.

The majority of structures can be made to resonate, i.e. to vibrate with excessive oscillatory motion. Resonant vibration is mainly caused by an interaction between the inertial and elastic properties of the materials within a structure. Resonance is often the cause of, or at least a contributing factor to many of the vibration and noise related problems that occur in structures and operating machinery. To better understand any structural vibration problem, the resonant frequencies of a structure need to be identified and quantified. Today, modal analysis has become a widespread means of finding the modes of vibration of a machine or structure (Figure 3). In every development of a new or improved mechanical product, structural dynamics testing on product prototypes is used to assess its real dynamic behavior.

2. The "Modal" Model

Modes are inherent properties of an elastic structure, and are determined by the material properties (mass, damping, and stiffness), and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape (i.e. the so-called "modal parameters"). If either the material properties or the boundary conditions of a structure change, its modes will change. For instance, if mass is added to a structure, it will vibrate differently. To understand this, we will make use of the concept of single and multiple-degree-of-freedom systems.

Figure 3: Modal analysis of a car body.

2.1. Single Degree of Freedom

A single-degree-of-freedom (SDOF) system (see Figure 4where the mass *m* can only move along the vertical x-axis) is described in the time domain by the following equation

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \qquad (1)$$

with *m* the mass, *c* the damping coefficient, and *k* the stiffness. This equation states that the sum of all forces acting on the mass m should be equal to zero with $f(t)$ an externally applied force, $-m\ddot{x}(t)$ the inertial force, $-c\dot{x}(t)$ the (viscous) damping (internal) force, and $-kx(t)$ the restoring force. The variable $x(t)$ stands for the position of the mass m with respect to its equilibrium point, i.e. the position of the mass when $f(t) \equiv 0$. Transforming (1) to the Laplace domain (assuming zero initial conditions) yields

$$Z(s)X(s) = F(s) \qquad (2)$$

with $Z(s)$ the dynamic stiffness

$$Z(s) = ms^2 + cs + k\,. \qquad (3)$$

The transfer function $H(s)$ between displacement and force, $X(s) = H(s)F(s)$, equals the inverse of the dynamic stiffness

$$H(s) = \frac{1}{ms^2 + cs + k}\,. \qquad (4)$$

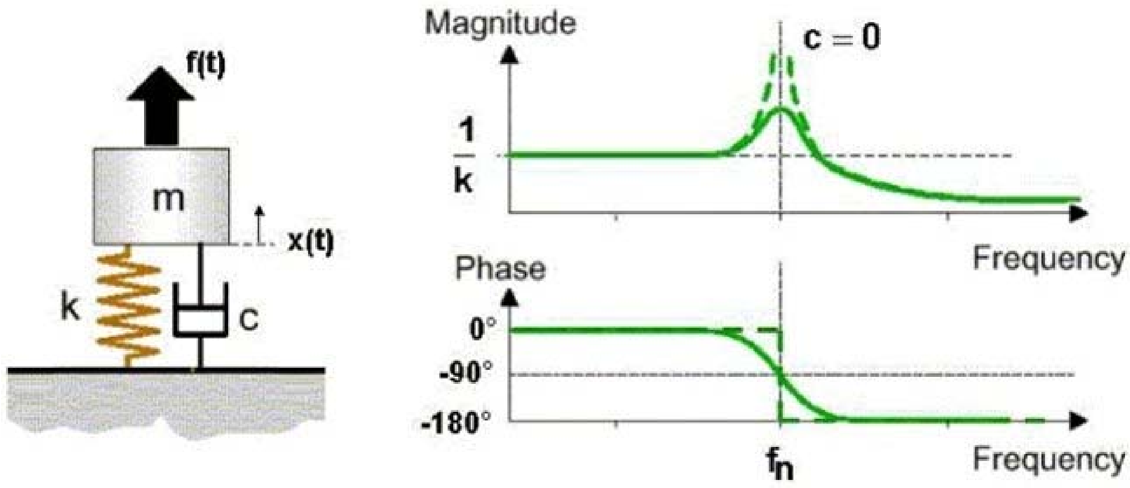

Figure 4: SDOF system.

The roots of the denominator of the transfer function, i.e. $d(s) = ms^2 + cs + k$, are the poles of the system. In mechanical structures, the damping coefficient c is usually very small ($\zeta \ll 1$) resulting in a complex conjugate pole pair

$$\lambda = -\sigma \pm \mathrm{i}\omega_d \qquad (5)$$

with $f_d = \omega_d/2\pi$ the damped natural frequency,

$f_n = \omega_n/2\pi$ the (undamped) natural frequency where $\omega_n = \sqrt{k/m} = |\lambda|$, and

$\zeta = c/2m\omega_n = \sigma/|\lambda|$ the damping ratio ($f_d = f_n\sqrt{1-\zeta^2}$).

If $c = 0$, the system is not damped and the poles become purely imaginary, $\lambda = \pm \mathrm{i}\omega_n$. If, for instance, a mass Δm is added to the original mass *m* of the structure, its natural frequency decreases to $\omega_n' = \sqrt{k/(m+\Delta m)}$.

The Frequency Response Function (FRF), denoted by $H(\omega)$, is obtained by replacing the Laplace variable *s* in (4) by $\mathrm{i}\omega$ resulting in

$$H(\omega) = \frac{1}{-m\omega^2 + \mathrm{i}c\omega + k} = \frac{1}{(k - m\omega^2) + \mathrm{i}c\omega} \qquad (6)$$

Clearly, if $c = 0$, then $H(\omega)$ goes to infinity for $\omega \to \omega_n = \sqrt{k/m}$ (see Figure 4).

Although very few practical structures could realistically be modeled by a single-degree-of-freedom (SDOF) system, the properties of such a system are important because those of a more complex multiple-degree-of-freedom (MDOF) system can always be represented as the linear superposition of a number of SDOF characteristics (when the system is linear time-invariant).

-
-
-

Bibliography

Ewins D.J. (2000). *Modal Testing: Theory, Practice and Application*, Hertfordshire: Research Studies Press. [In this book, all the steps involved in planning, executing, interpreting and applying the results from a modal test are described in straightforward terms.]

Ewins D.J. and Inman D.J., Editors (2001). *Structural Dynamics @ 2000: Current Status and Future Directions*, Baldock: Research Studies Press. [This book presents an integrated collection of contributions on structural dynamics.]

Heylen W., Lammens S. and Sas P. (1998). *Modal Analysis Theory and Testing*, KULeuven (ISBN 90-73802-61-X). [This book gives a good introduction to the theory as well as all practical aspects of experimental modal analysis.]

Inman D.J. (1994). *Engineering Vibration*, Englewood Cliffs: Prentice Hall. [This book gives an introduction to mechanical systems and vibration.]

Maia N.M.M. and Silva J.M.M., Editors (1997). *Theoretical and Experimental Modal Analysis*, Taunton: Research Studies Press. [This book takes an advanced and up-to-date look at modal analysis. The basics of vibration theory and signal processing are discussed.]

Pintelon R. and Schoukens J. (2001). *System Identification: A Frequency Domain Approach*, IEEE Press and John Wiley & Sons (ISBN 0-7803-6000-1). [This book presents a general approach to system identification, with both practical examples and theoretical discussions.]

Biographical Sketch

Patrick Guillaume was born in Anderlecht, Belgium, in 1963. He received the master degree in electro-mechanical engineering in 1987 from the Vrije Universiteit Brussel (VUB), Belgium. In 1987 he joined the Department of Electrical Engineering (ELEC) of the Vrije Universiteit Brussel where he received the Ph.D. degree in 1992. In 1996 he joined the Department of Mechanical Engineering (WERK) of the same university. He is currently associate professor and coordinator of the Acoustics & Vibration Research Group. His main research interests are situated in the field of system identification, signal processing and experimental (vibro-acoustic) modal analysis.