

## SUBSPACE IDENTIFICATION METHODS

**Katrien De Cock and Bart De Moor**

*K.U. Leuven, Department of Electrical Engineering (ESAT-SCD), Belgium*

**Keywords:** Systems, Discrete time systems, System identification, Subspace identification, Linear systems, Kalman filter, Numerical linear algebra, Orthogonal projection, Oblique projection, State space models, Time invariant systems, Multivariable models, Deterministic models, Stochastic models, Row space, State sequence, Singular value decomposition, QR factorization, LQ factorization, Least squares, Block Hankel matrix, Principal angles, Software, Positive realness, Stability, Bilinear systems, N4SID, MOESP, CVA

### Contents

1. Introduction
    - 1.1. State Space Models
    - 1.2. The Basic Idea behind Subspace Identification Algorithms
  2. Notation
    - 2.1. Block Hankel Matrices and State Sequences
    - 2.2. Model Matrices
  3. Geometric Tools
    - 3.1. Orthogonal Projections
    - 3.2. Oblique Projections
  4. Deterministic subspace identification
    - 4.1. Calculation of a State Sequence
    - 4.2. Computing the System Matrices
  5. Stochastic subspace identification
    - 5.1. Calculation of a State Sequence
    - 5.2. Computing the System Matrices
  6. Combined deterministic-stochastic subspace identification algorithm
    - 6.1. Calculation of a State Sequence
    - 6.2. Computing the System Matrices
    - 6.3. Variants
  7. Comments and perspectives
  8. Software
- Acknowledgements  
Glossary  
Bibliography  
Biographical Sketches

### Summary

This chapter gives a short introduction to and survey of subspace identification algorithms. Deterministic, stochastic and combined deterministic-stochastic subspace identification algorithms are treated. These methods estimate state sequences directly from the given data, either explicitly or implicitly, through an orthogonal or oblique projection of the row spaces of certain block Hankel matrices of data into the row

spaces of other block Hankel matrices, followed by a singular value decomposition (SVD) to determine the order, the observability matrix and /or the state sequence. The extraction of the state space model is then achieved through the solution of a least squares problem. Each of these steps can be elegantly implemented using well-known numerical linear algebra algorithms such as the singular value decomposition and the QR decomposition.

## 1. Introduction

This Section contains a description of the central ideas of this chapter. First, in Section 1.1, we describe state space models, which is the type of models that is delivered by subspace identification algorithms. In Section 1.2 we explain how subspace identification algorithms work.

### 1.1. State Space Models

Models in this chapter are lumped, discrete time, linear, time-invariant, state space models. From the number of epithets used, this might seem like a highly restricted class of models (especially the fact that they are linear), but, surprisingly enough, many industrial processes can be described very accurately by this type of models, especially locally in the neighborhood of a working point. Moreover, there is a large number of control system design tools available to build controllers for such systems and models.

Mathematically, these models are described by the following set of difference equations:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k, \\ y_k = Cx_k + Du_k + v_k, \end{cases} \quad (1)$$

with

$$\mathbf{E}\left[\begin{pmatrix} w_p \\ v_p \end{pmatrix} \begin{pmatrix} w_q^T & v_q^T \end{pmatrix}\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{pq} \geq 0. \quad (2)$$

$\mathbf{E}$  denotes the expected value operator and  $\delta_{pq}$  the Kronecker delta.

In this model, we have

- **vectors:** The vectors  $u_k \in \mathbf{R}^m$  and  $y_k \in \mathbf{R}^l$  are the observations at time instant  $k$  of respectively the  $m$  inputs and  $l$  outputs of the process. The vector  $x_k \in \mathbf{R}^n$  is the state vector of the process at discrete time instant  $k$  and contains the numerical values of  $n$  states.  $v_k \in \mathbf{R}^l$  and  $w_k \in \mathbf{R}^n$  are unobserved vector signals, usually called the measurement, respectively process noise. It is assumed that they are zero mean, stationary, white noise vector sequences. (The Kronecker delta in (2) means  $\delta_{pq} = 0$  if  $p \neq q$ , and

$\delta_{pq} = 1$  if  $p = q$ .) The effect of the process  $w_k$  is different from that of  $v_k$ :  $w_k$  as an input will have a dynamic effect on the state  $x_k$  and output  $y_k$ , while  $v_k$  only affects the output  $y_k$  directly and therefore is called a measurement noise.

- matrices:**  $A \in \mathbf{R}^{n \times n}$  is called the (dynamical) system matrix. It describes the dynamics of the system (as characterized by its eigenvalues).  $B \in \mathbf{R}^{n \times m}$  is the input matrix, which represents the linear transformation by which the deterministic inputs influence the next state.  $C \in \mathbf{R}^{l \times n}$  is the output matrix, which describes how the internal state is transferred to the outside world in the observations  $y_k$ . The term with the matrix  $D \in \mathbf{R}^{l \times m}$  is called the direct feedthrough term. The matrices  $Q \in \mathbf{R}^{n \times n}$ ,  $S \in \mathbf{R}^{n \times l}$  and  $R \in \mathbf{R}^{l \times l}$  are the covariance matrices of the noise sequences  $w_k$  and  $v_k$ . The block matrix in (2) is assumed to be positive definite, as is indicated by the inequality sign. The matrix pair  $\{A, C\}$  is assumed to be observable, which implies that all *modes* in the system can be observed in the output  $y_k$  and can thus be identified. The matrix pair  $\{A, [B \ Q^{1/2}]\}$  is assumed to be controllable, which in its turn implies that all *modes* of the system can be excited by either the deterministic input  $u_k$  and/or the stochastic input  $w_k$ .

A graphical representation of the system can be found in Figure 1.

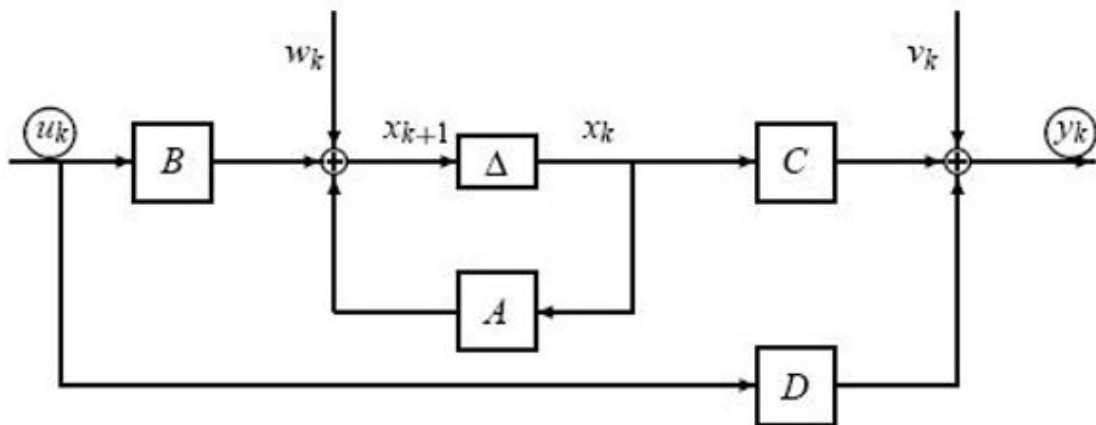


Figure 1: The (circled) vector signals  $u_k$  and  $y_k$  are available (observed) while  $v_k$ ,  $w_k$  are unknown disturbances. The symbol  $\Delta$  represents a delay. Note the inherent feedback via the matrix  $A$  (which represents the dynamics). Sensor or actual dynamics are completely contained in  $A$  too. It is assumed that  $u_k$  is available without measurement noise.

We are now ready to state the main mathematical problem of this chapter.

Given  $s$  consecutive input and output observations  $u_0, \dots, u_{s-1}$ , and  $y_0, \dots, y_{s-1}$ .  
Find an appropriate order  $n$  and the system matrices  $A, B, C, D, Q, R, S$ .

## 1.2. The Basic Idea behind Subspace Identification Algorithms

*The goal of this Section is to provide a verbal description of the main principles on which subspace identification algorithms are based. The mathematical derivations will be elaborated on in the next sections.*

Subspace identification algorithms are based on concepts from system theory, (numerical) linear algebra and statistics. The main concepts in subspace identification algorithms are

1. The *state sequence of the dynamical system* is determined first, directly from input/output observations, without knowing the model. That this is possible for the model class (1) is one of the main contributions of subspace algorithms, as compared to “classical” approaches that are based on an input-output framework. The difference is illustrated in Figure 2. So an important achievement of the research in subspace identification was to demonstrate how the Kalman filter states can be obtained directly from input-output data using linear algebra tools (QR and singular value decomposition) without knowing the mathematical model. An important consequence is that, once these states are known, the identification problem becomes a linear least squares problem in the unknown system matrices, and the process and measurement noise covariance matrices follow from the least squares residuals, as is easy to see from Eq. (1):

$$\underbrace{\begin{pmatrix} x_{i+1} & x_{i+2} & \cdots & x_{i+j} \\ y_i & y_{i+1} & \cdots & y_{i+j-1} \end{pmatrix}}_{\text{known}} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \underbrace{\begin{pmatrix} x_i & x_{i+1} & \cdots & x_{i+j-1} \\ u_i & u_{i+1} & \cdots & u_{i+j-1} \end{pmatrix}}_{\text{known}} + \begin{pmatrix} w_i & w_{i+1} & \cdots & w_{i+j-1} \\ v_i & v_{i+1} & \cdots & v_{i+j-1} \end{pmatrix}. \quad (3)$$

The meaning of parameters  $i$  and  $j$  will become clear henceforth.

Even though the state sequence can be determined explicitly, in most variants and implementations, this is not done explicitly but rather implicitly. Said in other words, the set of linear equations above can be solved ‘implicitly’ as will become clear below, without an explicit calculation of the state sequence itself. Of course, when needed, the state sequence can be computed explicitly.

The two main steps that are taken in subspace algorithms are the following.

- (a) Determine the model order  $n$  and a state sequence  $\hat{x}_i, \hat{x}_{i+1}, \dots, \hat{x}_{i+j}$  (estimates are denoted by  $\hat{\cdot}$ ). They are typically found by first projecting row spaces of

data block Hankel matrices and then applying singular value decomposition (see Sections 4, 5, 6).

(b) Solve a least squares problem to obtain the state space matrices:

$$\begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} = \min_{A,B,C,D} \left\| \begin{pmatrix} \hat{x}_{i+1} & \hat{x}_{i+2} & \cdots & \hat{x}_{i+j} \\ y_i & y_{i+1} & \cdots & y_{i+j-1} \end{pmatrix} - \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \hat{x}_i & \hat{x}_{i+1} & \cdots & \hat{x}_{i+j-1} \\ u_i & u_{i+1} & \cdots & u_{i+j-1} \end{pmatrix} \right\|_F^2, \quad (4)$$

where  $\|\cdot\|_F$  denotes the Frobenius-norm of a matrix. The estimates of the noise covariance matrices follow from

$$\begin{pmatrix} \hat{Q} & \hat{S} \\ \hat{S}^T & \hat{R} \end{pmatrix} = \frac{1}{J} \begin{pmatrix} \rho_{w_i} & \rho_{w_{i+1}} & \cdots & \rho_{w_{i+j-1}} \\ \rho_{v_i} & \rho_{v_{i+1}} & \cdots & \rho_{v_{i+j-1}} \end{pmatrix} \begin{pmatrix} \rho_{w_i} & \rho_{w_{i+1}} & \cdots & \rho_{w_{i+j-1}} \\ \rho_{v_i} & \rho_{v_{i+1}} & \cdots & \rho_{v_{i+j-1}} \end{pmatrix}^T, \quad (5)$$

where  $\rho_{w_k} = \hat{x}_{k+1} - \hat{A}\hat{x}_k - \hat{B}u_k$  and  $\rho_{v_k} = y_k - \hat{C}\hat{x}_k - \hat{D}u_k$  ( $k = i, \dots, i + j - 1$ ) are the least squares residuals.

2. Subspace system identification algorithms make full use of the well developed body of *concepts and algorithms from numerical linear algebra*. Numerical robustness is guaranteed because of

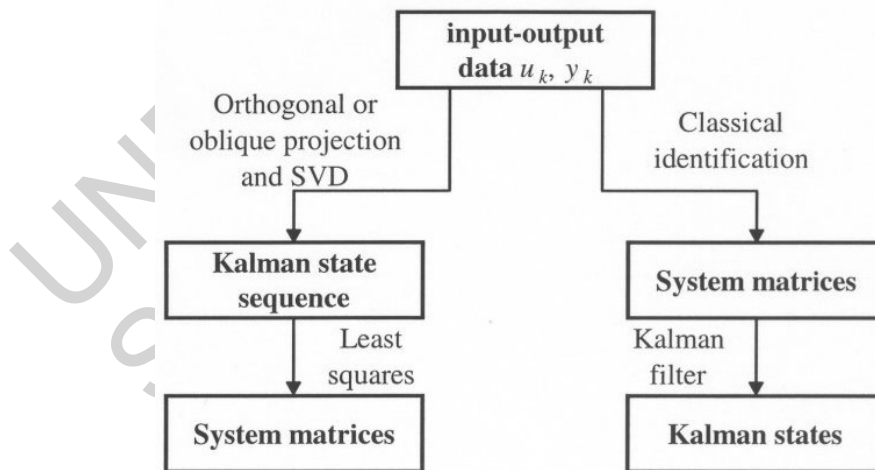


Figure 2: Subspace identification aims at constructing state space models from input-output data. The left hand side shows the subspace identification approach: first the (Kalman filter) states are estimated directly (either implicitly or explicitly) from input-output data, then the system matrices can be obtained. The right hand side is the classical approach: first obtain the system matrices, then estimate the states.

the well-understood algorithms, such as the QR-decomposition, the singular

value decomposition and its generalizations. Therefore, they are very well suited for large data sets ( $s \rightarrow \infty$ ) and large scale systems ( $m, l, n$  large). Moreover, subspace algorithms are not iterative. Hence, there are no *convergence* problems. When carefully implemented, they are computationally very efficient, especially for large datasets (implementation details are however not contained in this survey).

3. The conceptual straightforwardness of subspace identification algorithms translates into *user-friendly software implementations*. To give only one example: since there is no explicit need for parameterizations in the geometric framework of subspace identification, the user is not confronted with highly technical and theoretical issues such as canonical parameterizations. The number of user choices is greatly reduced when using subspace algorithms because we use full state space models and the only parameter to be specified by the user, is the order of the system, which can be determined by inspection of certain singular values.

## 2. Notation

In this section, we set some notation. In Section 2.1, we introduce the notation for the data block Hankel matrices and in Section 2.2 for the system related matrices.

### 2.1. Block Hankel Matrices and State Sequences

Block Hankel matrices with output and/or input data play an important role in subspace identification algorithms. These matrices can be easily constructed from the given input-output data. Input block Hankel matrices are defined as

$$U_{0|2i-1} \stackrel{\text{def}}{=} \begin{pmatrix} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{i-1} & u_i & u_{i+1} & \cdots & u_{i+j-2} \\ \hline u_i & u_{i+1} & u_{i+2} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & u_{i+3} & \cdots & u_{i+j} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{2i-1} & u_{2i} & u_{2i+1} & \cdots & u_{2i+j-2} \end{pmatrix} = \begin{pmatrix} U_{0|i-1} \\ U_{i|2i-1} \end{pmatrix} = \begin{pmatrix} U_p \\ U_f \end{pmatrix} \quad (6)$$

$$= \begin{pmatrix} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{i-1} & u_i & u_{i+1} & \cdots & u_{i+j-2} \\ u_i & u_{i+1} & u_{i+2} & \cdots & u_{i+j-1} \\ \hline u_{i+1} & u_{i+2} & u_{i+3} & \cdots & u_{i+j} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{2i-1} & u_{2i} & u_{2i+1} & \cdots & u_{2i+j-2} \end{pmatrix} = \begin{pmatrix} U_{0|i} \\ U_{i+1|2i-1} \end{pmatrix} = \begin{pmatrix} U_p^+ \\ U_f \end{pmatrix} \quad (7)$$

where:

- The number of block rows ( $i$ ) is a user-defined index which is large enough, i.e. it should at least be larger than the maximum order of the system one wants to identify. Note that, since each block row contains  $m$  (number of inputs) rows, the matrix  $U_{0|2i-1}$  consists of  $2mi$  rows.
- The number of columns ( $j$ ) is typically equal to  $s - 2i + 1$ , which implies that all  $s$  available data samples are used. In any case,  $j$  should be larger than  $2i - 1$ . Throughout this chapter, for statistical reasons, we will often assume that  $j, s \rightarrow \infty$ . For deterministic (noiseless) models, i.e. where  $v_k \equiv 0$  and  $w_k \equiv 0$ , this will however not be needed.
- The subscripts of  $U_{0|2i-1}, U_{0|i-1}, U_{0|i}, U_{i|2i-1}, etc...$  denote the subscript of the first and last element of the first column in the block Hankel matrix. The subscript “ $p$ ” stands for “past” and the subscript “ $f$ ” for “future”. The matrices  $U_p$  (the past inputs) and  $U_f$  (the future inputs) are defined by splitting  $U_{0|2i-1}$  in two equal parts of  $i$  block rows. The matrices  $U_p^+$  and  $U_f^-$  on the other hand are defined by shifting the border between past and future one block row down. The superscript “+” stands for “add one block row” while the superscript “-” stands for “delete one block row”.

They are defined as  $U_p^+ = U_{0|i}$  and  $U_f^- = U_{i+1|2i-1}$ .

The output block Hankel matrices  $Y_{0|2i-1}, Y_p, Y_f, Y_p^+, Y_f^-$  are defined in a similar way.

State sequences play an important role in the derivation and interpretation of subspace identification algorithms. The state sequence  $X_i$  is defined as:

$$X_i \stackrel{\text{def}}{=} (x_i \quad x_{i+1} \quad \dots \quad x_{i+j-2} \quad x_{i+j-1}) \in \mathbf{R}^{n \times j}, \quad (8)$$

where the subscript  $i$  denotes the subscript of the first element of the state sequence.

## 2.2. Model Matrices

Subspace identification algorithms make extensive use of the observability and of its structure. The extended ( $i > n$ ) observability matrix  $\Gamma_i$  (where the subscript  $i$  denotes the number of block rows) is defined as:

$$\Gamma_i \stackrel{\text{def}}{=} \begin{pmatrix} C \\ CA \\ CA^2 \\ \dots \\ CA^{i-1} \end{pmatrix} \in \mathbf{R}^{li \times n}. \quad (9)$$

We assume the pair  $\{A, C\}$  to be observable, which implies that the rank of  $\Gamma_i$  is equal to  $n$ .

## 3. Geometric Tools

In Section 3.1 through 3.2 we introduce the main geometric tools used to reveal some system characteristics. They are described from a linear algebra point of view, independently of the subspace identification framework we will be using in the next sections.

In the following sections we assume that the matrices  $A \in \mathbf{R}^{p \times j}$ ,  $B \in \mathbf{R}^{q \times j}$  and  $C \in \mathbf{R}^{r \times j}$  are given (they are dummy matrices in this section). We also assume that  $j \geq \max(p, q, r)$ , which will always be the case in the identification algorithms.

### 3.1. Orthogonal Projections

The orthogonal projection of the row space of  $A$  into the row space of  $B$  is denoted by  $A/B$  and its matrix representation is

$$A/B \stackrel{\text{def}}{=} AB^T (BB^T)^\dagger B, \quad (10)$$

where  $(\bullet)^\dagger$  denotes the Moore-Penrose Pseudo-inverse of the matrix  $(\bullet)$ .  $A/B^\perp$  is the projection of the row space of  $A$  into  $B^\perp$ , the orthogonal complement of the row space of  $B$ , for which we have  $A/B^\perp = A - A/B = A(I_j - B(BB^T)^\dagger B)$ . The projections  $\Pi_B$  and  $\Pi_{B^\perp}$  decompose a matrix  $A$  into two matrices, the row spaces of which are



orthogonal:

$$A = A\Pi_B + A\Pi_{B^\perp}. \quad (11)$$

The matrix representations of these projections can be easily computed via the LQ decomposition of  $\begin{pmatrix} B \\ A \end{pmatrix}$ , which is the numerical matrix version of the Gram-Schmidt orthogonalization procedure.

Let  $A$  and  $B$  be matrices of full row rank and let the LQ decomposition of  $\begin{pmatrix} B \\ A \end{pmatrix}$  be denoted by

$$\begin{pmatrix} B \\ A \end{pmatrix} = LQ^T = \begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} Q_1^T \\ Q_2^T \end{pmatrix}, \quad (12)$$

where  $L \in \mathbf{R}^{(p+q) \times (p+q)}$  is lower triangular, with  $L_{11} \in \mathbf{R}^{q \times q}$ ,  $L_{21} \in \mathbf{R}^{p \times q}$ ,  $L_{22} \in \mathbf{R}^{p \times p}$

and  $Q \in \mathbf{R}^{j \times (p+q)}$  is orthogonal, i.e.  $Q^T Q = \begin{pmatrix} Q_1^T \\ Q_2^T \end{pmatrix} \begin{pmatrix} Q_1 & Q_2 \end{pmatrix} = \begin{pmatrix} I_q & 0 \\ 0 & I_p \end{pmatrix}$ . Then, the

matrix representations of the orthogonal projections can be written as

$$A/B = L_{21}Q_1^T, \quad (13)$$

$$A/B^\perp = L_{22}Q_2^T. \quad (14)$$

TO ACCESS ALL THE 32 PAGES OF THIS CHAPTER,  
[Click here](#)

### Bibliography

Akaike H. (1974). Stochastic theory of minimal realization. *IEEE Transactions on Automatic Control* **19**, 667–674.

Akaike H. (1975). Markovian representation of stochastic processes by canonical variables. *SIAM Journal on Control* **13**(1), 162–173.

Aling H., Milletti U., Kosut R., Mesaros M., Van Overschee P., De Moor B. (1993). An interactive system identification module for Xmath. In *Proceedings of the 1993 American Control Conference (ACC)*

1993), pp. 3071–3075, San Francisco, California, USA.

Amirthalingam R., Lee J.H. (1999). Subspace identification based inferential control applied to a continuous pulp digester. *Journal of Process Control* **9**, 397–406.

Aoki M. (1987). *State Space Modeling of Time Series*. Berlin, Germany: Springer Verlag.

Arun K.S., Kung S.Y. (1990). Balanced approximation of stochastic systems. *SIAM Journal on Matrix Analysis and Applications* **11**, 42–68.

Basseville M., Abdelghani M., Benveniste A. (2000). Subspace-based fault detection algorithms for vibration monitoring. *Automatica* **36**, 101–109.

Bauer D. (1998). Some Asymptotic Theory for the Estimation of Linear Systems Using Maximum Likelihood Methods or Subspace Algorithms. Ph.D. thesis, Institut für Ökonometrie, Operations Research und Systemtheorie, Technische Universität Wien, Vienna, Austria.

Bauer D. (2001). Order estimation for subspace methods. *Automatica* **37**, 1561–1573.

Bauer D., Deistler M., Scherrer W. (1998). Asymptotic distributions of subspace estimates under misspecification. In *Proceedings of the International Symposium on the Mathematical Theory of Networks and Systems (MTNS 1998)*, pp. 699–702, Padova, Italy.

Bauer D., Deistler M., Scherrer W. (1999a). Consistency and asymptotic normality of some subspace algorithms for systems without observed inputs. *Automatica* **35**, 1243–1254.

Bauer D., Ljung L. (2002). Some facts about the choice of the weighting matrices in Larimore type of subspace algorithms. *Automatica* **38**(5).

Bittanti S., Lovera M. (1998). Assessing model uncertainty in subspace identification methods: a simulation study. In *Proceedings of the International Symposium on the Mathematical Theory of Networks and Systems (MTNS 1998)*, pp. 703–706, Padova, Italy.

Byrnes C.I., Lindquist A. (1997). On the partial stochastic realization problem. *IEEE Transactions on Automatic Control* **42**, 1049–1070.

Chen H., Maciejowski J. (2000a). An improved subspace identification method for bilinear systems. In *Proceedings of the 39th IEEE Conference on Decision and Control (CDC 2000)*, Sydney, Australia.

Chen H., Maciejowski J. (2000b). Subspace identification method for combined deterministic-stochastic bilinear systems. In *Proceedings of the 12th IFAC Symposium on System Identification (SYSID 2000)*, Santa Barbara, California, USA.

Chen H., Maciejowski J. (2000c). Subspace identification of deterministic bilinear systems. In *Proceedings of the American Control Conference (ACC 2000)*, vol. 3, pp. 1797–1801, Chicago, Illinois, USA.

Chiuso A., Picci G. (2001). Some algorithmic aspects of subspace identification with inputs. *International Journal of Applied Mathematics and Computer Science* **11**(1), 55–75. [In this paper a new structure for subspace identification algorithms is proposed to help fixing problems when certain experimental conditions cause ill-conditioning].

Cho Y. (1993). *Fast Subspace Based System Identification: Theory and Practice*. Ph.D. thesis, Information Systems Lab, Stanford University, Stanford, California, USA.

Cho Y., Xu G., Kailath T. (1994a). Fast identification of state space models via exploitation of displacement structure. *IEEE Transactions on Automatic Control* **AC-39**(10).

Cho Y., Xu G., Kailath T. (1994b). Fast recursive identification of state space models via exploitation of displacement structure. *Automatica, Special Issue on Statistical Signal Processing and Control* **30**(1), 45–59.

Chou C.T., Verhaegen M. (1997). Subspace algorithms for the identification of multivariable dynamic errors-in-variables models. *Automatica* **33**(10), 1857–1869.

Chou C.T., Verhaegen M.H. (1996). Subspace-based methods for the identification of multivariable dynamic errors-in-variables models. In *Proceedings of the 35th Conference on Decision and Control*

(CDC 1996), pp. 3636–3641, Kobe, Japan.

Chui N.L.C., Maciejowski J.M. (1996). Realization of stable models with subspace methods. *Automatica* **32**(11), 1587–1595. [In this paper the authors present algorithms to find stable approximants to a least-squares problem, which are then applied to subspace methods to ensure stability of the identified model].

Dahl'en A., Lindquist A., Mar'ı J. (1998). Experimental evidence showing that stochastic subspace identification methods may fail. *Systems & Control Letters* **34**, 303–312.

De Moor B. (1988). *Mathematical Concepts and Techniques for Modeling of Static and Dynamic Systems*. Ph.D. thesis, Department of Electrical Engineering, Katholieke Universiteit Leuven, Leuven, Belgium.

De Moor B. (1994). Numerical algorithms for state space subspace system identification. *Academia Analecta, Klasse der Wetenschappen, Koninklijke Akademie voor Wetenschappen* **55**(5).

De Moor B., Moonen M., Vandenberghe L., Vandewalle J. (1988a). The application of the canonical correlation concept to the identification of linear state space models. In J.L. A. Bensoussan, ed., *Analysis and Optimization of Systems*, pp. 1103–1114, Heidelberg, Germany: Springer Verlag.

De Moor B., Moonen M., Vandenberghe L., Vandewalle J. (1988b). Identification of linear state space models with singular value decomposition using canonical correlation concepts. In E. Deprettere, ed., *SVD and Signal Processing: Algorithms, Applications and Architectures*, pp. 161–169, North-Holland: Elsevier Science Publishers B.V.

De Moor B., Van Overschee P. (1994). Graphical user interface software for system identification. Tech. Rep. Report 94–06I, ESAT-SISTA, Department of Electrical Engineering, Katholieke Universiteit Leuven, Leuven, Belgium. [Award winning paper of the *Siemens Award 1994*].

De Moor B., Van Overschee P., Favoreel W. (1999). Algorithms for subspace state-space system identification: An overview. In B.N. Datta, ed., *Applied and Computational Control, Signals, and Circuits*, vol. 1, chap. 6, pp. 247–311, Birkhauser. [This chapter presents an overview of the field of subspace identification and compares it with the traditional prediction error methods. The authors present several comparisons between prediction error methods and subspace methods, including comparisons of accuracy and computational effort].

De Moor B., Vandewalle J. (1987). A geometrical strategy for the identification of state space models of linear multivariable systems with singular value decomposition. In *Proceedings of the 3rd International Symposium on Applications of Multivariable System Techniques*, pp. 59–69, Plymouth, UK.

Deistler M., Peterzell K., Scherrer W. (1994a). Consistency and relative efficiency of subspace methods. In *Proceedings of the 10th IFAC Symposium on System Identification (SYSID 1994)*, vol. 2, pp. 157–163, Copenhagen, Denmark.

Deistler M., Peterzell K., Scherrer W. (1994b). The relative efficiency of different subspace methods. In *Proceedings of the 10th IFAC Symposium on System Identification (SYSID 1994)*, vol. 2, pp. 158–163, Copenhagen, Denmark.

Desai U., Kirkpatrick R., Pal D. (1985). A realization approach to stochastic model reduction. *International Journal of Control* **42**(2), 821–838.

Desai U.B., Pal D. (1982). A realization approach to stochastic model reduction and balanced stochastic realization. In *Proceedings of the 21st Conference on Decision and Control*, pp. 1105–1112, Orlando, USA.

Desai U.B., Pal D. (1984). A transformation approach to stochastic model reduction. *IEEE Transactions on Automatic Control* **AC-29**(12), 1097–1100.

Falkus H., Van Overschee P., Murad G., Hakvoort H., Ludlage J. (1994). Advanced identification and robust control of a glass tube production process. In *Proceedings of the Third Philips Conference on Applications of Systems and Control Theory (PACT)*, Doorwerth, The Netherlands.

Faure P. (1976). Stochastic realization algorithms. In R. Mehra, D. Lainiotis, eds., *System Identification: Advances and Case Studies*, Academic Press.

Favoreel W. (1999). *Subspace Methods for Identification and Control of Linear and Bilinear Systems*.

Ph.D. thesis, Faculty of Engineering, K.U.Leuven, Leuven, Belgium. [Bilinear systems make a good trade-off between the precision of general nonlinear systems and the mathematical tractability of linear systems. In this thesis subspace identification algorithms for bilinear systems are introduced. They are a direct generalization of the existing linear subspace algorithms. The second part of the thesis deals with the analogies of subspace identification and linear optimal control. Using techniques from the field of subspace system identification, the author derives a new class of control algorithms that allow for the calculation of an optimal controller of an unknown plant, directly from measurements of the inputs and the outputs of the plant, without first modeling the plant].

Favoreel W., De Moor B. (1998). Subspace identification of bilinear systems. In *Proceedings of the International Symposium on the Mathematical Theory of Networks and Systems (MTNS 1998)*, Padova, Italy. [In this paper subspace algorithms for the identification of bilinear MIMO systems are described].

Favoreel W., De Moor B., Gevers M., Van Overschee P. (1999a). Closed-loop model-free subspace-based lqg-design. In *Proceedings of the IEEE Mediterranean Conference on Control and Automation*, Haifa, Israel. [Based on techniques from the field of subspace system identification, the paper gives a direct method to implement a finite horizon LQG-controller].

Favoreel W., De Moor B., Van Overschee P. (1997a). A bilinear extension of subspace identification for systems subject to white inputs. In *Proceedings of the 1997 American Control Conference (ACC 1997)*, pp. 607–611, Albuquerque, New Mexico, USA. [Linear subspace identification techniques are generalized to subspace algorithms for bilinear, discrete time, time invariant systems with white inputs.].

Favoreel W., De Moor B., Van Overschee P. (1997b). Subspace identification of balanced deterministic bilinear systems subject to white noise. In *Proceedings of the European Control Conference (ECC 1997)*, pp. 1–9, Brussels, Belgium.

Favoreel W., De Moor B., Van Overschee P. (1999b). Subspace identification of bilinear systems subject to white inputs. *IEEE Transactions on Automatic Control* **44**(6), 1157–1165. [The class of existing linear subspace identification techniques is generalized to subspace identification algorithms for bilinear systems.].

Favoreel W., De Moor B., Van Overschee P. (2000). Subspace state space system identification for industrial processes. *Journal of Process Control* **10**, 149–155. [A general overview of subspace system identification methods is given. A comparison between subspace identification and prediction error methods is made on the basis of computational complexity and precision of the methods by applying them on 10 industrial data sets].

Favoreel W., Van Huffel S., De Moor B., Sima V., M. V. (1999c). Comparative study between three subspace identification algorithms. In *Proceedings of the European Control Conference (ECC 1999)*, Karlsruhe, Germany. [A comparison is made between different subspace system identification techniques. The comparison is made on the basis of 15 publicly available practical data sets].

Golub G.H., Van Loan C.F. (1996). *Matrix Computations*. Baltimore: The Johns Hopkins University Press.

Haverkamp B. (2001). *State Space Identification: Theory and Practice*. Ph.D. thesis, Faculty of Information Technology and Systems, Delft University of Technology, Delft, The Netherlands.

Ho B.L., Kalman R.E. (1966). Effective construction of linear state-variable models from input/output functions. *Regelungstechnik* **14**(12), 545–548.

Jansson M., Wahlberg B. (1996). A linear regression approach to state-space subspace system identification. *Signal Processing* **52**(2), 103–129.

Jansson M., Wahlberg B. (1998). On consistency of subspace methods for system identification. *Automatica* **34**(12), 1507–1519.

Kalman R.E. (1960). A new approach to linear filtering and prediction problems. *Transactions of the American Society of Mechanical Engineers, Journal of Basic Engineering* **83**(1), 35–45.

Kalman R.E. (1963). Mathematical description of linear dynamical systems. *SIAM Journal on Control* **1**, 152–192.

Kalman R.E. (1981). Realization of covariance sequences. In *Proceedings of the Toeplitz Memorial*

Conference, Tel Aviv, Israel.

Katayama T., Picci G. (1999). Realization of stochastic systems with exogenous inputs and subspace identification methods. *Automatica* **35**(10), 1635–1652.

Kung S.Y. (1978). A new identification method and model reduction algorithm via singular value decomposition. In *Proceedings of the 12th Asilomar Conference on Circuits, Systems and Computations*, pp. 705–714.

Larimore W.E. (1983). System identification, reduced ordered filtering and modeling via canonical variate analysis. In *Proceedings of the 1983 American Control Conference (ACC 1983)*, vol. 2, pp. 445–451, San Francisco, California, USA.

Larimore W.E. (1990). Canonical variate analysis in identification, filtering and adaptive control. In *Proceedings of the 29th Conference on Decision and Control (CDC 1990)*, pp. 596–604, Hawaii, USA.

Larimore W.E. (1994). The optimality of canonical variate identification by example. In *Proceedings of the 10th IFAC Symposium on System Identification (SYSID 1994)*, pp. 151–156, Copenhagen, Denmark.

Lindquist A., Picci G. (1985). Realization theory for multivariate stationary gaussian processes. *SIAM Journal on Control and Optimization* **23**(6), 809–857.

Lindquist A., Picci G. (1991). A geometric approach to modelling and estimation of linear stochastic systems. *Journal of Mathematical Systems, Estimation, and Control* **1**(3), 241–333.

Lindquist A., Picci G. (1993). On “subspace methods” identification. In *Proceedings of the International Symposium on the Mathematical Theory of Networks and Systems (MTNS 1993)*, vol. 2, pp. 315–320, Regensburg, Germany.

Lindquist A., Picci G. (1994a). On ‘subspace methods’ identification and stochastic model reduction. In *Proceedings of the 10th IFAC Symposium on System Identification (SYSID 1994)*, vol. 2, pp. 397–403, Copenhagen, Denmark.

Lindquist A., Picci G. (1994b). On “subspace methods” identification and stochastic model reduction. In *Proceedings of the 10th IFAC Symposium on System Identification (SYSID 1994)*, vol. 2, pp. 397–404, Copenhagen, Denmark.

Lindquist A., Picci G. (1996). Canonical correlation analysis, approximate covariance extension, and identification of stationary time series. *Automatica* **32**, 709–733.

Liu K. (1992). Identification of multi-input and multi-output systems by observability range space extraction. In *Proceedings of the 31st Conference on Decision and Control (CDC 1992)*, pp. 915–920, Tucson, Arizona, USA.

Ljung L. (1991). A simple start-up procedure for canonical form state space identification, based on subspace approximation. In *Proceedings of the 30th IEEE Conference on Decision and Control (CDC 1991)*, pp. 1333–1336, Brighton, UK.

Ljung L., McKelvey T. (1996). Subspace identification from closed loop data. *Signal Processing* **52**, 209–215.

Lovera M. (1997). *Subspace Identification Methods: Theory and Applications*. Ph.D. thesis, Department of Electronics and Information, Politecnico di Milano, Milan, Italy.

Lovera M., Gustafsson T., Verhaegen M. (2000). Recursive subspace identification of linear and non-linear Wiener state-space models. *Automatica* **36**, 1639–1650.

Lovera M., Verhaegen M., Chou C.T. (1998). State space identification of MIMO linear parameter varying models. In *Proceedings of the International symposium on the mathematical theory of networks and systems (MTNS 1998)*, pp. 839–842, Padova, Italy.

McKelvey T. (1995). *Identification of State-Space Models from Time and Frequency data*. Ph.D. thesis, Department of Electrical Engineering, Linköping University, Sweden.

McKelvey T., Akcay H. (1995). System identification with periodic excitation signals: A subspace based algorithm. In *Proceedings of 3rd European Control Conference (ECC 1995)*, pp. 423–428, Rome, Italy.

- McKelvey T., Akcay H., Ljung L. (1996a). Subspace-based identification of infinite-dimensional multivariable systems from frequency-response data. *Automatica* **32**(6), 885–902.
- McKelvey T., Akcay H., Ljung L. (1996b). Subspace-based multivariable system identification from frequency response data. *IEEE Transactions on Automatic Control* **41**(7), 960–979.
- Moonen M., De Moor B., Ramos J., Tan S. (1992). A subspace identification algorithm for descriptor system. *Systems & Control Letters* pp. 47–52.
- Moonen M., De Moor B., Vandenberghe L., Vandewalle J. (1989). On-and off-line identification of linear state-space models. *International Journal of Control* **49**(1), 219–232.
- Moonen M., De Moor B., Vandewalle J. (1991). SVD-based subspace methods for multivariable continuous time system identification. In G. Rao, N. Sinha, eds., *Identification of Continuous-time Systems*, pp. 473–488, Kluwer Academic Publications.
- Moonen M., Ramos J. (1993). A subspace algorithm for balanced state space system identification. *IEEE Transactions on Automatic Control* **38**, 1727–1729.
- Moonen M., Vandewalle J. (1990). A QSVD approach to on-and off-line state space identification. *International Journal of Control* **51**(5), 1133–1146.
- Oku H., Nijse G., Verhaegen M., Verdult V. (2001). Change detection in the dynamics with recursive subspace identification. In *Proceedings of the 40th IEEE Conference on Decision and Control (CDC 2001)*, Orlando, Florida.
- Ottersten B., Viberg M. (1994). A subspace based instrumental variable method for state space system identification. In *Proceedings of the 10th IFAC Symposium on System Identification (SYSID 1994)*, vol. 2, pp. 139–144.
- Paternell K., Scherrer W., Deistler M. (1996). Statistical analysis of novel subspace identification methods. *Signal processing* **52**, 161–177.
- Picci G., Katayama T. (1996). Stochastic realization with exogenous inputs and ‘subspace-methods’ identification. *Signal Processing* **52**(2), 145–160.
- Silverman L. (1971). Realization of linear dynamical systems. *IEEE Transactions on Automatic Control* **AC-16**, 554–567.
- Stoica P., Jansson M. (2000). MIMO system identification: state-space and subspace approximations versus transfer function and instrumental variables. *IEEE Transactions on Signal Processing* **48**(11), 3087–3099.
- Stoica P., Sorelius J. (1999). Subspace-based parameter estimation of symmetric noncausal autoregressive signals from noisy measurements. *IEEE Transactions on Signal Processing* **47**(2), 321–331.
- Van Gestel T., Suykens J., Van Dooren P., De Moor B. (2001). Identification of stable models in subspace identification by using regularization. *IEEE Transactions on Automatic Control* **46**(9), 1416–1420. [This paper shows how one can impose stability to the model that is identified with a subspace algorithm. The method proposed is based on regularization].
- Van Overschee P., De Moor B. (1991). Subspace algorithms for the stochastic identification problem. In *Proceedings of the 30th Conference on Decision and Control (CDC 1991)*, pp. 1321–1326, Brighton, England.
- Van Overschee P., De Moor B. (1992). Two subspace algorithms for the identification of combined deterministic stochastic systems. In *Proceedings of the 31st Conference on Decision and Control (CDC 1992)*, pp. 511–516, Tucson, Arizona, USA.
- Van Overschee P., De Moor B. (1993a). N4SID: Numerical algorithms for state space subspace system identification. In *Proceedings of the 12th World Congress of the International Federation of Automatic Control, IFAC*, vol. 7, pp. 361–364, Sydney, Australia.
- Van Overschee P., De Moor B. (1993b). Subspace algorithms for the stochastic identification problem. *Automatica* **29**, 649–660. [In this paper a subspace algorithm is derived to consistently identify stochastic state space models from given output data].

Van Overschee P., De Moor B. (1993c). Subspace identification of a glass tube manufacturing process. In *Proceedings of the second European Control Conference (ECC 1993)*, pp. 2338–2343, Groningen, The Netherlands.

Van Overschee P., De Moor B. (1994). N4SID – Subspace algorithms for the identification of combined deterministic-stochastic systems. *Automatica* **30**(1), 75–94. [In this paper two subspace algorithms for the identification of mixed deterministic-stochastic systems are derived].

Van Overschee P., De Moor B. (1995). A unifying theorem for three subspace system identification algorithms. *Automatica, Special Issue on Trends in System Identification* **31**(12), 1853–1864. [The authors indicate and explore similarities between three different subspace algorithms for the identification of combined deterministic-stochastic systems. It is shown that all three algorithms are special cases of one unifying scheme].

Van Overschee P., De Moor B. (1996a). Continuous-time frequency domain subspace system identification. *Signal Processing, Special Issue on Subspace Methods, part II: System Identification* **52**(2), 179–194.

Van Overschee P., De Moor B. (1996b). *Subspace Identification for linear systems: Theory – Implementation – Applications*. Dordrecht, The Netherlands: Kluwer Academic Publishers. [In this book the theory of subspace identification algorithms is presented in detail].

Van Overschee P., De Moor B., Boyd S., Aling H., R. K. (1994). A fully interactive system identification module for Xmath (ISID). In *Proceedings of the 10th IFAC Symposium on System Identification (SYSID 1994)*, vol. 4, p. 1, Copenhagen, Denmark.

Van Overschee P., De Moor B., Suykens J. (1991). Subspace algorithms for system identification and stochastic realization. In *Proceedings of the 35th Conference on Decision and Control (CDC 1996)*, pp. 589–595, Kobe, Japan.

Vandaele P., Moonen M. (1999). A stochastic subspace algorithm for blind channel identification in noise fields with unknown spatial color. In *Proceedings of the IEEE Conference on Acoustics, Speech and Signal Processing (ICASSP 1999)*, Phoenix, Arizona, USA. [In this paper a method based on subspace techniques is presented for the blind estimation of a communication channel].

Verdult V. (2002). *Nonlinear System Identification: A State-space Approach*. Ph.D. thesis, Faculty of Applied Physics, University of Twente, Twente, The Netherlands. [This thesis deals with the identification of nonlinear systems. It provides a general framework for subspace identification of linear parameter-varying state-space systems and also discusses subspace identification methods for bilinear systems. For both model classes also nonlinear optimization-based identification methods are presented. Furthermore, a new method for the identification of local linear state-space systems is given.].

Verdult V., Verhaegen M. (1999). Subspace-based identification of MIMO bilinear systems. In *Proceedings of the European Control Conference 1999 (ECC 1999)*, Karlsruhe, Germany.

Verdult V., Verhaegen M. (2000). Identification of multivariable linear parameter-varying systems based on subspace techniques. In *Proceedings of the 39th IEEE Conference on Decision and Control*, Sydney, Australia.

Verdult V., Verhaegen M. (2001). Identification of multivariable bilinear state space systems based on subspace techniques and separable least squares optimization. *International Journal of Control* **74**(18), 1824–1836.

Verdult V., Verhaegen M. (2002). Subspace identification of multivariable linear parameter-varying systems. *Automatica* **38**(5).

Verhaegen M. (1993). Application of a subspace model identification technique to identify lti systems operating in closed-loop. *Automatica* **29**(4), 1027–1040.

Verdult V., Verhaegen M. (1994a). Identification of the deterministic part of MIMO state space models given in innovations form from input-output data. *Automatica* **30**(1), 61–74.

Verdult V., Verhaegen M. (1994b). Identification of the deterministic part of MIMO state space models given in innovations form from input-output data. *Automatica, Special Issue on Statistical Signal Processing and Control* **30**(1), 61–74.

- Verdult V., Verhaegen M. (1996). A subspace model identification solution to the identification of mixed causal, anti-causal lti systems. *SIAM Journal on Matrix Analysis and Applications* **17**(2), 332–347.
- Verhaegen M., Dewilde P. (1992a). Subspace model identification part 1. the output-error state-space model identification class of algorithms. *International journal of control* **56**(5), 1187–1210.
- Verdult V., Verhaegen M. (1992b). Subspace model identification part 2. analysis of the elementary output-error state-space model identification algorithm. *International journal of control* **56**(5), 1211–1241.
- Verdult V., Verhaegen M. (1993). Subspace model identification part 3. analysis of the ordinary output-error state-space model identification algorithm. *International journal of control* **56**(3), 555–586.
- Verhaegen M., Westwick D. (1996). Identifying MIMO Hammerstein systems in the context of subspace model identification methods. *International Journal of Control* **63**(2), 331–349.
- Verhaegen M., Yu X. (1995). A class of subspace model identification algorithms to identify periodically and arbitrarily time-varying systems. *Automatica* **31**(2), 201–216.
- Viberg M. (1994). Subspace methods in system identification. In *Proceedings of the 10th IFAC Symposium on System Identification (SYSID 1994)*, pp. 1–12, Copenhagen, Denmark.
- Viberg M., Ottersten B., Wahlberg B., Ljung L. (1991). A statistical perspective on state-space modeling using subspace methods. In *Proceedings of the 30th Conference on Decision and Control (CDC 1991)*, pp. 1337–1342, Brighton, England.
- Viberg M., Ottersten B., Wahlberg B., Ljung L. (1993). Performance of subspace based state space system identification methods. In *Proceedings of the 12th World Congress of the International Federation of Automatic Control, IFAC*, vol. 7, pp. 369–372, Sydney, Australia.
- Westwick D., Verhaegen M. (1996). Identifying MIMO Wiener systems using subspace model identification methods. *Signal Processing* **52**, 235–258.
- Zeiger H., McEwen A. (1974). Approximate linear realization of given dimension via Ho's algorithm. *IEEE Transactions on Automatic Control* **19**, 153.

### Biographical Sketches

**Bart De Moor** was born Tuesday July 12, 1960 in Halle, Belgium. He is married and has three children. In 1983, he obtained his Master (Engineering) Degree in Electrical Engineering at the Katholieke Universiteit Leuven, Belgium, and a PhD in Engineering at the same university in 1988. He spent 2 years as a Visiting Research Associate at Stanford University (1988-1990) at the departments of EE (ISL, Prof. Kailath) and CS (Prof. Golub). Currently, he is a full professor at the Department of Electrical Engineering (<http://www.esat.kuleuven.ac.be>) of the K.U.Leuven.

His research interests are in numerical linear algebra and optimization, system theory and identification, quantum information theory, control theory, data-mining, information retrieval and bio-informatics, areas in which he has (co)authored several books and hundreds of research papers (consult the publication search engine at <http://www.esat.kuleuven.ac.be/sista-cosic-docarch/template.php>).

Currently, he is leading a research group of 39 PhD students and 8 postdocs and in the recent past, 16 PhDs were obtained under his guidance. He has been teaching at and been a member of PhD jury's in several universities in Europe and the US. He is also a member of several scientific and professional organizations.

His work has won him several scientific awards (Leybold-Heraeus Prize (1986), Leslie Fox Prize (1989), Guillemin-Cauer best paper Award of the IEEE Transaction on Circuits and Systems (1990), Laureate of the Belgian Royal Academy of Sciences (1992), bi-annual Siemens Award (1994), best paper award of Automatica (IFAC, 1996), IEEE Signal Processing Society Best Paper Award (1999). He is an associate editor of several scientific journals.

From 1991-1999 he was the chief advisor on Science and Technology of several ministers of the Belgian Federal Government and the Flanders Regional Governments.



He was and/or is in the board of 3 spin-off companies ([www.ipcos.be](http://www.ipcos.be), [www.data4s.com](http://www.data4s.com), [www.tml.be](http://www.tml.be)), of the Flemish Interuniversity Institute for Biotechnology ([www.vib.be](http://www.vib.be)), the Study Center for Nuclear Energy ([www.sck.be](http://www.sck.be)) and several other scientific and cultural organizations.

Full details on his CV can be found at [www.esat.kuleuven.ac.be/~demoor](http://www.esat.kuleuven.ac.be/~demoor).

**Katrien De Cock** received a Master's degree in Electrical Engineering in 1996, a Teaching Degree in Applied Sciences in 1997 and a PhD degree in Engineering in 2002 from the K.U.Leuven in Belgium. Since January 2003, she has been a Postdoctoral Research Associate at the Department of Electrical & Computer Engineering of Stony Brook University, NY. Her current research projects deal with biomedical signal processing.

Her research interests are stochastic modeling of enzyme reactions, system identification, with subspace identification techniques in particular, distance measures for signals and models, canonical correlation analysis for stochastic processes, information theory, fault detection, and identification of stochastic models for network traffic.