IDENTIFICATION OF LINEAR SYSTEMS IN TIME DOMAIN

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Contents

Summary

In a series of articles, basic principles and results of linear system identification techniques in the time domain are described. This powerful methodology for modeling dynamic systems has found applications in many areas. Various ways to introduce parameters to be estimated in the models will be described as well as some standard estimation methods.

1. What Is System Identification?

The field of modeling dynamic systems from experimental data is called *system identification*. This is an interdisciplinary area where major developments have been carried out in control theory, signal processing, statistics and time series analysis, as well as in various application areas.

A dynamic system can be conceptually described as in Figure 1. The system is controlled by input variables $u(t)$, which often can be designed by the user. It is also affected by the disturbances $v(t)$. In some signal processing applications, the inputs may be absent. The output signals are measured variables, which provide useful information about the system. For a *dynamic* system, the control action at time t will influence the output at future time instants.

1.5. The Need of Mathematical Models

Mathematical models of dynamic systems are needed in many areas and for various reasons.

There are many industrial processes, for example production of paper, iron, glass or chemical compounds that must be controlled in order to run safely and efficiently. For an accurate regulator design, some type of model of the process is needed. The models can be of various types and of various degrees of sophistication. For design of a simple regulator, it may be sufficient to know the crossover frequency and the phase margin in a Bode plot (showing the frequency response). In other cases, such as the design of an optimal controller, the designer will need a much more detailed model, which also includes the properties of the disturbances.

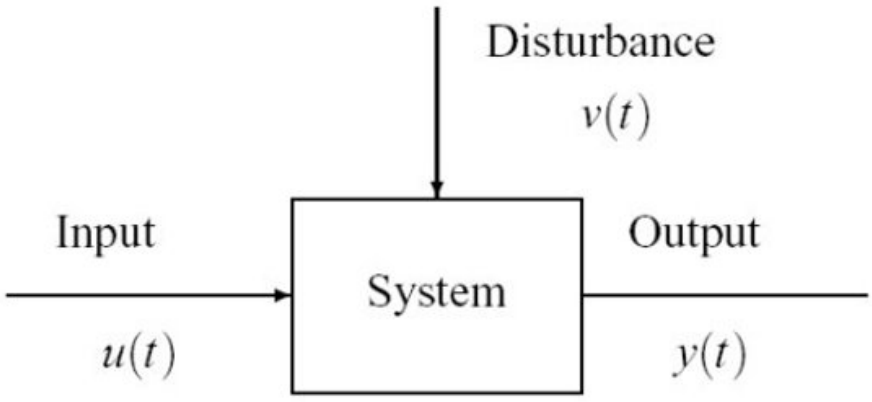

Figure 1: A dynamic system with input u(t), output y(t) and disturbance v(t), where t denotes time.

Signal processing is nowadays applied in many areas such as forecasting, data communication, speech processing, radar, sonar and electrocardiogram analysis. The recorded data are filtered in some way and a good design of the filter should reflect the properties (such as high-pass characteristics, low-pass characteristics, existence of

resonance frequencies, etc) of the signal. To describe such spectral properties, a model of the signal is needed.

Often the primary aim of modeling is to aid in design. In other cases, the *knowledge* of a model can itself be the purpose, as for example when describing the effect of a drug. If the models can explain measured data satisfactorily, they might also be used to explain and understand the observed phenomena.

In a more general sense, modeling is used in many branches of science as an aid to describe and understand reality.

It may also be interesting to model a technical system that does not exist, but may be constructed at some time in the future. In addition, in such a case, the purpose of modeling is to *gain insight* into and knowledge of the dynamic behavior of the system.

An example is a large space structure, where the dynamic behavior cannot be deduced by studying structures on earth, because of gravitation and atmospheric effects. Needless to say, for examples like this, the modeling must be based on theory and *a priori* knowledge, since experimental data are not available.

1.6. Classification of Models

Models of dynamic systems can be of many kinds.

- *Mental, intuitive or verbal models*. For example, this is the form of "model" used when driving a car ("turning the wheel causes the car to turn", "pushing the brake decreases the speed", etc).
- *Graphs and tables*. A Bode plot of a servo system is a typical example of a model in a graphical form. The step response, i.e. the output of a process excited with a step as input, is another type of model in graphical form. Such models can give useful information about the dynamic properties of a process.
- *Mathematical models*. Although graphs may also be regarded as "mathematical" models, here mathematical models are confined to differential and difference equations. Such models are very well suited to the analysis, prediction and design of dynamic systems, regulators and filters. This type of model is predominantly used in system identification.

It should be stressed that although speaking generally about systems with inputs and outputs, the discussion here is to a large extent applicable also to pure time series analysis with only one signal being available.

Signal models for time series can be useful in the design of spectral estimators, predictors or filters that adapt to the signal properties.

1.7. Mathematical Modeling

Basically, there are two ways of constructing mathematical models:

- *Mathematical modeling*. This is an analytic approach. Basic laws from physics (such as Newton's laws and balance equations) are used to describe the dynamic behavior of a phenomenon or a process.
- *System identification*. This is an experimental approach. Some experiments are performed on the system; a model is then fitted to the recorded data by assigning suitable numerical values to its parameters.

System identification is mostly applied to "black box" models. However, it is also possible to combine the two techniques, and include detailed *a priori* information about structure and parameter values when fitting a model to data.

Models obtained by system identification have the following properties, in contrast to models based solely on mathematical modeling:

- They are less general (they are valid for a certain working point, a certain type of input, a certain process, etc).
- They give little physical insight, since in most cases the parameters of the model have no direct physical meaning. The parameters are used only as tools to give a good description of the system's overall behavior.
- They are relatively easy to construct and to use. (Enough physical insight may not be available for a pure mathematical modeling.)

1.8. Applying System Identification

An identification experiment is performed by exciting the system (using some sort of input signal such as a step, a sinusoid, or a random signal) and observing its input and output over a time interval.

These signals are normally recorded and stored in a computer for subsequent "information processing". Then, a parametric model of the process is fitted to the recorded input and output sequences.

The first step is to determine an appropriate form of the model (typically a linear difference equation of a certain order). As a second step, a statistically based method is used to estimate the unknown parameters of the model (such as the coefficients in the difference equation).

The estimation of structure and parameters are often done iteratively. A tentative structure is first chosen and the corresponding parameters are estimated.

The model obtained is then tested to see whether it is an appropriate representation of the system. If this is not the case, a more complex model structure must be considered, its parameters estimated, the new model validated, etc.

The procedure is illustrated in Figure 2. Note that the "restarts" after the model validation phase give an iterative scheme.

It is stressed that model validation is a key step when applying system identification. In

this paper, we focus on parameter estimation methods, while model validation is treated elsewhere.

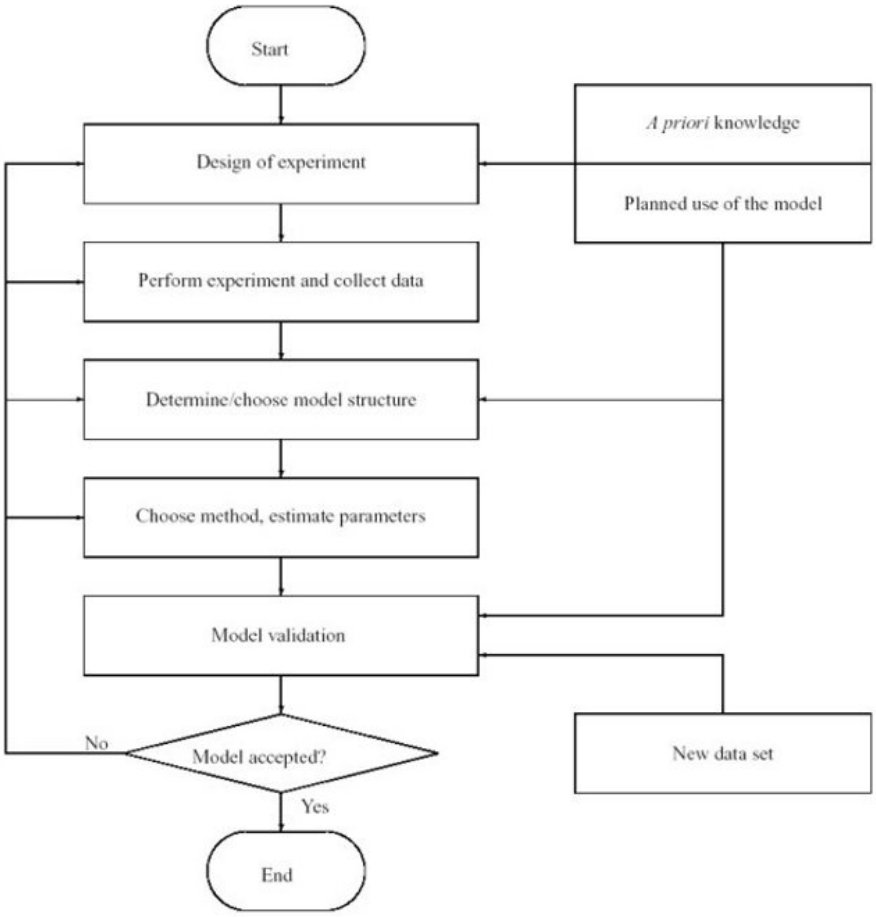

Figure 2: Schematic flowchart of system identification

2. The Setup

2.3. Some Basic Concepts

This section introduces some basic concepts that are valuable when describing and analyzing identification methods. The importance of these concepts will be demonstrated later by some simple examples.

The result of an identification experiment will be influenced by (at least) the following four factors, which will be discussed further in what follows.

- *The system S.* The physical reality that provides the experimental data will generally be referred to as the *process*. In order to perform a theoretical analysis of an identification, it is necessary to introduce assumptions on the data. The word *system* will be used in such cases, to denote a mathematical description of the process. In practice, where real data are used, the system is unknown and can even be an idealization. For simulated data, however, it is not only known but also used directly for the data generation in the computer.

The way 'system' is used here, is fairly standard within the literature on system identification. In some other contexts, it might fit better to label the idealized mathematical description as 'process model' or 'system model'. Then, of course the term 'true system' would just mean the 'system model with the true parameter vector'.

Note that to *apply* identification techniques it is *not* necessary to know the system. The system concept is used here only for investigating how different identification methods behave under various circumstances. For that purpose, the concept of a "system" will be most useful.

- *The model structure M*. Sometimes nonparametric models are applied. Such models are described by curves, functions or tables. A step response is a simple example. It is a curve that carries some information about the characteristic properties of a system. Impulse responses and frequency diagrams (Bode plots) are other examples of nonparametric models. In many cases, however, it is relevant to deal with *parametric models*. Such models are characterized by a parameter vector, which will be denoted by θ. The corresponding model will be denoted $M(\theta)$. When θ is varied over some set of feasible values, one obtains a model set (a set of models), or a model structure M.
- *The identification method I*. A large variety of identification methods have been proposed in the literature. Some important ones will be discussed later. It is worth mentioning here that several proposed methods could and should be regarded as versions of the same basic approach, tied to different model structures, even if they were originally introduced and are known under different names.
- *The experimental condition X*. In general terms, X is a description of how the identification experiment is carried out. This includes the selection and generation of the input signal, possible feedback loops in the process, the sampling interval, prefiltering of the data prior to estimation of the parameters, etc.

Of the four concepts, S, M, I, X, the system S must be regarded as fixed. It is "given" in the sense that its properties cannot be changed by the user. The experimental condition X is determined when the data are collected from the process. It can often be influenced to some degree by the user.

However, there may be restrictions - such as safety considerations or requirements of "nearly normal" operations - that prevent a free choice of the experimental condition X. Once the data are collected, the user can still choose the identification method I and the model structure M. Several choices of I and M can be tried on the same set of data until a satisfactory result is obtained.

2.4. Identifiability

The concept of identifiability can be introduced in a number of ways, but the following is convenient for the present purposes.

When an identification method I is applied to a parametric model structure M, the resulting estimate is denoted by $\hat{\theta}(N; S, M, I, X)$. Clearly, the estimate will depend not only on I and M but also on the number of data points N, the true system S and the experimental condition X.

To describe what we want to identify, assume that the true system S is linear, discrete time and can be described as

$$\begin{aligned} &y(t) = G_s(q^{-1})u(t) + H_s(q^{-1})e_s(t) \\ &Ee_s(t)e_s^T(t') = \Lambda_s\delta_{t,t'}. \end{aligned} \qquad (1)$$

Then, the set of "desirable filters", $D_T(S, M)$ can be formally defined as

$$D_T(S,M) = \{\theta \mid G(q^{-1};\theta) \equiv G_s(q^{-1}), \quad H(q^{-1};\theta) \equiv H_s(q^{-1}), \Lambda(\theta) \equiv .\Lambda_s\}. \qquad (2)$$

This set thus describes precisely those parameter vectors that give a perfect description of the process. Three cases may occur:

1. $D_T(S, M)$ is empty. Then, the model parameterization is too simple to describe the system. This is called *underparameterization*. (It is, of course, the most likely outcome in practice.)
2. $D_T(S, M)$ has precisely one point, say θ_o. This is the best possible case. We can call θ_o the true *parameter vector*.
3. $D_T(S, M)$ consists of more than one point. This is called *overparametrization*. This situation occurs if a too high model order is chosen (leading to pole zero cancellations) or if too many free parameters are entered in a state space model.

Now assume that the set $D_T(S, M)$ is non-empty. We then say that the system S is *system identifiable* under M, I and X, abbreviated SI(M, I, X), if

$$\hat{\theta}(N; S, M, I, X) \to D_T(S, M) \quad \text{as } N \to \infty \qquad (3)$$

(with probability one). If $D_T(S, M)$ contains more than one point then the shortest distance between the estimate $\hat{\theta}$ and the set $D_T(S, M)$ of all parameter vectors describing $G(q^{-1})$ and $H(q^{-1})$ exactly, tends to zero as the number of data points tends to infinity.

We say that the system S is *parameter identifiable* under M, I and X, abbreviated PI(M, I, X), if it is SI(M, I, X) and $D_T(S, M)$ consists of exactly one point. This is the ideal case. If the system is PI(M, I, X) then the parameter estimate $\hat{\theta}$ will be unique for large values of N and also consistent (i.e. $\hat{\theta}$ converges to the true value, as given by the definition of $D_T(S, M)$).

Here the concept of identifiability has been separated into two parts. Firstly, the convergence of the parameter estimate $\hat{\theta}$ to the set $D_T(S, M)$ (i.e. system identifiability)

is a property that basically depends on the identification method I and the experimental condition X. Secondly, the role of the model structure M lies in the set $D_T(S, M)$.

Other treatments of identifiability consider basically the properties of the set $D_T(S, M)$, which ideally should have just one point, corresponding to the true parameter vector. Such an approach may be useful in the deterministic case. It would then describe the possibility to recover the true system properties uniquely from the input-output data, without specifying any particular identification method.

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Biographical Sketch

Torsten Söderström was born in Malmö, Sweden, in 1945. He received the MSc degree (`civilingenjör') in engineering physics in 1969 and the PhD degree in automatic control in 1973, both from Lund Institute of Technology, Lund, Sweden. He is a Fellow of IEEE.

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Dr Söderström is the author or coauthor of more than 200 technical papers. His main research interests are in the fields of system identification, signal processing, and adaptive systems. He is the (co)author

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Within IFAC (International Federation of Automatic Control) he has served in several capacities including Automatica editor for the area of System Parameter Estimation since 1992, Council member 1996-2002, member of Executive Board and Chair of Awards Committee 1999-2002, IPC chairman of the SYSID'94 symposium, and guest associate editor or editor for three special issues of Automatica.