NONPARAMETRIC SYSTEM IDENTIFICATION

H. Kashiwagi
Kumamoto University, Japan

Keywords: correlation method, Gaussian signal, identification, kernel method, M-sequence, nonlinear system, nonparametric method, pseudorandom sequence, orthogonal expansion, Volterra kernel, Wiener kernel, white noise

Contents

1. Introduction
2. Representation of Nonlinear Systems
3. Identification of Wiener Kernels
3.1. Wiener's Orthogonal Expansion Method
3.2. Lee–Schetzen's Method
4. Identification of Volterra Kernels
4.1. Hooper–Gyftopoulos Method
4.2. Watanabe–Stark's Method
4.3. Kashiwagi–Sun's Method
5. Frequency Domain Approach
Glossary
Bibliography
Biographical Sketch

Summary

This article presents a survey of various methods for nonparametric identification of nonlinear systems. Nonparametric identification methods are those that measure Wiener kernels or Volterra kernels, since an output of a nonlinear system can be described by the convolution integral of Wiener or Volterra kernels and the system input. Section 1 highlights the representation methods of nonlinear systems by kernels including mutual relationships between Wiener kernels and Volterra kernels.

Section 2 describes identification methods for Wiener kernels including Wiener's orthogonal expansion method and Lee-Schetzen's correlation method. Section 3 covers the identification method of Volterra kernels including Hooper-Gyftopoulos' correlation method, Watanabe-Stark's othonormal basis function method, and Kashiwagi-Sun's M-sequence correlation method. For each method, comments are made on the computational load from a practical application point of view.

1. Introduction

System identification methods are divided into two groups: parametric and nonparametric. Parametric methods identify system model with an underlying mathematical structure that is associated with a coefficient set or parameters, whereas nonparametric methods model a system directly with its responses.

Let us take an example in a linear system. A method for obtaining a transfer function of a system is a parametric method. The system parameters in this case are coefficients of the transfer function, and the number of parameters is less than or equal to $2n + 1$ where n is the order of the system. In the same way, a state equation or a difference equation method belongs to the parametric group. In contrast, a method for obtaining an impulse response, step response or frequency response of the system belongs to the nonparametric group.

Similarly, identification methods for nonlinear systems are also divided into the parametric and nonparametric groups. Nonparametric methods of nonlinear system identification include those system representation methods using Volterra kernels or Wiener kernels. Hence a nonparametric method for nonlinear system identification usually means a method for obtaining Volterra kernels or Wiener kernels.

In this article, identification methods for obtaining Volterra kernel or Wiener kernel of a nonlinear control system are described in detail. (For parametric methods of nonlinear system identification, see articles under the topic entitled *Identification of Nonlinear Systems.*)

2. Representation of Nonlinear Systems

Let $u(t)$ be an input to a nonlinear system and $y(t)$ be its output. Then the output $y(t)$ can be in general written as follows.

$$y(t) = \sum_{i=0}^{\infty} H_i\left[u(t)\right] \tag{1}$$

where

$$\begin{aligned} H_i[u(t)] = \int_0^{\infty}\int_0^{\infty} \dots \int_0^{\infty} h_i(\tau_1,\tau_2,\dots,\tau_i)u(t-\tau_1)u(t-\tau_2) \\ \dots\dots u(t-\tau_i)d\tau_1 d\tau_2 \dots d\tau_i \end{aligned} \tag{2}$$

and $h_i(\tau_1,\tau_2,\dots\tau_i)$ is called the Volterra kernel of the *i*-th order.

The basis of Eq. (1) is due to Frechet (1910) who showed that any continuous functional can be represented by a series of functionals of integer order whose convergence is uniform on all compact sets of continuous functions. Hence Eq. (1) applies to those nonlinear systems whose output is continuous for a continuous input. Therefore we should note that Eq. (1) does not apply to those nonlinear systems that have a multi-valued nonlinearity like hysteresis, backlash or an on-fading memory. However we should also note that most nonlinear systems are considered to be representable by Eq. (1).

Another representation method for nonlinear system via kernel method, when the input $u(t)$ is a white Gaussian signal, is the Wiener kernel method as shown in the following equation.

$$y(t) = \sum_{n=0}^{\infty} G_n\left[k_n; u(t)\right] \tag{3}$$

where G_n is called the Wiener G-functional, with orthogonal properties given by the next two equations.

$$\overline{G_m\left[k_m; u(t)\right] G_n\left[k_n; u(t)\right]} = 0, \qquad \text{for } m \neq n \tag{4}$$

$$\overline{H_m\left[u(t)\right] G_n\left[k_n; u(t)\right]} = 0, \qquad \text{for } m < n \tag{5}$$

Here k_n is called the *n*-th order Wiener kernel and the bar over $G_m G_n$ denotes the time average over the interval $(-\infty, \infty)$. Eq. (4) shows that Wiener G-functionals are orthogonal to each other. Equation (5) shows that $G_n\left[k_n; u(t)\right]$ is orthogonal to $H_m\left[u(t)\right]$ when $m < n$. The first four terms of the Wiener G-functional are

$$G_0\left[k_0; u(t)\right] = k_0$$

$$G_1\left[k_1; u(t)\right] = \int_{-\infty}^{\infty} k_1(\tau_1) u(t-\tau_1) d\tau_1$$

$$\begin{aligned} G_2\left[k_2; u(t)\right] = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_2(\tau_1, \tau_2) u(t-\tau_1) u(t-\tau_2) d\tau_1 d\tau_2 \\ &- A\int_{-\infty}^{\infty} k_2(\tau_1, \tau_1) d\tau_1 \end{aligned} \tag{6}$$

$$\begin{aligned} G_3\left[k_3; u(t)\right] = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k_3(\tau_1, \tau_2, \tau_3) u(t-\tau_1) \\ &u(t-\tau_2) u(t-\tau_3) d\tau_1 d\tau_2 d\tau_3 \\ &+ \int_{-\infty}^{\infty} k_{1(3)}(\tau_1) u(t-\tau_1) d\tau_1 \end{aligned}$$

Here $k_{1(3)}(\tau_1)$ is called the derived Wiener kernel from the third Wiener kernel $k_3(\tau_1,\tau_2,\tau_3)$ given by the next equation.

$$k_{1(3)}(\tau_1) = -3A\int_{-\infty}^{\infty} k_3(\tau_1,\tau_2,\tau_2)d\tau_2 \qquad (7)$$

In this way, $k_{i(j)}$ denotes the *i*-th degree kernel derived from the *j*-th degree kernel.

A is the power of the input white Gaussian signal. We should note that the Wiener G-functional representation of a nonlinear system was originally developed for white Gaussian input, but it is also applicable to a non-white or non-Gaussian input by the use of a prewhitening technique.

Given the input white Gaussian signal, what is the relationship between the Volterra kernel $h_i(\tau_1,\tau_2,...\tau_i)$ in Equation (1) and the Wiener kernel $k_i(\tau_1,\tau_2,...\tau_i)$ in Eq. (3)? When we compare Eq. (1) and Eq. (3), we get

$$\begin{aligned} h_0 &= k_0 + k_{0(2)} + k_{0(4)} + \ldots \\ h_1 &= k_1 + k_{1(3)} + k_{1(5)} + \ldots \\ h_2 &= k_2 + k_{2(4)} + \ldots \\ h_3 &= k_3 + k_{3(5)} + \ldots \end{aligned} \qquad (8)$$

Therefore when we wish to identify a nonlinear system using a white Gaussian input by a kernel method, obtaining Volterra kernels is equivalent to obtaining Wiener kernels, since these two representations have one to one correspondence.

-
-
-

Bibliography

Barker H.A. and Pradisthayon T. (1970). High-order autocorrelation functions of pseudorandom signals based on m-sequences,. *Proceedings of the Institution of Electrical Engineers*, **117** (9), pp. 1857–1863. [This paper discusses some properties of high-order autocorrelation functions of M-sequence which are necessary for obtaining the second Volterra kernel of a nonlinear system.]

Billings S.A. (1980). Identification of nonlinear systems—a survey, *The Institution of Electrical Engineers Proceedings*, **127**, Part D (6), pp. 272–285. [This survey presents nonlinear system identification algorithms and related topics including not only nonparametric Wiener and Volterra kernel methods but also block-oriented systems, bilinear systems, etc.]

French A.S. and Butz E.G. (1973). Measuring the Wiener kernels of a non-linear system using the fast Fourier transform algorithm, *International Journal of Control*, **17** (3), pp. 529–539. [This paper describes a method for measuring the Wiener kernels of a nonlinear system via frequency domain treatment of kernels.]

Hooper R.J. and Gyftopoulos E.P. (1966). On the measurement of characteristic kernels of a class of nonlinear systems. *Proceedings of a Symposium on Neutron Noise, Waves and Pulse Propagation*, University of Florida, February 14–16, 1966. Gainesville, pp. 343–353. [This paper first proposed a method of obtaining up to second order Volterra kernels by use of pseudorandom ternary signal.]

Hung G. (1977). Introductory review—the kernel identification method (1910–1977)—review of theory, calculation, application, and interpretation, *Mathematical Biosciences*, **37**, pp. 135–190. [This paper systematically reviews the theory, calculation, application, and interpretation of the kernel identification methods from 1910 to 1977.]

Kashiwagi H. (1996). *M-sequence and its Applications*, Japan, 204 pp. Shoukoudo Co. [This book shows the theory of M-sequence and its applications to measurement and control systems including nonlinear system identification.] (in Japanese)

Kashiwagi H. and Sun Yeping (1994). A method for identifying Volterra kernels on nonlinear systems and its applications. *Proceedings of Asian Control Conference*, Tokyo, Japan, July 27–30, 1994, pp. 401–404. [This paper proposes a method of Volterra kernel identification of nonlinear system by use of the M-sequence correlation method.]

Schetzen M. (1980). *The Volterra and Wiener Theories of Nonlinear Systems*, 531 pp. New York: John Wiley & Sons. [This book presents all the essential aspects of the Volterra and Wiener theories of nonlinear system so that most readers can easily understand the contents.]

Schetzen M. (1981). Nonlinear system modeling based on the Wiener theory. *Proceedings of the Institution of Electrical and Electronic Engineers*, **69** (12), pp. 1557–1573. [This paper presents a tutorial of nonlinear system modeling methods based on Wiener's theory, discussing the basic concepts that underlie Wiener's theory.]

Watanabe A. and Stark L. (1975). Kernel method for nonlinear analysis: identification of a biological control system, *Mathematical Biosciences*, **27**, pp. 99–108. [This paper describes a method for computing kernels of a nonlinear system subjected to random inputs, by means of a finite-dimensional approximation combined with the least-square error method, with some application results to physiological control systems.]

Wiener N. (1958). *Nonlinear Problems in Random Theory*, 131 pp. New York: MIT Press and John Wiley & Sons. [This book presents the basic theory on nonlinear problems in random theory, and proposes the use of Wiener kernel for representing nonlinear system behavior.]

Biographical Sketch

Professor **Hiroshi Kashiwagi** was born in 1939 in Kumamoto, Japan. He received BE, ME and PhD degrees in measurement and control engineering from the University of Tokyo, Japan, in 1962, 1964, and 1967 respectively.

In 1967, he became an Associate Professor at Kumamoto University. From 1973 to 1974, he served as a visiting Associate Professor at Purdue University, Indiana, USA. He became a Professor at Kumamoto University in 1976. He served as a member of Board of Trustees of SICE (Society of Instrument and Control Engineers, Japan), General Chairman of SICE 1992 held in Kumamoto, General Chairman of ICAUTO 1995 (International Conference of Automation) held in Indore, India, in 1995, and General Chairman of APCCM 1998, 2000, and 2002 (Asia Pacific Conference on Control and Measurement) held in China in 1998, 2000, and 2002, respectively.

In 1994, he was awarded SICE Fellow for his contributions to the field of measurement and control engineering through his various academic activities. He also received the Gold Medal Prize at ICAUTO 1995 held in India. In 1997, he was awarded "Best Book Award" from SICE for his new book entitled

"M-sequence and its Application" written in Japanese and published in 1996 by Shoukoudo Publishing Co. in Japan. In 1999, he received the "Best Paper Award" from SICE for his paper "M-transform and its application to system identification".

His research interests include signal processing and applications, especially pseudorandom sequence and its applications to measurement and control engineering including nonlinear system identification.