IDENTIFICATION IN THE FREQUENCY DOMAIN

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Contents

Summary

This article reviews techniques to identify the frequency domain properties of linear and nonlinear systems from measured physical input/output data. Practical spectral analysis procedures are discussed that are valid with random data to predict the frequency response functions in single-input/single-output (SI/SO) linear models and in multiple-input/single-output (MI/SO) linear models. Special procedures are discussed for five types of nonlinear system model: Volterra nonlinear models, Hammerstein nonlinear models, Wiener nonlinear models, SI/SO nonlinear models, and models with nonlinear feedback.

Formulas to identify Volterra nonlinear models are very complicated to apply and difficult to interpret. Formulas to identify Hammerstein and Wiener nonlinear models are limited in scope because of an assumption of Gaussian input data and restrictions on zero-memory nonlinear operations. Formulas to identify SI/SO nonlinear models are valid for data with arbitrary probability and spectral properties, and with arbitrary zero-memory nonlinear operations.

These results are simple to apply and interpret for wide classes of nonlinear system problems because the SI/SO nonlinear models can be replaced by equivalent MI/SO linear models. Practical direct MI/SO techniques can be used to solve the SI/SO nonlinear models, and practical reverse MI/SO techniques can be used to solve models

with nonlinear feedback.

1. Introduction

This article is concerned with the identification of frequency domain properties in linear and nonlinear systems using measured physical input/output random data. Two main properties distinguish nonlinear systems from linear systems. First, nonlinear systems do not satisfy the additive and homogeneous properties of linear systems. Second, the passage of Gaussian input data through nonlinear systems produces non-Gaussian output data. A general spectral analysis technique exists to identify the frequency response functions in linear systems. No general technique exists that applies to all nonlinear systems. Instead, special techniques are required for particular types of nonlinear system model that can occur. This article reviews techniques for five types of nonlinear system model deemed to be important in many engineering and scientific applications.

The first three types of nonlinear model are Volterra, Hammerstein, and Wiener nonlinear models. Formulas to identify the frequency properties in Volterra nonlinear models are known only for Gaussian input data, and require the computation of multidimensional frequency response and spectral density functions. Formulas to identify the frequency properties in Hammerstein and Wiener nonlinear models are based on time domain iterative techniques that usually assume Gaussian input data and restrict the results to zero-memory nonlinear systems that are polynomials with constant coefficients.

The last two types of nonlinear model are SI/SO nonlinear models that can be solved by direct MI/SO techniques, and models with nonlinear feedback that can be solved by reverse MI/SO techniques. These practical techniques are very broad in scope because:

- the input and output data can have arbitrary probability and spectral properties,
- the zero-memory nonlinear systems are not restricted in form, and
- standard programs exist that include coherence functions and error analysis criteria to evaluate the results.

2. Linear System Identification

A fundamental understanding is required on how to compute and apply spectral density functions from measured data to identify the frequency response functions in SI/SO linear models and in MI/SO linear models. Knowledge of these linear techniques is essential in solving the SI/SO nonlinear models in Section 3.3, and the models with nonlinear feedback in Section 3.4.

2.1. SI/SO Linear Models

The main SI/SO linear model of concern in this article is pictured in Figure 1 for the case of a SI/SO linear model with output noise. This model, as well as other less important types of SI/SO linear model that might be of interest for special applications, are discussed in much detail in the relevant titles cited in the Bibliography.

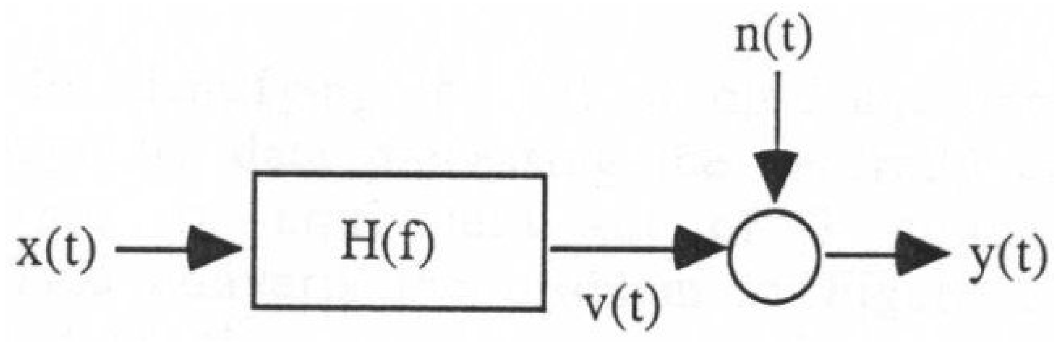

Figure 1. SI/SO linear model with output noise

In Figure 1, the terms are:

$x(t)$ = measured input
$y(t)$ = measured total output $= v(t) + n(t)$
$v(t)$ = unmeasured linear system output
$n(t)$ = unmeasured output noise
$H(f)$ = linear system frequency response function

The optimum linear system frequency response function for Figure 1 is defined as the frequency response function that minimizes the spectral density function of the output noise. It is computed by the cross-spectral density function between the input and output records $G_{xy}(f)$, divided by the autospectral density function of the input record $G_{xx}(f)$, namely:

$$H(f) = [G_{xy}(f)/G_{xx}(f)] \ .$$

This simple, practical spectral analysis formula applies to general SI/SO linear models with output noise where the measured physical input/output data can have arbitrary probability and spectral properties.

The output noise $n(t)$ in Figure 1 and in succeeding models represents all possible deviations from the model, including any unspecified nonlinear effects. With the optimum linear system, the computed output records $v(t)$ and $n(t)$ are automatically uncorrelated.

2.2. MI/SO Linear Models

A typical MI/SO linear model of concern in this article is pictured in Figure 2 for the case of a three-input/single-output linear model with output noise. A general MI/SO linear model with output noise can have an arbitrary number of input records.

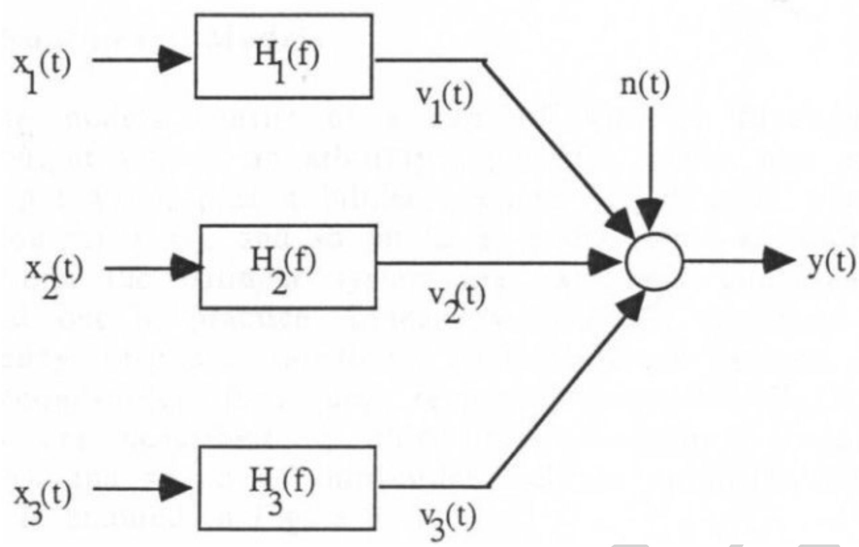

Figure 2. MI/SO linear model with output noise

In Figure 2, the terms are:

$x_1(t),\ x_2(t),\ x_3(t)$ = measured or computed inputs

$y(t)$ = measured total output = $v(t)+n(t)$

$v(t) = v_1(t)+v_2(t)+v_3(t)$ = sum of linear system outputs

$n(t)$ = unmeasured output noise

$H_1(f),\ H_2(f),\ H_3(f)$ = linear system frequency response functions

The key to identifying the set of optimum linear systems in Figure 2 is to change by data processing the generally correlated set of three input records into an equivalent set of three mutually uncorrelated input records. This converts the problem in Figure 2 into three separate SI/SO linear models that can be solved separately as per the spectral formula for Figure 1. Recommended practical steps to perform this work for general MI/SO linear models are discussed fully and applied to many problems in literature. Available standard programs to solve Figure 2 proceed as follows:

1. The first step is to compute a new set of mutually uncorrelated input records from the original input records using conditioned spectral density functions and the simple SI/SO spectral analysis approach in Section 2.1.
2. To produce the same total output record $y(t)$ and the same output noise $n(t)$, if the mutually uncorrelated input records and the original input records are different, then these mutually uncorrelated input records must pass through a different set of linear systems $\{L_i(f)\}$ instead of the previous set of linear systems $\{H_i(f)\}$ shown in Figure 2.
3. Apply the simple SI/SO spectral analysis formula in Section 2.1 to identify the optimum linear system $\{L_i(f)\}$ between each of the

uncorrelated input records and the total output record $y(t)$.

4. The set of optimum linear systems $\{H_i(f)\}$ between each of the original input records and the total measured output record $y(t)$ can now be computed from the $\{L_i(f)\}$ systems by straightforward algebraic equations.

Note that this practical procedure applies to general MI/SO linear system problems where the measured physical input/output random data can have arbitrary probability and spectral properties. As shown in the next section, this same approach can be used to identify the frequency properties in a large class of SI/SO nonlinear models.

3. Nonlinear System Identification

Frequency domain techniques to identify five types of nonlinear system models will now be discussed. These five types are:

1. Volterra nonlinear models
2. Hammerstein nonlinear models
3. Wiener nonlinear models
4. SI/SO nonlinear models
5. Models with nonlinear feedback.

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Biographical Sketch

Julius S. Bendat is an internationally recognized authority in the field of random data analysis techniques and their applications to many engineering problems. He is an independent mathematical consultant for a wide range of organizations in both industry and government, and has presented short courses and seminars in the United States and 25 other countries on the topics of analyzing and identifying linear and nonlinear systems from random data. Contributions include: (a) the development of practical error analysis criteria to design experiments and evaluate results from measured random data, (b) practical procedures to analyze the frequency domain properties of random data through multiple-input/output linear systems, (c) practical ways to predict the response of known linear and nonlinear systems to random data, and (d) practical techniques to identify the frequency domain properties of unknown linear and nonlinear systems from measured input/output random data. He is the author of *Nonlinear System Techniques and Applications* (1998), and coauthor with A.G. Piersol of *Random Data Analysis and Measurement Procedures*, Third Edition (2000) and *Engineering Applications of Correlation and Spectral Analysis*, Second Edition (1993). Experience includes: President, Measurement Analysis Corporation; Senior Staff, Thompson Ramo Wooldridge Corporation; and Research Engineer, Northrop Aircraft Corporation. He received his Ph.D. in Mathematics from the University of Southern California, USA.