PRACTICAL ISSUES OF SYSTEM IDENTIFICATION

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Summary

System Identification concerns the problem of building mathematical models of dynamical systems. This involves a fair amount of theory and algorithms. Equally important, though, is the practical side of the methodology. This chapter deals with the issues that are essential to construct a good model in practice. Such issues include the problem of input and experiment design. Typical essential features of the input are discussed as well as examples of commonly used inputs. Next comes the question of how to condition the measured signals. This involves issues of removing trends and disturbances outside of the frequency ranges of interest for the model. The most demanding task is to find a suitable model structure, guided by information in the observed data. A first cut methodology for this is described. For a successful application, it is in the end a matter of combining intuition, and information from various data test. Two applications, a fighter aircraft and a buffer vessel in process industry illustrate the process.

1. The Framework

System Identification is both a science and an art. In several articles in this encyclopedia, the science of System Identification has been developed. This involves techniques for parameter estimation, the statistical framework, choice of model structures, techniques for non-parametric models etc. The *art* of System Identification concerns how all these techniques are applied in practice: What problems have to be solved, what choices have to be made when the user faces a real-life plant and need a mathematical model of how it works. Some issues that are critical for this situation will be reviewed in this article.

1.4. Starting Point

We consider the following set-up for system identification. The set of models that can be used consists of linear time invariant (LTI) descriptions of dynamical systems. They will generally be described as

$$y(t) = G(q, \theta)u(t) + H(q, \theta)e(t) \tag{1}$$

Here $y(t)$ is the output at time t, $u(t)$ is the input signal at time t, and $e(t)$ is a disturbance source, typically described as a sequence of independent random variables. q is the shift operator, and G and H are rational transfer functions in q. In other words, Eq. (1) describes a set of linear difference equations relating the input, output, and disturbances to each other.

The signals y and u are observed, while e is not measurable. The measurements are made in discrete time, and they will generally just be enumerated as $Z^N = \{y(1), u(1), y(2), u(2), ..., y(N), u(N)\}$. The model could very well be multivariable, i.e. $y(t)$ and $u(t)$ could be vectors containing several input and output variables.

The system identification problem is to

- generate a suitable input signal u
- measure the corresponding y
- find an appropriate model parameterization (structure) in Eq. (1)
- determine the “best values” of the corresponding parameters θ
- determine if the resulting model is adequate for its intended purpose

1.5. Some Typical Model Structures

The general model in Eq. (1) contains all possible LTI models. It is just a question of how to parameterize the transfer functions. See other entries in this encyclopedia for more details on how this can be done. Some common special cases are

- ARX models: $A(q)y(t) = B(q) + e(t)$. This corresponds to the parameterization of $G(q, \theta)$ as a rational function

  $$G(q, \theta) = \frac{B(q)}{A(q)} \tag{2}$$

  with the parameters θ being the coefficients of the numerator and denominator polynomials. This structure has a noise model

$$H(q,\theta) = \frac{1}{A(q)} \tag{3}$$

that does not have any extra degree of freedom. The most important reason for the use of such ARX models is that θ can be estimated by a simple linear least squares method.

- Output Error (OE) models: These use a fixed noise model $H(q, \theta) = 1$ and thus assume all measurement error to be white noise at the output of the system.
- State-space models: These correspond to the parameterization of G and H in terms of matrices A, B, C, D, K as in
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$$G(q,\theta) = C(\theta)(qI - A(\theta))^{-1}B(\theta) + D(\theta) \tag{4}$$

$$H(q,\theta) = C(\theta)(qI - A(\theta))^{-1}K(\theta) + I \tag{5}$$

This means that the input-output relationship can be written as a state-space model (in innovations form):

$$x(t+1) = A(\theta)x(t) + B(\theta) + K(\theta)e(t) \tag{6}$$

$$y(t) = C(\theta)x(t) + D(\theta) + e(t) \tag{7}$$

The parameterization of the state space matrices can be done in an arbitrary way. They could, for example be constructed from an underlying continuous-time model with parameter entries corresponding to unknown constants of physical significance.

1.6. Estimating the Parameters

There are several possibilities for estimating the parameters in Eq. (1). A generic method is the *prediction error approach*: Form the error between the model output $G(q, \theta)u(t)$ and the measured output $y(t)$ and filter it with the inverse noise model:

$$\varepsilon(t,\theta) = H^{-1}(q,\theta)(y(t) - G(q,\theta)u(t)) \tag{8}$$

Note that ε can be seen as an estimate of e in Eq. (1). Then, select the parameter estimate $\hat{\theta}_N$ so that the size of these weighted errors becomes as small as possible:

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N}\sum_{t=1}^{N} |\,\varepsilon(t,\theta)\,|^2 \tag{9}$$

The minimization of this criterion, and hence the computation of the estimate must normally be made by iterative numerical search. It is essentially only the ARX model that allows a closed form expression for $\hat{\theta}_N$.

For the state-space model in Eq. (6) there is another possibility to estimate the system matrices, known as *subspace methods*. In short, this method first estimates the states x from ARX- like expressions, and then treats Eq. (6) as a linear regression with x (pretended as) known. The advantage is a numerically efficient, non-iterative algorithm.

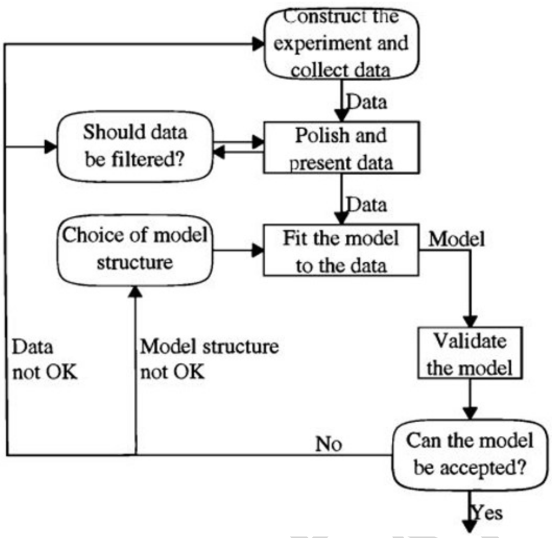

Figure 1: Identification cycle. Rectangles: the computer's main responsibility. Ovals: the user's main responsibility.

2. The User and the System Identification Problem

To turn to the art of System Identification, we can pose as users faced with a physical process with which to experiment. The aim is to construct a reliable mathematical model. This task involves several subproblems:

1. Select an input signal to apply to the process
2. Collect the corresponding output data
3. Scrutinize the obtained data to find out if some preprocessing will be necessary
4. Specify a model structure.
5. Let the computer deliver the best model in this structure, when applied to the collected data.
6. Evaluate the properties of this model.
7. Test a new structure, go to step 4.
8. If the models obtained in this way are not adequate, go back to step 3 to try some other data preprocessing, or to step 1 to carry out a new experiment with another and more "revealing" input.

See Figure 1 which illustrates this process.

2.2. The Tool: Interactive Software

To have any chance of success it will be necessary to have good computer support. Figure 1 shows (in rectangles) what support the computer can supply. The techniques and algorithms behind these rectangles belong to the "science" of System Identification. The remaining decisions, which have to be taken by the user, (marked in ovals in the figure) are what this article will focus on.

The first thing that requires help is to compute the model and to evaluate its properties. There are now many commercially available program packages for identification that supply such help. They typically contain the following routines:

A. *Handling of data, plotting, and the like*
Filtering of data, removal of drift, choice of data segments, and so on.

B. *Non-parametric identification methods*
Estimation of covariances, Fourier transforms, correlation and spectral analysis, and so on.

C. *Parametric estimation methods*
Calculation of parametric estimates in different model structures.

D. *Presentation of models*
Simulation of models, estimation and plotting of poles and zeros, computation of frequency functions and plotting in Bode diagrams, and so on.

E. *Model validation*
Computation and analysis of residuals ($\varepsilon(t, \hat{\theta}_N)$); comparison between different models' properties, and the like.

The existing program packages differ mainly by various user interfaces and by different options regarding the choice of model structure according to item C.

One of the most used packages is MathWork's SYSTEM IDENTIFICATION TOOLBOX (SITB), which is used together with MATLAB. The command structure is given by MATLAB's programming environment with the workspace concept and MACRO possibilities in the form of m-files. SITB gives the possibility to use all model structures of the black-box type, Eq. (1), with an arbitrary number of inputs. ARX-models and state-space models with an arbitrary number of inputs and outputs are also covered. Moreover, the user can define arbitrary tailor-made linear state-space models in discrete and continuous time as in Eq. (6). A Graphical User Interface helps the user both to keep track of identified models and to guide him or her to available techniques.

The remainder of this article will deal with the issues in the ovals of Figure 1: In Section 3 the choice of input will be discussed, while Section 4 deals with data preprocessing. The difficult task to find a good model structure is then discussed in Section 5. Finally, two applications will be reviewed in Section 6.

3. Choice of Input Signals

The requirement from the previous section that the data should be informative means for open loop operation that the input should be persistently exciting (p.e.) of a certain order; i.e. that it contains sufficiently many distinct frequencies. This leaves a substantial amount of freedom for the actual choice, and we shall in this section discuss good and typical choices of input signals.

For the identification of linear systems, there are three basic facts that govern the choices:

1. The asymptotic properties of the estimate (bias and variance) depend only on the *input spectrum*-not the actual waveform of the input.
2. The input must have limited amplitude: $\underline{u} \leq u(t) \leq \overline{u}$. The *crest factor* measures how well a given signal utilizes such a given amplitude span: It is essentially defined as the maximum amplitude divided by the standard deviation of the signal.
3. Periodic inputs may have certain advantages.

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Biographical Sketch

Lennart Ljung received his Ph.D. in Automatic Control from the Lund Institute of Technology in 1974. Since 1976 he has been Professor of the chair of Automatic Control In Linkoping, Sweden, and is currently Director of the Competence Center “Information Systems for Industrial Control and Supervision” (ISIS). He has held visiting positions at Stanford and MIT and has written several books on System Identification and Estimation. He is an IEEE Fellow and an IFAC Advisor as well as a member of the Royal Swedish Academy of Sciences (KVA), a member of the Royal Swedish Academy of Engineering Sciences (IVA), and an Honorary Member of the Hungarian Academy of Engineering. He has been elected to the US National Academy of Engineering in 2004. He has received honorary doctorates from the Baltic State Technical University in St Petersburg, and from Uppsala University. In 2002 he received the Quazza Medal from IFAC.