DECOUPLING CONTROL

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Contents

Summary

This article studies the problem of decoupling. This general term can be divided into two main sub-categories: dynamic and static decoupling. Dynamic decoupling is more general and guarantees that under any operating conditions, the manipulated variables influence independently the respective outputs. Static decoupling on the other hand is concerned only with the problem of steady-state, hence decoupling is guaranteed only

for a special choice of inputs - step changes and in general operating regime interactions between outputs occur.

The term decoupling generally means diagonal decoupling, i.e. each input/output is independent. When certain inputs/outputs are grouped together, various decoupling regimes can be obtained: block decoupling when some subsystems are independent, or triangular decoupling where some outputs influence the other outputs but not vice versa. The last part of the article is devoted to the decoupling issues in process control design where several simplifications lead to specialized control strategies.

The questions that will be answered when investigating decoupling control design are:

- Is it possible to decouple the given system?
- What kind of decoupling can be realized?
- What kind of controller is required ?
- What are consequences and undesired effects of decoupling ?

1. Introduction

Multi-input multi-output control systems occur very frequently in practice and constitute difficult control problems for the operating personnel.

Consider for example a distillation column where control over concentration of multiple products is desired and the operators can change various flows within the column. However, change of any manipulated variable influences all concentrations and it is difficult to manipulate all flows simultaneously to obtain the desired effect. Other common examples may include control of flying objects (aircrafts, helicopters, landing modules), electrical devices (turbines), and many more, where in many situations it is required to change several variable simultaneously to produce desired values of controlled variables.

A suitable operating practice is to find pairs of the input-output variables where the corresponding manipulated variable has the maximum effect on the output variable and to design simple controllers for each pair. In this phase of control design, techniques of loop selection are used. If the choice of input-output pair is difficult to perform, it is possible to insert a precompensator that makes the system diagonally dominant. Pursuing this idea further, a some way of compensation can be sought that makes the compensated closed-loop system totally diagonal. This is called decoupling. In this case each output variable is influenced only by one manipulated variable and the problem of multivariable control is then reduced to a series of singlevariable control problems that are more easily solved.

This article studies various categories and special cases of decoupling. In the first part multivariable system description and some selected compensation strategies are introduced that will be used throughout the article. Next, motivation for decoupling control is shown on control of a heat exchanger. Next sections serve a theoretical foundations of the results presented for the heat exchanger. These include dynamic, static decoupling and process control decoupling strategies: The first of them deals with dynamic decoupling. It begins with the conditions that are to be satisfied in order to achieve decoupled closed-loop system. Selected choices of dynamic compensation are then discussed starting from the most versatile dynamic feedback with input dynamics and then showing which are conditions that enable to use less general compensation strategies as linear state feedback and linear output feedback. The first subsections treat diagonal decoupling, afterwards also block-diagonal and triangular decoupling strategies are proposed. Next section deals with static decoupling and shows again conditions that assure it. Finally, the last section gives an overview of process control decoupling strategies.

1.2. Preliminaries

See also Chapter *Control of Linear Multivariable Systems* for the definition of further symbols and notations.

1.1.1. Multivariable System Description

A linear time invariant deterministic continuous-time multivariable system with p outputs y, m inputs u, and n states x can be described as (see *Description and Classification.*)

- State-space system $\left[A, B, C, D\right]$ (see *Canonical State Space Representation and Feedback.*)

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where $\left[A, B, C, D\right]$ are matrices of dimensions $A[n,n]$, $B[n,m]$, $C[p,n]$, and $D[p,m]$.

- Transfer function $T\left(s\right)$

$$y = T\left(s\right)u\,, \tag{2}$$

where $T(s)[p,m]$ is a matrix with entries transfer functions corresponding to the appropriate input/output pair.

- Matrix fraction description $R(s)$, $P(s)$ with z being the partial state (see *Polynomial and Matrix Fraction Description.)*

$$y = R\left(s\right)z, \quad u = P\left(s\right)z, \tag{3}$$

where $R(s)$, $P(s)$ are polynomial matrices of dimensions $P[m,m]$, $R[p,m]$ respectively, $P(s)$ is column proper with column degrees $d_i, i = 1,...m$.

The relation between these forms is

$$T(s) = C(sI - A)^{-1} B + D = R(s) P(s)^{-1} \tag{4}$$

1.1.2. Control Structures Used for Decoupling

The controllers that will be used for decoupling design can be divided into four categories (reference output denoted by *r*):

- Static feedforward (SFF) $u = Gr$
- Constant linear output feedback (LOF) $u = Hy + Gr$
- Linear state feedback (LSF) $u = Fx + Gr = H(s)y + Gr$
- Linear state feedback with input dynamics (LSFID)

1.1.3. Square and Non-square Systems

Decoupling is mainly used for square systems, i.e. systems with equal number of inputs and outputs. In general, we can distinguish three cases: $m > p$, $m < p$, and $m = p$.

If there are more outputs than inputs $(m < p)$, the fundamental problem is to define decoupling. A possible approach is to define subsystems where some subset of inputs influence only a subset of outputs without affecting other outputs, hence to divide the original (m, p) system into smaller (m_i, p_i) subsystems.

The methods given in the subsequent sections can then be applied to each subsystem. If it is possible to decouple all subsystems independently, then it is possible to decouple them simultaneously.

The situation when there are more inputs than outputs $(m > p)$is in fact a more useful as with the equal number of inputs and outputs. Clearly, in the former case there are more degrees of freedom to obtain a decoupled closed-loop system.

1.1.4. Problem Formulation

The requirement for a various versions of dynamic decoupling is closely related to constraining the closed-loop transfer function to some particular form. More specifically, the system will be diagonally decoupled if its transfer function matrix is diagonal and of full rank. The requirement of triangularly decoupled system is equivalent to the transfer function matrix being (lower) triangular and full rank. Finally, a system is said to be block decoupled if its transfer function matrix is block diagonal and has full rank.

2. Control of a Heat Exchanger

2.1. Model

To explain the ideas of decoupling, let us start with an example of a heat exchanger control. We consider a heat exchanger in form of a sphere tank with the diameter r, supplied by input flow of cold liquid with temperature ϑ_0 and electrically heated. Within the exchanger, level h and temperature ϑ are to be controlled, the manipulated variables are input flowrate q_0 and heat power ω, and the disturbance is the inlet temperature ϑ_0. Output flowrate of the heated medium q_1 depends on the height of the liquid in the tank and is given as $q_1 = k\sqrt{h}$. We assume for simplicity that the height of the liquid is smaller than r, the exchanger is well mixed and insulated, and that heat capacity c_p and density ρ are constant.

With these assumptions, material and energy balances of the heat exchanger are as follows

$$q_0\rho = q_1\rho + \rho\frac{dV}{dt} \tag{5}$$

$$q_0\rho c_p\vartheta_0 + \omega = q_1\rho c_p\vartheta + \rho c_p\frac{dV\vartheta}{dt} \tag{6}$$

where V is the volume of the liquid inside of the tank depending on actual liquid level h and for a sphere it is given as

$$V(h) = \frac{1}{3}\pi h^2(3r - h) \tag{7}$$

After some manipulations, the differential equations describing dynamic behavior of controlled variables are given as

$$\frac{dh}{dt} = \frac{1}{V'(h)}\left(q_0 - k\sqrt{h}\right) \tag{8}$$

$$\frac{d\vartheta}{dt} = \frac{1}{V(h)}\left[\frac{\omega}{\rho c_p} + q_0\left(\vartheta_0 - \vartheta\right)\right] \tag{9}$$

where $V'(h) = \frac{dV(h)}{dh}$. The actual values of all parameters have been chosen as $r = 2$ m, $\rho = 1kg/m^3$, $c_p = 4.2J/kg/K$, $k = 0.05m^{2.5}/\text{s}$. The steady state values of input signals are $q_0^s = 0.05m^3/\text{s}$, $\omega^s = 1J/\text{s}$, $\vartheta_0^s = 300\,\text{K}$.

From the control point of view, the heat exchanger represents nonlinear 2-input, 2-output system of a triangular structure as the height h is not coupled with the heat power ω. Response of the exchanger to step changes in manipulated variables is shown in Fig. 1. As expected, the height is not influenced by the step change of the heat power at time $t = 1000$ s whereas temperature is influenced by both input flowrate q_0 change at time $t = 20$ s and the heat power at time $t = 1000$ s.

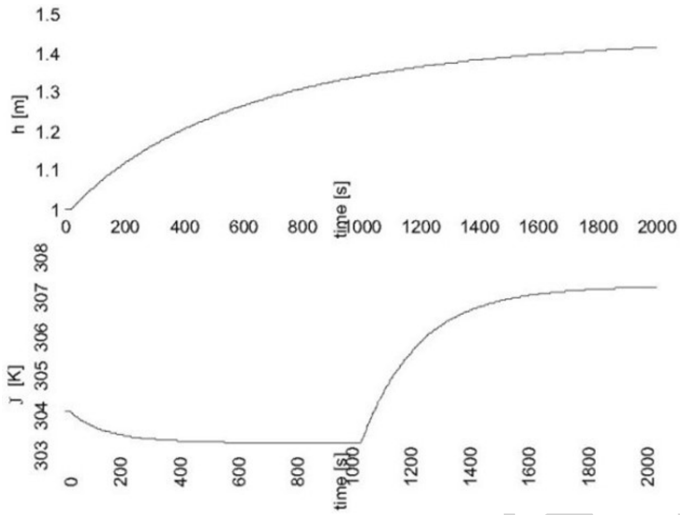

Figure 1: Step responses of the heat exchanger

Steady state of the exchanger is given as

$$h^s = \left(\frac{q_0^s}{k}\right)^2 \tag{10}$$

$$\vartheta^s = \vartheta_0^s + \frac{\omega^s}{q_0^s \rho c_p} \tag{11}$$

For the control design purposes, a linearized model in deviation variables will be developed. Let us denote the deviations from steady state $u_1 = q_0 - q_0^s$, $u_2 = \omega - \omega^s$, $x_1 = h - h^s$, $x_2 = \vartheta - \vartheta^s$ and assume that the inlet temperature is constant.

After the Taylor expansion of the nonlinear elements, the original nonlinear model can be approximated in the neighborhood of the steady state by a linear state space model of the form

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{12}$$

or, in the transfer function matrix as

$$T(s) = \begin{pmatrix} \dfrac{b_{11}}{s - a_{11}} & 0 \\ \dfrac{b_{21}}{s - a_{22}} & \dfrac{b_{22}}{s - a_{22}} \end{pmatrix} \tag{13}$$

We can see that the system states are already decoupled and that the interactions are caused only by the matrix B. Moreover, as in the nonlinear case, the transfer function matrix $T(s)$ is triangular and the output y_1 is decoupled from the input u_2.

2.2. Static Decoupling

Let us consider the simplest decoupling strategy – static decoupling. In this case we desire to have a situation when a step change in the static, steady state level of each input is reflected by a corresponding change in the steady-state level of the corresponding output and only that output.

A simple feedforward solution as described in a more detail in section 4 specifies a constant precompensator matrix G that manipulates reference signals and is given as the inverse of the matrix $T(0)$

$$G = \begin{pmatrix} -\dfrac{b_{11}}{a_{11}} & 0 \\ -\dfrac{b_{21}}{a_{22}} & -\dfrac{b_{22}}{a_{22}} \end{pmatrix}^{-1} = \begin{pmatrix} -\dfrac{a_{11}}{b_{11}} & 0 \\ \dfrac{a_{11}b_{21}}{b_{11}b_{22}} & -\dfrac{a_{22}}{b_{22}} \end{pmatrix} \tag{15}$$

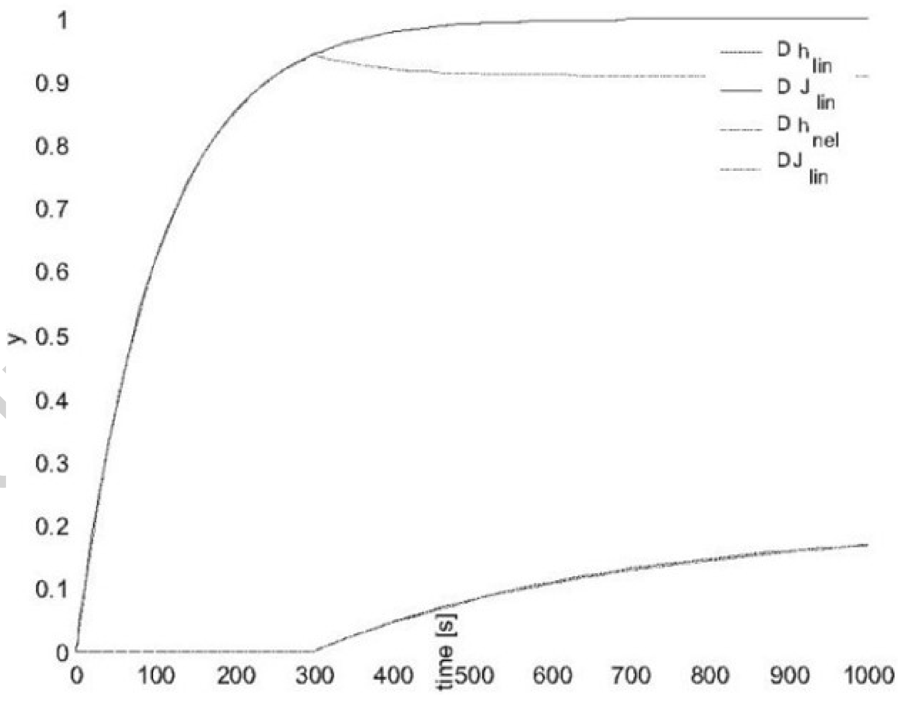

Figure 2: Static decoupling control design for the heat exchanger

Figure 2 compares behavior of the linear and nonlinear models. For the linear model, this control design resulted in full dynamic decoupling (not only in steady states, but also during transients, see section 3) as the coupling effects are not caused by the system matrix A. The nonlinear model, however, differs from the linear model significantly at time $t = 300$ s and still exhibits some small coupling effects.

2.3. Dynamic Decoupling

Even if we have seen that dynamic decoupling is not necessary in this case, we can design a controller that guarantees decoupling behavior all the time also for more complex processes.

The most general procedure is linear state feedback with input dynamics described by the algorithm 1 in section 3.1. The closed-loop poles have been specified as

$$\hat{P}(s) = \mathrm{diag}(s + 0.05, s + 0.05) \tag{16}$$

and the results are both for linear and nonlinear models are shown in Fig. 3. We can see that response speed is much improved. However, the problem with nonlinear model still remains.

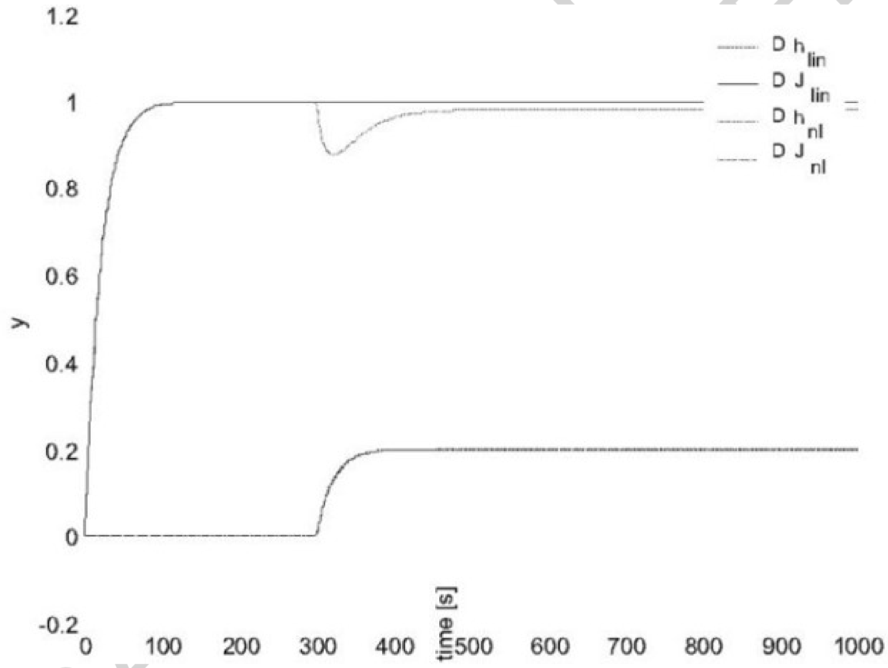

Figure 3: Dynamic decoupling design for the heat exchanger

Very often, the real input signals are constrained in the magnitude. The effect of the constraints is much more important in the decoupled closed-loop system.

Fig. 4 shows both manipulated and controlled variables when upper level constraint has been imposed as $u_1 < 0.05$. We can observe, that even in the linear case, the closed-loop behavior exhibits couplings at time $t = 300$ s.

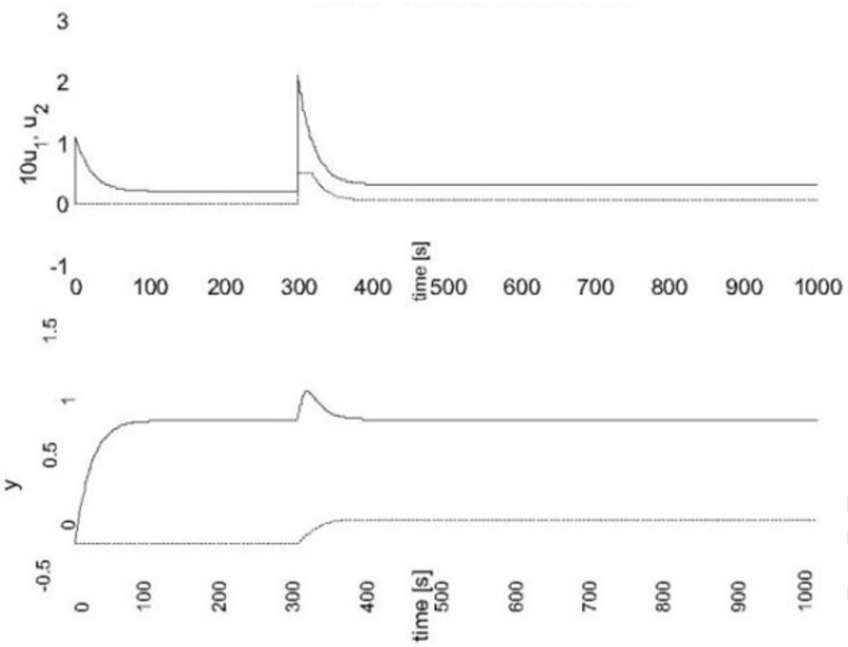

Figure 4: Saturation effects from input clipping for the heat exchanger

There are several approaches that can counteract this situation. In Fig. 5, the second input is scaled down by the same amount as the first input. We can see that decoupled behaviour is again established.

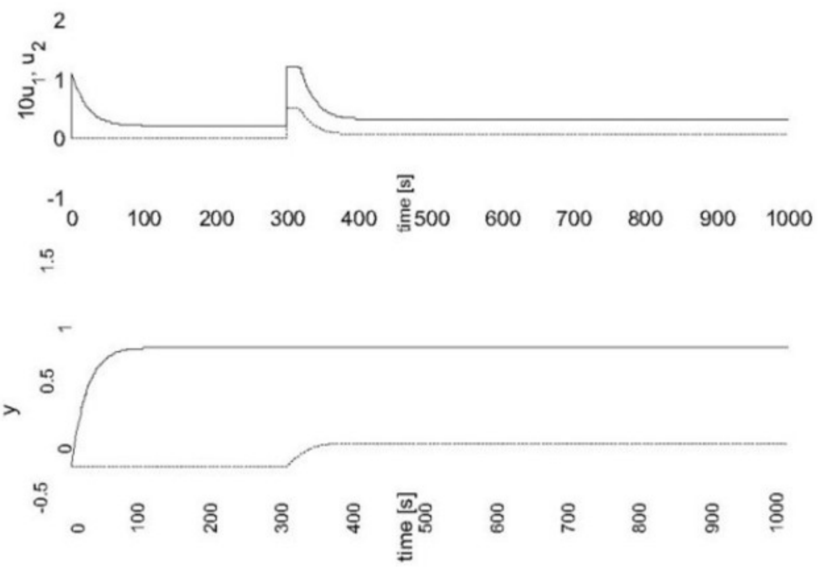

Figure 5: Input scaling in the presence of constraints for the heat exchanger

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Biographical Sketch

Miroslav Fikar received the Ing. degree from the Faculty of Chemical Technology (CHTF), Slovak University of Technology in Bratislava in 1989, Dr. in 1994, and Doc. (Associate Professor) in 2000. Since 1989 he has been with the Department of Process Control CHTF STU. In 1999 he was awarded the Alexander von Humboldt fellowship. He also worked at Technical University Lyngby, Technische Universitaet Dortmund, CRNS-ENSIC Nancy, Ruhr Universitaet Bochum, and others. The publication activity of Dr. Fikar includes more than 70 works and he is co-author of three books. In his scientific work, he deals with predictive control, constraint handling, system identification, optimization, and process control.