ADAPTIVE DUAL CONTROL

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Contents

Summary

The paper discusses the solution to the optimal adaptive control problem over an extended time horizon. This leads to a controller that has dual features, i. e.; it uses control actions as well as probing actions. The solution of the optimal dual control problem is intractable from a computational point of view. Approximations to obtain simpler suboptimal dual controllers are thus important.There are many different approaches concerning how to obtain suboptimal dual controllers. Many of the approximations use a cautious controller as a starting point and introduce different active probing features. This can be done by including a term in the loss function that reflects the quality of the estimates of the parameters of the process. To introduce a dual feature this term must be a function of the control signal that is going to be determined and it should also contain information about the quality of the parameter estimates. The suboptimal dual controllers should also be such that they easily can be used for higher-order systems.

1. Introduction

In all control problems there are certain degrees of uncertainty with respect to the process to be controlled. The structure of the process and/or the parameters of the process may vary in an unknown way. There are several ways to handle these types of uncertainties in the process. Feedback in itself makes the closed loop system, to some extent, insensitive against process variations. Fixed parameter controllers can also be designed to make the closed loop system robust against process variations. Such controllers must, by nature, be conservative in the sense that the bandwidth of the closed loop system has to be decreased to reduce the influence of the variation in the process. Another way to handle uncertainties is to use an adaptive controller. In the adaptive controller, there are attempts to identify or estimate the unknown parameters of the process.

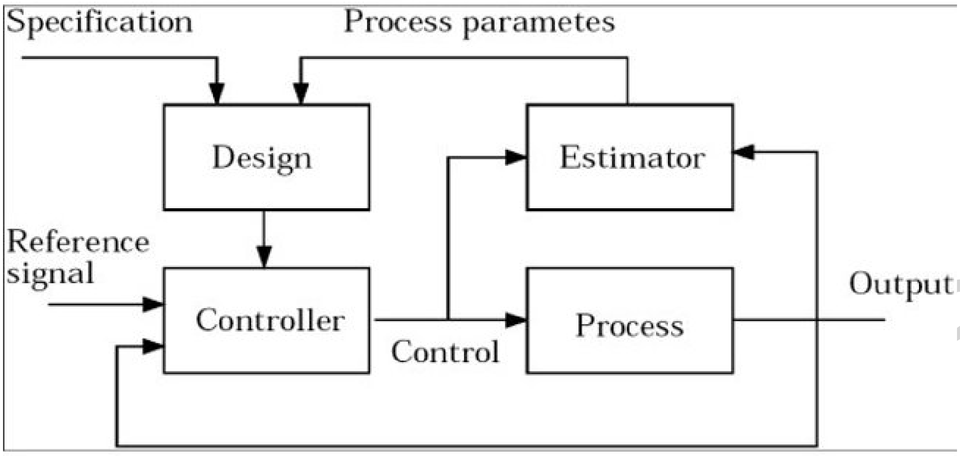

Figure 1: Self-tuning adaptive control system.

Most adaptive controllers have the structure shown in Figure 1, which is a self-tuning adaptive control system. The inputs and the outputs of the process are fed to the estimator block, which delivers information about the process to the controller design block. The design block uses the latest process information to determine the parameters of the controller. The adaptive controller thus consists of an ordinary feedback loop and a controller parameter-updating loop. Different classes of adaptive controllers are obtained depending on the process information that is used in the controller and how this information is utilized.

To obtain good process information it is necessary to perturb the process. Normally, the information about the process will increase with the level of perturbation. On the other hand, the specifications of the closed loop system are normally such that the output should vary as little as possible. There is thus a conflict between information gathering and control quality. This problem was introduced and discussed by A.A. Feldbaum in a sequence of four seminal papers from 1960 and 1961, see the references. Feldbaum's main idea is that in controlling the unknown process it is necessary for the controller to have dual goals. First, the controller must control the process as well as possible. Second, the controller must inject a probing signal or perturbation to get more information about the process. By gaining more process information, better control can be achieved in future time. The compromise between probing and control or in Feldbaum's terminology investigating and directing leads to the concept of *dual control*. Feldbaum showed that a functional equation gives the solution to the dual control problem. The derivation is based on dynamic programming and the resulting functional equation is often called the *Bellman equation*. The solution to this equation is intractable from a numerical point of view and only a few very simple examples have been solved, analytically or numerically. There is thus a great need for different approximations that can lead to simpler suboptimal solutions with dual features. In the suboptimal dual controllers, it is necessary to introduce both cautious and probing features. Both parts of the control action can be obtained in numerous ways and different proposed schemes would be classified into a handful of principles. This article gives an overview of adaptive dual control. To do so, it is also necessary to introduce some concepts from the general field of adaptive control.

2. Stochastic Adaptive Control

To formulate the adaptive dual control problem we must specify the model for the process, the admissible control signals, and the specifications (loss function) for the closed loop system. Introduce the following notations: $y(k)$ is the process output, $u(k)$ is the control signal, $\theta(k)$ is a vector of the unknown parameters of the process, $\hat{\theta}(k)$ is the current estimate of the process parameters, and $P(k)$ is the parameter uncertainty. Inputs up to time $k-1$ and outputs up to time k are collected into the vector

$$Y_k = [y(k) \;\; y(k-1) \;\; u(k-1) \ldots y(0) \;\; u(0)] \tag{1}$$

It is assumed that the process is described by the discrete time model

$$y(k+1) = f(u(k), Y_k, \theta(k), \zeta(k)) \tag{2}$$

where $\zeta(k)$ is a stochastic process driving the process and/or the parameters of the process. The probability distribution of ζ is assumed known. This implies that the output at the next sampling instance: $k + 1$ is a possibly nonlinear function of the control signal to be determined at time k, some, not necessarily all, of the elements in Y_k and of the unknown process parameters. It is assumed that the function $f(\cdot)$ is known. This implies that the structure of the process is known but that there are unknown parameters, $\theta(k)$. The admissible controllers are causal functions $g(\cdot)$ of all information gathered up to time k, i. e. Y_k. If the parameters of the process are known the control signal at time k is also allowed to be a function of $\theta(k)$.

The performance of the closed loop system is measured by a loss function that should be as small as possible. Assume that the loss function to be minimized is

$$J_N = E\left\{ \frac{1}{N} \sum_{k=1}^{N} h(y(k), u(k-1), y_r(k), k) \right\} \tag{3}$$

where y is the process output, y_r is the reference signal, $h(\cdot)$ is a positive convex function, and E denotes mathematical expectation taken over the distribution of ζ. This is called an *N-stage criterion*. The loss function should be minimized with respect to the admissible control signals

$u(0)$, $u(1)$, ..., $u(N-1)$. A simple example of the loss function is

$$J_N = E\left\{ \frac{1}{N} \sum_{k=1}^{N} (y(k) - y_r(k))^2 \right\} \tag{4}$$

The parameters of the process can be described in several different ways, for instance, as

- Random walk
- Random walk with local and global trends
- Jump changes
- Markov chain

Random walk implies that the parameters are drifting due to an underlying stochastic process. In the Markov chain model, the parameters are changing between a finite number of possible outcomes. Depending on the type of variation of the process parameters, it is necessary to use different estimation methods. It is thus assumed that the parameter variation is described in stochastic terms where the probability distribution of the process is known. Different types of prediction error methods can be used to obtain the probability distribution of the parameters. If the process is linear in the parameters and if the parameter variations can be described by a Gaussian process then the distribution is fully characterized by the mean value $\hat{\theta}(k)$ and the co-variance matrix $P(k)$. The covariance matrix is used as a measure of the uncertainty of the parameter estimates. The future behavior of $P(k)$ depends on the choice of the control signal.

The model, with the description of its parameter variations, the admissible control laws, and the loss function are now specified. The adaptive control problem has been transformed into an optimization problem, where the control signals over the control horizon have to be determined. One of the difficulties in the optimization problem is to anticipate how the future behavior (or formally the behavior of the distribution function) of the parameter estimates will be influenced by the choice of the control signals. The controllers minimizing Eq. (3) are very different if $N = 1$ or if N is large. The stochastic adaptive control problem can be attacked in many different ways. Many adaptive controllers are based on the *separation principle*. This implies that the unknown parameters are estimated separately from the design part. The separation is sometimes optimal and is in other cases used as an assumption. The separation principle holds, for instance, for the Gaussian case and when the process is linear in the unknown parameters, and the loss function is a quadratic function.

Assume that for the known parameter case the optimal controller is

$$u(k) = g_{known}(Y_k, \theta(k)) \tag{5}$$

The simplest adaptive controller is thus obtained by estimating the unknown process parameters $\hat{\theta}(k)$ and then use them as if they were the true ones, i. e. use the controller

$$u(k) = g_{known}(Y_k, \hat{\theta}(k)) \tag{6}$$

An adaptive controller of this kind is said to be based on the *certainty equivalence principle*. Self-tuning controllers are, in general, of this kind. The control actions determined in the design block, when using the certainty equivalence principle, do not take any active actions that will influence the uncertainty. An optimal adaptive controller should also consider the quality of the parameter estimates when designing the controller. Poor estimates, or information, should lead to other control actions than good estimates. A simple modification of the certainty equivalence controller is obtained by minimizing the loss function in Eq. (3) which is only one step ahead. This leads to a controller that also uses the uncertainties of the parameter estimate. This type of controller is called a *cautious controller*. The cautious controller has the form

$$u_{cautious}(k) = g_{cautious}(Y_k, \hat{\theta}(k), P(k)) \tag{7}$$

The cautious controller obtained when the control horizon in Eq. (3) is $N = 1$, which is sometimes also called a myopic controller, since it is shortsighted and looks only one step ahead. The cautious controller hedges against poor process knowledge. A consequence of this caution is that the gain in the controller is decreased. With small control signals, less information will be gained about the process and the parameter uncertainties may increase, and even smaller control signals will be generated. This vicious circle leads to *turn-off* of the control. This problem mainly occurs for systems with strongly, time-varying parameters. An adaptive control scheme is sometimes also denoted *weakly dual;* if it uses the model uncertainties when deriving the control signal. The certainty equivalence and the cautious controllers do not deliberately take any measure to improve the information about the unknown process parameters. They are thus non-dual adaptive controllers. The learning is “accidental” or “passive”, i. e. there is no intentional probing signal introduced.

Example

Consider an integrator process in which the gain is changing in a stochastic way, i. e. we have the model

$$y(k) - y(k-1) = \theta(k)u(k-1) + e(k) \tag{8}$$

where $e(k)$ is white noise. The gain of the integrator is modeled as

$$\theta(k+1) = \varphi\theta(k) + \nu(k) \tag{9}$$

where φ is known and $\nu(k)$ is white noise.

The certainty equivalence controller that minimizes the variance of the output is given by

$$u(k) = -\frac{1}{\hat{\theta}(k+1)} y(k) \tag{10}$$

It is immediately clear that this controller is not good when $\hat{\theta} = 0$. The cautious controller is

$$u(k) = -\frac{\hat{\theta}(k+1)}{\hat{\theta}^2(k+1) + p_\theta(k+1)} y(k) \tag{11}$$

where p_θ is the uncertainty of the estimate $\hat{\theta}$. By including the parameter uncertainty, the gain in the controller is decreased when p_θ becomes large. The cautious controller is also less sensitive than the certainty equivalence controller to parameter errors when $\hat{\theta}(k+1)$

is small. The gain in the cautious controller approaches zero when p_θ increases, i. e. there is a possibility that the control action is turned off when the excitation of the process decreases. The cautious controller approaches the certainty equivalence controller when p_θ approaches zero.

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Bibliography

Allison B.J., Ciarniello J.E., Tessier P.J.C., and Dumont G.A. (1995). Dual Adaptive Control of Chip Refiner Motor Load. *Automatica*, **31**(8), 1169-1184. [This is an application of suboptimal dual control.]

Aström K.J. and Wittenmark B. (1971). Problems of Identification and Control. *Journal of Mathematical Analysis and Applications*, **34**, 90-113. [One example of a numerically calculated dual controller.]

Feldbaum A.A. (1960-1961). Dual Control Theory. I-IV. *Automation Remote Control*, **21**, 874-880; **22,** 1033-1039, 1-12, 109-11. [The seminal papers that introduce the concept of dual control.]

Filatov N. M. and Unbehauen H. (2000). Survey of Adaptive Dual Control Methods. *IEEE Proc. Control Theory Appl*. **147**, 118-128. [Survey article.]

Wittenmark B. (1975). Stochastic Adaptive Control Methods: A Survey. *Int. J. Control*, **21**, 705-730. [Survey article.]

Wittenmark B. (1995). Adaptive Dual Control Methods: An Overview. 5*th IFAC Symposium on Adaptive Systems in Control and Signal Processing*. pp. 67-72, Budapest, Hungary. Invited paper TFR 91-72. [Survey article.]

Biographical Sketch

Björn Wittenmark was born in Växjö, Sweden in 1943. He obtained the M.Sc. degree in Electrical Engineering in 1966 and his Ph.D. in Automatic Control in 1973, both from Lund Institute of Technology. Since 1989 he has been a Full Professor at the Department of Automatic Control at Lund Institute of Technology, Lund, Sweden. His main research interests are in the fields of adaptive control, digital control, and process control. He has written numerous papers in these areas and is the co-author of eight books. He has written the book *Computer Controlled Systems* and *Adaptive Control* (both co-authored with Karl J. Åström). Wittenmark became a Fellow of IEEE in 1991.