ADAPTIVE NONLINEAR CONTROL

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Summary

In the 1990's adaptive nonlinear control was one of the most active areas of control research. Using the approach called "backstepping," it is now possible to design adaptive controllers for systems with significant nonlinearities and large modeling uncertainties arising in several areas of mechanical, aerospace, electrical, and chemical engineering. This tutorial presents the most practical among the results of adaptive nonlinear control in a fashion that emphasizes the basic design steps and highlights fundamental nonlinear features.

1. Introduction

The term 'adaptive control' encompasses a family of algorithms developed for plants whose models contain unknown constant parameters. While 'adaptive linear control' is applicable to most time-invariant linear models, 'adaptive nonlinear control' is more recent and is restricted to special classes of nonlinear models. In both linear and nonlinear models the unknown parameters are assumed to appear linearly.

The adaptive control problem for nonlinear models is much more difficult than for linear models. In nonlinear models the state may escape to infinity in finite time, so that exponentially fast parameter estimation may not be fast enough for stabilization. Either the parameter update law is to be faster, or the non-adaptive version of the controller is to guarantee boundedness in the presence of unknown parameters and parameter estimation transients. The first possibility is addressed in the sections describing the 'tuning function design', while the 'modular design' section briefly describes the second possibility. Both designs are recursive and expand the idea of 'backstepping' which is explained on a non-adaptive example in the next section.

2. Backstepping

Backstepping is a design procedure applicable to nonlinear models in a 'triangular' form. It will now be illustrated using the system

$$\dot{x}_1 = x_2 + \varphi(x_1)^{\mathrm{T}}\theta, \quad \varphi(0) = 0 \tag{1}$$

$$\dot{x}_2 = u, \tag{2}$$

where θ is a *known* parameter vector and $\varphi(x_1)$ is a smooth nonlinear function. Our goal is to stabilize the equilibrium $x_1 = x_2 = 0$. First, the state x_2 is treated as a **virtual control** for the x_1-equation (1), and a **stabilizing function**

$$\alpha_1(x_1) = -c_1 x_1 - \varphi(x_1)^{\mathrm{T}}\theta, \quad c_1 > 0 \tag{3}$$

is designed assuming that $x_2 = \alpha_1(x_1)$ can be implemented. Since this is not the case, we define

$$z_1 = x_1 \tag{4}$$

$$z_2 = x_2 - \alpha_1(x_1). \tag{5}$$

The complete system (1), (2) is expressed in the z-coordinates as

$$\dot{z}_1 = \dot{x}_1 = x_2 + \varphi^{\mathrm{T}}\theta = z_2 + \alpha_1 + \varphi^{\mathrm{T}}\theta = -c_1 z_1 + z_2 \tag{6}$$

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = u - \frac{\partial \alpha_1}{\partial x_1}(x_2 + \varphi^{\mathrm{T}}\theta), \tag{7}$$

where $\dot{\alpha}_1$ is implemented analytically, without a differentiator. For the system (6)-(7) we now design a control law $u = \alpha_2(x_1, x_2)$ to render the time derivative of a Lyapunov function

$$V(x_1, x_2) = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{8}$$

negative definite:

$$\begin{aligned}\dot{V} &= z_1\left(-c_1 z_1 + z_2\right) + z_2\left[u - \frac{\partial \alpha_1}{\partial x_1}(x_2 + \varphi^{\mathrm{T}}\theta)\right]. \\ &= -c_1 z_1^2 + z_2\left[u + z_1 - \frac{\partial \alpha_1}{\partial x_1}(x_2 + \varphi^{\mathrm{T}}\theta)\right].\end{aligned} \tag{9}$$

A simple way to achieve negativity of $\dot{V}$ is to employ u to make the bracketed expression equal to $-c_2 z_2$ with $c_2 > 0$, namely,

$$u = \alpha_2(x_1, x_2) = -c_2 z_2 - z_1 + \frac{\partial \alpha_1}{\partial x_1}(x_2 + \varphi^{\mathrm{T}}\theta). \tag{10}$$

This simple choice may not be the best choice because it cancels some terms which may contribute to the negativity of $\dot{V}$. Backstepping design offers enough flexibility to avoid cancellation. However, for the sake of clarity, we continue with this control law, which yields

$$\dot{V} = -c_1 z_1^2 - c_2 z_2^2. \tag{11}$$

This means that the equilibrium $z = 0$ is globally asymptotically stable and the same is true about $x = 0$. The resulting closed-loop system in the z-coordinates is

$$\begin{bmatrix}\dot{z}_1 \\ \dot{z}_2\end{bmatrix} = \begin{bmatrix}-c_1 & 1 \\ 1 & -c_2\end{bmatrix}\begin{bmatrix}z_1 \\ z_2\end{bmatrix}. \tag{12}$$

This completes a non-adaptive backstepping design. An adaptive version of backstepping, called 'tuning function design', which intertwines parameter update design steps with those of control law design, is presented in the next section.

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Biographical Sketches

Petar V. Kokotovic, native of Belgrade, Yugoslavia, was invited to the United States in 1965 to present a series of seminars on control system design. Soon after he joined the University of Illinois, where he remained for 25 years and held the endowed Grainger Chair. In 1991 he joined the University of California, Santa Barbara, as Co-director of the newly formed Center for Control Engineering and Computation.

In the 1960's, Kokotovic developed the sensitivity points method, a precursor to adaptive control, still in use for automatic tuning of industrial controllers. In the 1970's, he pioneered singular perturbation techniques for multi-time-scale design of control systems and flight trajectories, which found widespread applications. One of them led to the discovery of a fundamental relationship between the slow and fast phenomena and strong and weak connections in dynamic networks. In the 1980's, Kokotovic and coworkers identified the main forms of adaptive systems instability and introduced redesigns which made adaptive controllers more robust. As a long-term industrial consultant, Kokotovic contributed to the design of the first computer controls for car engines at Ford, and to power system stability analysis at General Electric. Kokotovic's current research is in nonlinear control, both robust and adaptive. He initiated the development of a popular nonlinear recursive design back-stepping, and its use for robust nonlinear control of jet engines, as a part of a multi-university AFOSR-PRET Center.

Professor Kokotovic supervised 30 Ph.D. students and 15 postdoctoral researchers. With them he co-authored numerous papers and ten books, four of which appeared in 1995-96. Professor Kokotovic is a fellow of the IEEE and a member of the National Academy of Engineering. He received an Eminent Faculty Award, two Outstanding IEEE Transactions Paper Awards (1983 and 1993), and delivered the 1991 IEEE Control Systems Society Bode Prize Lecture. In 1990 he was awarded the triennial Quazza Medal by the International Federation of Automatic Control, and in 1995 he received the IEEE Control Systems Field Award.

Miroslav Krstic is Professor and Vice Chair in the Department of Mechanical and Aerospace Engineering at University of California, San Diego. Prior to moving to UCSD, he was Assistant Professor in the Department of Mechanical Engineering and the Institute of Systems Research at University of Maryland. He got his PhD in Electrical Engineering from University of California at Santa Barbara, under Petar Kokotovic as his advisor, and received the UCSB Best Dissertation Award. Krstic is an IEEE Fellow and has received the National Science Foundation Career Award, Office of Naval Research Young Investigator Award, and is the first recipient of the Presidential Early Career Award for Scientists and Engineers (PECASE) in the area of control theory. He is a recipient of several paper prize awards, including the George S. Axelby Outstanding Paper Award of IEEE Transactions on Automatic Control, and the O. Hugo Schuck Award for the best paper at American Control Conference.

Krstic is a co-author of the books „Nonlinear and Adaptive Control Design“ (Wiley, 1995), „Stabilization of Nonlinear Uncertain Systems“ (Springer-Verlag, 1998), and „Flow Control by Feedback“ (Springer-Verlag, 2002). He is a co-author of two patents on control of aeroengine compressors and combustors. He has served as Associate Editor for the „IEEE Transactions“ on Automatic Control}, „International Journal of Adaptive Control and Signal Processing“, „Systems and Control Letters“, and „Journal for Dynamics of Continuous, Discrete, and Impulsive Systems“. Krstic is a Vice President for Technical Activities and a member of the Board of Governors of the IEEE Control Systems Society. His research interests include nonlinear, adaptive, robust, and stochastic control theory for finite dimensional and distributed parameter systems, and applications to propulsion systems and flows.