MODEL-BASED PREDICTIVE CONTROL

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Contents

Summary

MBPC is a feedback-control methodology suitable to enforce efficiently hard constraints on the variables of the controlled system. It is shown that the method hinges upon a constrained open-loop optimal control problem along with the adoption of the so-called receding-horizon control strategy. In the important case of time-invariant linear saturated ANCBI systems, MBPC algorithms can be devised with the property of ensuring global feasibility/stability. Considerations on how to deal with disturbances and model uncertainties are also given. A presentation of a simplified form of MBPC, *viz*. the PCG, is finally discussed.

1. Introduction

Model-Based Predictive Control (MBPC) is conceptually a natural method for generating feedback control actions for linear and nonlinear plants subject to pointwise-in-time input and/or state-related constraints. A human being, for instance, while driving a vehicle, generates steering-wheel commands, by forecasting or *predicting* over a finite time-horizon, the (possible) vehicle state-evolutions, on the basis of vehicle current state and dynamics, and a *virtual* or potential steering-wheel command sequence.

Then, one, among such sequences, is sorted out, which fulfills safety constraints and meets performance requirements. Only a short initial portion of such a sequence is applied by the driver to the steering wheel, while its remaining part is discarded. After such an initial portion is applied, the driver repeats the whole operation by restarting predictions over a or *receded* time-horizon from the updated vehicle state as determined by the applied command. MBPC complies with the same logical scheme: the control sequence is computed by solving online, over a finite control horizon, an open-loop optimal control problem, given the plant dynamical model and current state. Though this computation hinges upon an open-loop control problem, MBPC yields a feedback-control action. Indeed, similarly to the driver behavior, in a discrete-time setting, only the first control of the open-loop control sequence is applied to the plant, and, according to the *receding horizon control* philosophy, the whole optimization cycle is repeated at the subsequent time-instant, based on the new plant-state.

Because it involves a control horizon made up by only a finite number of time-steps, MBPC can be often calculated online, by existing optimization routines, so as to minimize a performance index, in the presence of hard constraints on the time evolutions of input and/or state. MBPC ability of handling constraints is of paramount importance whenever constraints are part of the control design specifications. In fact, constraints are typically present in applications, as they stem from actuators' saturations and/or physical, safety or economical requirements. Despite the importance of constraints, there is a shortage of control methods for handling them effectively.

The main reason for the interest of control engineers in MBPC is therefore its ability to systematically and effectively handle hard constraints. An important observation in this connection is that, in contrast to MBPC, in feedback-control systems of more traditional type, *e.g*., LQG or H_∞ control, constraints are indirectly enforced, by imposing, whenever possible, a conservative behavior at a performance-degradation expense. Other instances where MBPC can be advantageously used comprise unconstrained plants for which offline computation of a control law is a difficult task as compared with online computations via receding-horizon control.

MBPC appears to have been proposed independently by several people, more or less simultaneously. It is not an easy matter to trace back its origin by looking at dates of related publications as the pioneers were mostly control practitioners who implemented MBPC well before the first publications appeared in the late seventies/early eighties. The early motivations for proposing MBPC were essentially twofold. On one side, the emphasis was on a control methodology, which would be applied to problems for which standard industrial controllers, *e.g*., PID, were inadequate.

Such a control methodology had to be based on intuitive concepts and offer ease of tuning. Within this context, constraint handling and optimality were not the main goals. On the other side, the emphasis was on an optimal plant operation under constraints, and control signal computations by repeatedly solving in real-time linear programming problems. To what nowadays we call MBPC, early publications gave various names, *e.g*., *Dynamic Matrix Control*, *Model Predictive Heuristic Control*, *Receding Horizon Feedback Control*, *Heuristically Enhanced Feedback Control*. A few patents related to early MBPC techniques were released starting from 1976.

Among the advanced control methodologies, MBPC is the one, which has made the most significant impact on industrial control engineering. So far, it has been applied mainly in the petrochemical industry, even if it is being increasingly introduced in other sectors of the process industry. The main reason for its success in these applications are:

1. The processes are typically slow so that there is enough time for the online required computations;
2. MBPC can take into account actuator limitations;
3. Relatively to conventional control, MBPC allows operation closer to constraints, which often yields more profitable production.

The success of MBPC may even become more significant in the future as an increasing use of nonlinear dynamic models is taking place in the process industry. These models are obtained by mathematically describing the chemical and physical transformations occurring inside the process. Then, there is a clear potential for future synergy of nonlinear models with MBPC in that nonlinear models can provide more accurate predictions of process behavior in a nonlinear regime.

Significant use of MBPC has also been made in adaptive control during the last two decades. Adaptive control of non-minimum phase plants required in fact the use of underlying control laws more sophisticated than minimum-variance control but simple enough to be synthesized in real-time on the grounds of continuously updated plant identified models. Remarkable examples of MBPC use in adaptive control are GPC and MUSMAR, both developed during the eighties.

The presentation of MBPC given hereafter aims at enlightening the main features of the approach, related well-established feasibility/stability constructive arguments, and current open problems. Consideration will be also given to the *command governor*, a specific control architecture of practical interest, which, though introduced independently of MBPC, in its recent developments has taken advantage of using conceptual tools of predictive control. For more specialized topics, the reader is referred to the three article level contributions dealing with MBPC, *viz.*:***"MBPC for Linear Systems"****; **"MBPC for Nonlinear Systems"**; and **"Adaptive Predictive Control"**.*

The presentation is organized as follows: Section 0 sets up the general ingredients of the constrained open-loop optimal control problem underlying any MBPC scheme. Section 0 describes the earliest and simplest form of a stabilizing MBPC algorithm. Section **Error! Reference source not found.** introduces a convenient form of a set-membership (ellipsoidal) terminal state-constraint devised so as to improve in terms of feasibility the algorithm of Section **Error! Reference source not found.**.

Section **Error! Reference source not found.** extends the scheme of Section **Error! Reference source not found.** by considering a state-dependent ellipsoidal constraint, which allows one to get *global* feasibility/stability whenever such a property is achievable in principle. Section **Error! Reference source not found.** describes how to deal with constant disturbances and nonzero setpoints, as well as model uncertainties of polytopic type. Section **Error! Reference source not found.** describes predictive reference governors. In Section **Error! Reference source not found.**, a brief assessment of the current status of MBPC concludes the contribution.

2. The Constrained Open-Loop Optimal Control (COLOC) Problem

In MBPC, the system to be controlled (plant) is usually represented by an ordinary differential equation. However, as the control is normally piecewise constant, the plant is most of the times described in terms of a difference equation

$$x(k+1) = \varphi(x(k), u(k)) \tag{1}$$

$$y(k) = \eta(x(k)) \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state at time k, $u(k) \in \mathbb{R}^m$ the input, $y(k) \in \mathbb{R}^p$ a state-related vector connected to performance requirements (see Eq. (6) below), φ is assumed to be continuous at the origin with ($\varphi(0_X, 0_U) = 0_X$) and $\eta(0_X) = 0_Y$. The plant input and state sequences are required to satisfy the constraints

$$u(k) \in U \tag{3}$$

$$x(k) \in X \tag{4}$$

where, usually, U is a convex and compact subset of $\mathbb{R}^m$, and X is a convex and closed subset of $\mathbb{R}^n$, both sets containing the origin in their interior. For the event (x, t) (*viz*., for state x at time t), the cost is defined by

$$J(x, t, \mathbf{u}) = \sum_{k=t}^{t+N-1} l(x(k), u(k)) + L(x(t+N)) \tag{5}$$

where $\mathbf{u} := \{u(t), u(t+1), \ldots, u(t+N-1)\}$ and $x(k) = x^{\mathbf{u}}(k; (x, t))$, the latter notation denoting the state at time k resulting from state x at time $t \le k$ and a control sequence $\mathbf{u}$. The terminal time $t + N$ increases with time t and, consequently, is referred to as a *receding horizon*. Various choices for the instantaneous loss l and the terminal loss L are in principle possible. However, according to the usual MBPC choice, hereafter the loss functions will be taken to be quadratic

$$\left.\begin{aligned} l(x(k), u(k)) &:= \| y(k) \|^2_{\psi_y} + \| u(k) \|^2_{\psi_u} \\ L(x(t+N)) &:= \| x(t+N) \|^2_{\psi_N} \end{aligned}\right\} \tag{6}$$

where $\| v \|^2_{\psi} := v'\psi v$, the prime denotes transpose, $\psi_y = \psi_y' > 0$, $\psi_u = \psi_u' > 0$ and $\psi_N = \psi_N' \ge 0$. In general, a terminal-state constraint

$$x(t+N) \in X_N \tag{7}$$

is also imposed.

At the event (x, t), the COLOC problem $P(x, t)$ is to find, provided it exists, the optimal (*virtual*) control sequence

$$\mathbf{u}^{\circ}(x, t) := \{u^{\circ}(t; (x, t)), u^{\circ}(t+1; (x, t)), \ldots, u^{\circ}(t+N-1; (x, t))\} \tag{8}$$

which minimizes $J(x, t, \mathbf{u})$ subject to the control, state and terminal-state constraints, and yields the value function

$$V(x,t) := J(x,t,\mathbf{u}^{\circ}(x,t)) \tag{9}$$

According to the receding-horizon mode of operation, only the first control $u°(t; (x, t))$ is applied to the plant input at time t. In such a way, a feedback-control action is obtained

$$u(t) = c(x,t) := u^{\circ}(t;(x,t)) \tag{10}$$

Since $\varphi(\cdot,\cdot)$, $\eta(\cdot)$, $l(\cdot,\cdot)$ and $L(\cdot)$ are time-invariant, problems $P(x, t)$ are time-invariant in that

$V(x, t) = V(x, 0)$ and $c(x, t) = c(x, 0)$. Consequently, it suffices at each event (x, t) to solve $P_N(x) := P(x, 0)$. Problem $P_N(x)$ is therefore as follows:

$$P_N(x): \qquad V_N(x) \;=\; \min_{\mathbf{u}}\{J_N(x,\mathbf{u}) \mid \mathbf{u}\in U_N(x)\} \tag{11}$$

$$J_N(x,\mathbf{u}) \;:=\; \sum_{k=0}^{N-1} l(x(k),u(k)) + L(x(N)) \tag{12}$$

where $\mathbf{u} = \{u(0), u(1), \ldots, u(N-1)\}$, $x(k) = x^{\mathbf{u}}(k; (x, 0))$ and $U_N(x)$ is the set of *feasible* control sequences, *viz*. sequences satisfying the control, state and terminal-state constraints. Because N is finite, the minimum exists provided that $\varphi(\cdot,\cdot)$ and $h(\cdot)$ are continuous, U compact, X and X_N closed, and $U_N(x)$ non-empty. At the event (x, t), $P_N(x)$ is solved yielding the optimizing (virtual) control sequence

$$\mathbf{u}^{\circ}(x) = \{u^{\circ}(0;x), u^{\circ}(1;x), \ldots, u^{\circ}(N-1;x)\} \tag{13}$$

the optimal (virtual) state trajectory

$$\mathbf{x}^{\circ}(x) = [x^{\circ}(0;x) = x, x^{\circ}(1;x), \ldots, x^{\circ}(N;x)\} \tag{14}$$

and the value function

$$V_N(x) = J_N(x,\mathbf{u}^{\circ}(x)) \tag{15}$$

The first control in the optimizing sequence $\mathbf{u}^{\circ}(x)$ is applied to the plant input at the time t, and the MBPC action results

$$c_N(x) = u^{\circ}(0;x) \tag{16}$$

It is to be underlined that MBPC computes numerically online at the event (x, t) the optimal control action Eq. (16) rather than computing offline the optimal control law $c_N(\cdot)$. It would be more convenient to explicitly compute offline, once for all, $c_N(\cdot)$ via dynamic programming. As this is usually very hard of even impossible, MBPC computes at the event (x, t) the optimal control action $c_N(x)$ rather than pre-computing the optimal control law $c_N(\cdot)$.

3. Zero Terminal-State MBPC

This can be regarded as the earliest and, conceptually, the simplest form of MBPC, which guarantees stability to the controlled system, whenever feasibility is satisfied. Here, $X_N = \{0_X\}$ and, hence, the terminal-state constraint is the equality constraint

$$x(N) = 0_X \tag{17}$$

Given the plant state x, here $U_N(x)$, the set of feasible control sequences, is the set of sequences, which drive the plant initial state x to the zero-state in N steps with no constraint violation. Assume that $U_N(x)$ is non-empty, and satisfaction of all other conditions for the existence of the optimizing control sequence $\mathbf{u}^\circ(x)$ in Eq.(13).

Then, asymptotic stability of the plant fed by the MBPC action can be easily proved by the following direct argument. Let $\bar{\mathbf{u}}(x)$ be the control sequence obtained from $\mathbf{u}^\circ(x)$ by deleting its first control and inserting the zero-control 0_U in its final position

$$\bar{\mathbf{u}}(x) := \{u^\circ(1;x), u^\circ(2;x), \ldots, u^\circ(N-1;x), 0_U\} \tag{18}$$

This is a control sequence again of length N which is feasible for $x^\circ(1;x) = \varphi(x, u^\circ(0;x))$. In fact, $x^\circ(1;x)$ is driven by $\bar{\mathbf{u}}(x)$ to 0_X in $N - 1$ steps, and held at 0_X at the N-th step because

$$\varphi(0_X, 0_U) = 0_X \tag{19}$$

Specifically, the state trajectory over N steps resulting from the initial state $x^\circ(1; x)$ and the control sequence $\bar{\mathbf{u}}(x)$ is

$$\bar{\mathbf{x}}(x) := \{x^\circ(1;x), \ldots, x^\circ(N-1;x), 0_X, 0_X\} \tag{20}$$

Moreover,

$$\begin{aligned} J_N(x^\circ(1;x), \bar{\mathbf{u}}(x)) &= J_N(x, \mathbf{u}^\circ(x)) - l(x, u^\circ(0;x)) \\ &= V_N(x) - \| y(0) \|^2_{\psi_y} - \| u(0) \|^2_{\psi_x} \end{aligned} \tag{21}$$

where $y(0) = \eta(x)$ is the initial plant output and $u(0) = c_N(x) = u^\circ(0;x)$ is the effective input supplied at time 0 by MBPC to the plant. Then, if $x(1) = \varphi(x, u(0))$ denotes the *effective* plant state at time 1 in the MBPC-controlled system, according to Eq. (11) and Eq. (21) we get

$$V_N(x(1)) \le V_N(x(0)) - \| y(0) \|_{\psi_y}^2 - \| u(0) \|_{\psi_u}^2$$

Going to the generic time k, $k = 0, 1, \ldots$, for the closed-loop system we have

$$V_N(x(k)) - V_N(x(k+1)) \ge \| y(k) \|_{\psi_y}^2 + \| u(k) \|_{\psi_u}^2 \qquad (22)$$

where $u(k)$, $x(k)$ and $y(k)$ denote, respectively, the effective plant input, state and output at time k in the MBPC-controlled system. Eq. (22) shows that $\{V_N(x(k))\}_{k=0}^{\infty}$ is a monotonically non-increasing sequence. Hence, being $V_N(x(k))$ nonnegative, as $k \to \infty$ it converges to $V_N(x(\infty))$, $0 \le V_N(x(\infty)) \le V_N(x(0))$. Consequently, summing both sides of Eq. (22) from $k = 0$ to $k = \infty$, we get

$$\infty > V(x(0)) - V(x(\infty)) \ge \sum_{k=0}^{\infty} \left[\| y(k) \|_{\psi_y}^2 + \| u(k) \|_{\psi_u}^2 \right] \qquad (23)$$

This, in turn, implies as $\psi_y > 0$ and $\psi_u > 0$

$$\lim_{k\to\infty} y(k) = 0_Y \quad \text{and} \quad \lim_{k\to\infty} u(k) = 0_U \qquad (24)$$

Then, under a detectability condition on Eq. (1) and Eq. (2), for $\forall x \in U_N(x) \neq \emptyset$ one can conclude asymptotic stability of the closed-loop system: stability that can be seen to be of exponential type if the plant is linear, *viz*.

$$\left.\begin{aligned} x(k+1) &= \Phi x(k) + Gu(k) \\ y(k) &= Hx(k) \end{aligned}\right\} \qquad (25)$$

The foregoing stability proof hinges upon the following two crucial points:

$$U_N(x) \neq \emptyset \qquad (26)$$

$$\mathbf{u}^{\circ}(x) \in U_N(x) \Rightarrow \bar{\mathbf{u}}(x) \in U_N(x^{\circ}(1; x)) \qquad (27)$$

Now, feasibility condition Eq. (26) can be lost in zero terminal-state MBPC, because of the need of driving the plant state to 0_X in a finite time. If the initial state is far from 0_X, this brings about the use of large plants' inputs which can violate possible input saturation constraints. Various approaches can be adopted so as to enlarge the set of admissible states (states such that Eq. (26) is fulfilled). *E.g.*, given a plant state $x \neq 0_X$ and an integer N_1 such that $U_{N_1}(x) = \emptyset$, one can always find, if the plant is controllable and only input saturation constraints are present, a possibly large but finite integer N_2, $N_2 > N_1$, so as to make $U_{N_2}(x)$ non-empty. The disadvantage with this approach is that N_2 can be too large, and, hence, the associated COLOC problem $P_{N_2}(x)$ too complex for an online solution for the available computing power and sampling time.

Hereafter, forms of MBPC will be presented which are particularly tailored for both solving the feasibility problem and yielding a highly performing closed-loop system. In so doing, for the sake of simplicity, we shall address linear plants of the form Eq. (23) in the presence of only input-saturation constraints. The reason for the latter choice is that, while hard bounds on the manipulated variables are typically dictated by physical constraints (e.g., limited power of actuators): state-related constraints are frequently of the "soft-type" and, hence, can be properly addressed by penalizing additional state-related variables in the cost. The reason for concentrating hereafter on linear plants is to simplify the exposition and call the reader's attention more directly on features, *e.g*. set-membership terminal constraints, which are also equally important in MBPC of nonlinear systems.

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Biographical Sketch

Edoardo Mosca obtained his Dr. Eng. degree in Electronics Engineering from the University of Rome "La Sapienza". He then spent four years, from 1964 to 1968, in industry where he worked on the research and development of advanced radar systems. Thereafter, he held academic positions at the University of Michigan, Ann Arbor, Michigan, and McMaster University, Ontario, Canada. Since 1972, he has been with the Engineering Faculty, University of Florence, Italy: from 1972 to 1975 as an Associate Professor, and since 1975 as a full Professor of Control Engineering. He is the author of more than 140 research papers spanning several diversified fields such as radar signal synthesis and processing, radio communications, system identification, adaptive, predictive, and switching supervisory control. He is the author of a book, *Optimal, Predictive, and Adaptive Control*, Prentice Hall, 1995. He is also the editor of the following journals: *European Journal of Control*, *International Journal of Adaptive Control and Signal Processing*, Wiley, and *IEE Proceedings-Control Theory and Applications*. He is a Council member of EUCA (European Union Control Association); the Italian NMO representative in IFAC (International Federation of Automatic Control); a Council member of IFAC; and a Fellow of the IEEE (Institute of Electrical and Electronics Engineers).