CONTROLS OF LARGE-SCALE SYSTEMS

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Summary

A system is sometimes considered to be large scale if it can be partitioned or decoupled into a number of subsystems, that is, small-scale systems. Another viewpoint is that a system is termed large scale if its dimensions are so great that conventional techniques of modeling, analysis, control, design, optimization, estimation, and computation fail to give reasonable solutions with reasonable efforts.

A third definition is based on the notion of *centrality*. Until the advent of large-scale systems, almost all control systems analysis and design procedures were limited to having system components and information flow from one point to another localized or centralized in one geographical location or center, such as a laboratory.

Thus, another definition is a system in which the concept of centrality fails. This can-be due to a lack either of centralized computing capability or of a centralized information structure. Large-scale systems appear in such diversified fields as sociology, business, management, the economy, the environment, energy, data networks, computer networks, power systems, flexible space structures, internet-based systems, transportation, aerospace, and navigational systems.

1. Historical Background

Since the 1950s, system theory has evolved from a semi-heuristic discipline directed toward the design and analysis of electronic and/or aerospace systems consisting of a handful of components into a very sophisticated theory capable of treating complex and large systems with myriad applications. The theory must deal not only with electronic and aerospace systems, the complexities of which have increased by several orders of magnitude, but also with a vast number of real-life systems in society, the economy, industry, and government.

Initially, system engineers attempted to cope with this increasing system complexity through the development of sophisticated numerical techniques in order to apply classical systems theory to large systems. This approach, however, soon reached a point of diminishing returns, and it became apparent that new system theoretical techniques would be necessary to handle large and complex systems. Although many such techniques are still being developed and require a great deal of fine-tuning, it is generally accepted that a key to the successful treatment of a large-scale system is to exploit fully its structural interconnection. This exploitation traditionally takes place in two ways: through the full use of, say, sparse matrix techniques or through the "decomposition" of a larger system into a finite number of smaller systems.

2. Modeling and Model Reduction

The first step in any scientific or technological study of a system is to design a mathematical model of the real problem. In any modeling tasks, two often conflicting factors prevail: simplicity and accuracy. On the one hand, if a system model is oversimplified, presumably for the sake of computational effectiveness, incorrect conclusions may be drawn from it in representing an actual real system. On the other hand, a highly detailed model can lead to unnecessary complications, and even if a feasible solution is attainable, the amount of detail generated may be so vast that further investigations on the system behavior are impossible and the practical value of the model becomes questionable. Clearly, a mechanism by which a compromise can be made between a complex, more accurate model and a simple, less accurate model is needed. Creating such a mechanism is not a simple undertaking. (See *Mathematical Modeling*.)

In the area of large-scale systems there have been three general classes of modeling techniques. These are *aggregation, perturbation,* and *descriptive variable* schemes. An aggregate model of a system is described by a "coarser" set of variables. The underlying reason for aggregating a system model is to retain the key qualitative properties of the system, such as stability, which can be viewed as a natural process through the second method of Lyapunov. In other words, the stability of a system described by several state variables is fully represented by a single variable — the Lyapunov function. Figure 1 presents a pictorial representation of the aggregation process. The system on the left is described by four variables (circles), and the system on the right represents an aggregated model in which two variables now describe the system. Variable 1, called $z1$, is an average of the first two variables of the full model, while the second aggregated variable $z2$ is an average of the third and fourth variables.

Another approach for large-scale system modeling is perturbation, which is based on ignoring certain interactions of the dynamic or structural nature of a system. Here again, however, the key system properties must not be sacrificed for the sake of reduced computations. Although both perturbation and aggregation schemes tend to reduce the computations needed and perhaps provide a simplification of structure, there has been no firm evidence that they are the most desirable for large-scale systems.

A new type of large-scale system modeling is the *descriptive variable* scheme. Here the fundamental principle is that the accuracy of a given large-scale system model is most likely preserved if the system is represented by the actual physical or economical variables that describe the operation of the system—hence the name *descriptive variable.*

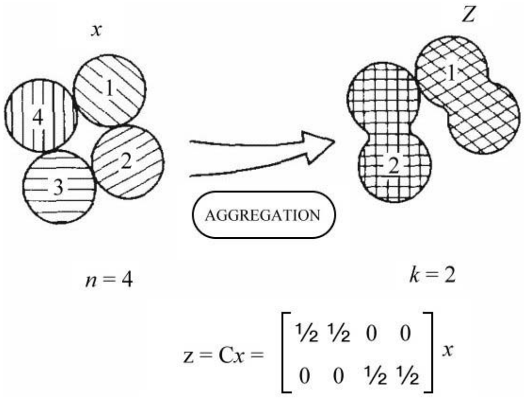

Figure. 1. Pictorial representation of the aggregation process.

This section is devoted to an examination of aggregation and perturbation, methods viewed as modeling alternatives for large-scale systems.

2.1. Aggregation

Aggregation has long been a technique for analyzing static economic models. The modern treatment of aggregation is based on the formulations of Malinvaud, which are shown in Fig. 2. In this diagram, X, Y, Z, and V are topological (or vector) spaces, and f represents a linear continuous map between the exogenous variable $x \varepsilon X$ and endogenous variable $y \varepsilon Y$. The aggregation procedures $h: X \to Z$ and $g: Y \to V$ lead to aggregated variables $z \in Z$ and $v \varepsilon V$. The map $k: Z \to V$ represents a simplified or an aggregated model. The aggregation is said to be "perfect" when k is chosen such that the relation

$$g\, f(x) = k\, h(x) \tag{1}$$

holds for all $x \in X$. The notion of perfect aggregation is an idealization at best, and in practice it is approximated through two alternative procedures, according to econometricians. These are (a) to impose some restrictions on f, g, and h while leaving X unrestricted and (b) to require Eq. (1) to hold on some subset of X.

2.1.1 Balanced Aggregation

One of the main shortcomings of model reduction methods is the lack of a strong numerical tool to go with the well-developed theory. For example, the minimal realization theory of Kalman offers a clear understanding of the internal structure of linear systems. The associated discussion on controllability, observability, and minimal realization often illustrate the points, but numerical algorithms are adequate only for low-order textbook examples. Furthermore, there has been little connection made between minimal realization, controllability and observability, and model reduction on the other hand. In this section we propose the *Principal Component Analysis* of statistics along with some algorithms for the computation of "singular value decomposition" of matrices to develop a model reduction scheme which makes the most controllable and observable modes of the system transparent. Under a certain matrix transformation, the system is said to be "balanced" and the most controllable and observable modes would become prime candidates for reduced-order model states.

Consider an asymptotically stable, controllable and observable linear time invariant system

(A, B, C) defined by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2a}$$

$$y(t) = Cx(t), \tag{2b}$$

where x, u, y are state, input and output vectors, and A, B, and C are $n \times n$ system, $n \times m$ input and $r \times n$ output matrices, respectively. The balanced matrix method is based on the simultaneous diagonalization of the positive definite controllability and observability Gramians of Eq. (2) which satisfy the following Lyapunov-type equations:

$$G_c A^T + AG_c + BB^T = 0 \tag{3}$$

$$G_0 A + A^T G_0 + C^T C = 0. \tag{4}$$

The balance approach of model reduction is essentially the computation of a similarity transformation matrix S such that both G_c and G_0 become equal and diagonal, that is, balanced. This transformation matrix is given by,

$$S = V\, D\, P\, \Sigma^{-1/2}, \tag{5}$$

where orthogonal matrices V and P satisfy the following symmetric eigenvalue/ eigenvector problems.

$$V^T G_c V = D^2 \tag{6}$$

and

$$P^T \left[(VD)^T G_0 (VD) \right] P = \Sigma^2 \tag{7}$$

$$\begin{aligned} \Sigma &= S^T G_0 S = S^{-1} G_c \left(S^{-1}\right)^T \\ &= \operatorname{diag}\left(\sigma_1, \sigma_2, \ldots, \sigma_n\right). \end{aligned} \tag{8}$$

Here D is a diagonal matrix like Σ. The diagonal elements of Σ have the property that $\sigma_1, \geq \sigma_2, \ldots, \geq \sigma_n > 0$ and are called *second-order modes* of the system. Using the transformation $\hat{x} = S^{-1}x$, one obtains the following full-order equivalent system,

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u = \left[\begin{array}{c|c} F & \hat{A}_{12} \\ \hline \hat{A}_{21} & \hat{A}_{22} \end{array}\right] \hat{x} + \left[\begin{array}{c} G \\ \hline \hat{B}_2 \end{array}\right] u \tag{9}$$

$$y = \hat{C}\hat{x} = \left[H \mid \hat{C}_2 \right] \hat{x}, \tag{10}$$

where

$$\hat{A} = S^{-1}AS, \hat{B} = S^{-1}B \text{ and } \hat{C} = CS \tag{11}$$

Now, if $\sigma_r \gg \sigma_r + 1$ for a given r, and internally dominant reduced-order model of order r can be obtained from Eqs. (9) and (10) by

$$\dot{z} = F_z = Gu \tag{12}$$

$$y = Hz, \tag{13}$$

where (F, G, K) matrices represent the desired reduced order model.

Although this partitioning of second-order models leading to a reduced and residual model are somewhat arbitrary, grouping the most controllable and observable modes together does provide a reasonable criterion for model reduction.

2.2. Perturbation

The basic concept of perturbation methods is the approximation of a system's structure by neglecting certain interactions within the model that lead to lower order. From a large-scale system modeling viewpoint, perturbation methods can be considered to be approximate aggregation techniques.

Two basic classes of perturbation are applicable to large-scale system modeling: "weakly coupled" models and "strongly coupled" models. This classification is not universally accepted, but a great number of authors have adapted it; others refer to these classes as non-singular (regular) and singular perturbations.

2.2.1. Weakly Coupled Models

In many industrial control systems certain dynamic interactions are neglected to reduce the computational burden for system analysis, design, or both. This is practiced in chemical process control and space guidance, for example, where different subsystems are designed for flow, pressure, and temperature control in an otherwise coupled process or for each axis of a three-axis attitude control system. The computational advantages of neglecting weakly coupled subsystems, however, are offset by a loss of overall system performance. In this section weakly coupled models for large-scale linear systems are introduced.

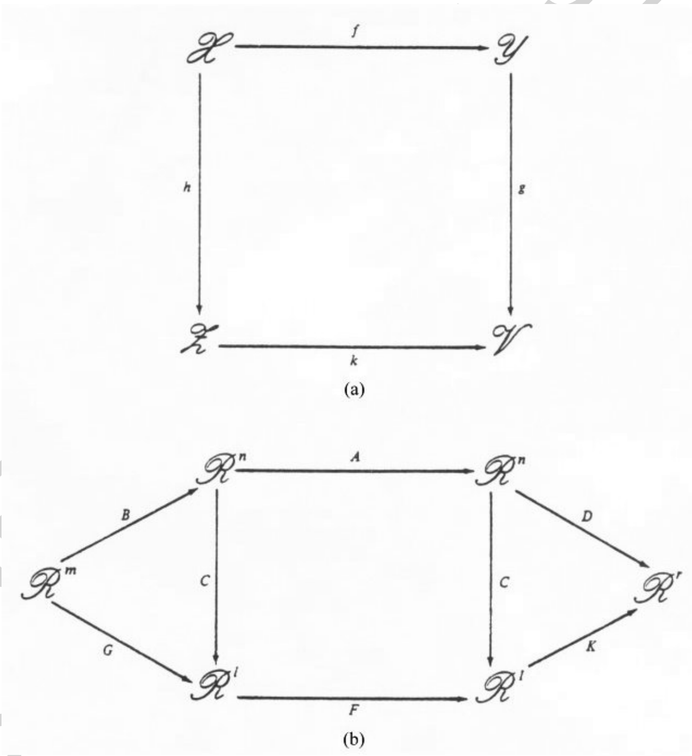

Figure 2. Pictorial representation of aggregation, (a) A static system; (b) a dynamic linear system.

Consider the following linear large-scale system split into two subsystems:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & \varepsilon A_{12} \\ \varepsilon A_{21} & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 & \varepsilon B_{12} \\ \varepsilon B_{21} & B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{14}$$

It is clear that when $\varepsilon = 0$, this system decouples into two subsystems,

$$\begin{aligned} \dot{\hat{x}}_2 &= A_1\hat{x}_1 + B_1\hat{u}_1 \\ \dot{\hat{x}}_2 &= A_2\hat{x}_2 + B_2\hat{u}_2 \end{aligned}, \qquad (15)$$

which correspond to two approximate aggregated models, one for each subsystem. In this way the computation associated with simulation and design will be reduced drastically, especially for large-system order n and k greater than two subsystems. In view of the decentralized structure of large-scale systems (Section 4), these two subsystems can be designed separately in a decentralized fashion, as shown in Fig. 3.

Research on weakly coupled systems has followed two main lines. The first is to set $\varepsilon = 0$ in Eq. (2) and try to find a quantitative measure of the resulting approximation when in fact $\varepsilon \neq 0$ under actual conditions. Such measures usually correspond to a loss of performance for a linear optimal control problem. Our focus here is not on the loss of optimality due to decomposition; rather, our object is to introduce conditions under which a system can be considered weakly coupled.

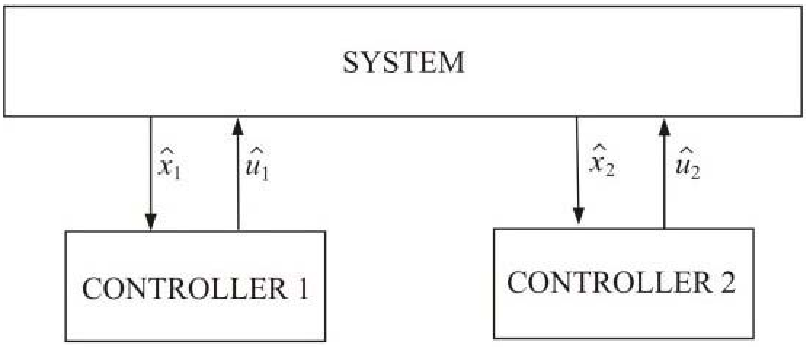

Figure 3. Decentralized control structure for two weakly coupled subsystems.

The second line of research is to exploit such a system in an algorithmic fashion in order to find an approximate optimal feedback gain through a MacLaurin's series expansion of the accompanying Riccati matrix in the coupling parameter ε. It has been shown that retaining k terms of the Riccati matrix expansion would give an approximation of order $2k$ to the optimal cost.

3. Strongly Coupled Models

Strongly coupled models are those with variables of highly distinct speeds. Such models are based on the concept of *singular perturbation*, which differs from regular perturbation (weakly coupled systems) in that perturbation is to the left of the system's state equation, that is, a small parameter multiplying the time derivative of the state vector. In practice many systems, most of them large in dimension, possess fast-changing variables displaying a singularly perturbed characteristic. For example, in a power system, the frequency and voltage transients vary from a few seconds in generator regulators, shaft-stored energy, and speed governor motion to several minutes in prime mover motion, stored thermal energy, and load voltage regulators.

Similar time-scale properties prevail in many other practical systems and processes, such as industrial control systems (e.g., cold rolling mills), biochemical processes, aircraft and rocket systems, and chemical diffusion reactions. In fact, some of the "order reduction" techniques that were discussed can be explained in terms of singular perturbation.

Consider a singularly perturbed system described by

$$\dot{x}(t) = A_1 x(t) + A_{12} z(t) + B_1 u(t)$$
$$x(t_0) = x_0 \qquad (16a)$$

$$\varepsilon \dot{z}(t) = A_{21} x(t) + B_2 u(t)$$
$$z(t_0) = z_0 . \qquad (16b)$$

If A_2 is nonsingular, as $\varepsilon \to 0$, Eq. (16) becomes

$$\dot{x}(t)\left(A_1 - A_{12}^{-1} A_{21}\right)\hat{x} + \left(B_1 - A_{12}^{-1} A_2\right)\hat{u} \qquad (17)$$

$$\hat{z}(t) = -A_2^{-1}\hat{x} - A_2^{-1} B_2 \hat{u} . \qquad (18)$$

Equation (17) is an approximate aggregated model in which the *n* eigenvalues of the original system are, in effect, approximated by the *l* eigenvalues of the $\left(A_1 - A_{12} A^{-1} A_{21}\right)$ matrix in Eq. (17). This observation follows the same line of argument in discussions of conditions for weakly coupled systems. A very important phenomenon associated with a singularly perturbed system is the existence of so-called *boundary layers*. In going from Eq. (16) to Eq. (17) the initial condition of $z(t)$ is lost and the values of $\hat{z}(t_0)$ and $z(t)_0 = z_0$ are in general different; the difference is termed a *left-side boundary layer*, which corresponds to the fast transients of Eq. (16). Figure 4 shows the boundary layer phenomenon for the fast state $z(t)$.

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Biographical Sketch

Mo Jamshidi (Fellow IEEE, Fellow ASME, Fellow AAAS) received the Ph.D. degree in electrical engineering from the University of Illinois at Urbana-Champaign in February 1971. He holds an honorary doctorate degree from Azerbaijan National University, Baku, Azerbaijan, 1999. Currently, he is the Regents Professor of Electrical and Computer Engineering, the AT&T Professor of Manufacturing Engineering and founding Director of Center for Autonomous Control Engineering (ACE) at the University of New Mexico, Albuquerque, NM, USA. He is a Senior Research Advisor at US Air Force Research Laboratory, KAFB, NM. He has also been an advisor for the NASA Headquarters Code K Minority Business Utilization. He was on the advisory board of the NASA JPL's Pathfinder Project mission, which landed on Mars on July 4, 1997 and a member of the NASA JPL Surface Systems Track Review Board. He was on the USA National Academy of Sciences NRC's Integrated Manufacturing Review Board. Previously he spent 6 years at US Air Force Phillips (formerly Weapons) Laboratory working on large-scale systems, control of optical systems and adaptive optics. He has been a consultant with Department of Energy's Los Alamos National Laboratory and Oak Ridge National Laboratory. He has worked in various academic and industrial positions at various national and international locations including with IBM and GM Corporations.

He has close to 500 technical publications including 48 books and edited volumes. Six of his books have been translated into at least one foreign language. He is the Founding Editor or co-founding editor or Editor-in-Chief of 5 journals (including Elsevier's *International Journal of Computers and Electrical Engineering*) and one magazine (*IEEE Control Systems Magazine*). He has been on the executive editorial boards of a number of journals and two encyclopedias. He was the series editor for ASME Press Series on Robotics and Manufacturing from 1988 to 1996 and Prentice Hall Series on Environmental and Intelligent Manufacturing Systems from 1991 to 1998. In 1986 he helped launch a specialized Symposium on robotics which was expanded to International Symposium on Robotics and Manufacturing (ISRAM) in 1988, and since 1994 it was expanded into World Automation Congress (WAC) where it now encompasses five main symposia and forum on Robotics, Manufacturing, Automation, Control, Soft Computing, Multimedia and Image Processing. He has been the General Chairman of WAC from its inception.

Dr. Jamshidi is a Fellow of the IEEE for contributions to "Large-scale systems theory and applications and engineering education", a Fellow of the ASME for contributions to "Control of robotic and manufacturing systems," Fellow of the AAAS - the American Association for the Advancement of Science for contributions to "Complex large-scale systems and their applications to controls and optimization," an Associate Fellow of Third World Academy of Sciences (Trieste, Italy), Member of Russian Academy of Nonlinear Sciences, Associate Fellow, Hungarian Academy of Engineering, Corresponding members of Persian Academies of Science and Engineering, a member of the New York Academy of Sciences and recipient of the IEEE Centennial Medal and IEEE Control Systems Society Distinguished Member Award and the IEEE CSS Millennium Award. He is currently on the Board of Governors of the IEEE Society on Systems, Man and Cybernetics. He is an Honorary Professor at three Chinese Universities. He is on the Board of Nobel Laureate Glenn T. Seaborg Hall of Science for Native American Navajo Nation. He is also on the board of Directors of the Sassoon Group of Corporations (Miami, FL). He is listed in a number of biographical (Who's Who) volumes.