LQ-STOCHASTIC CONTROL

João Miranda Lemos
INESC-ID/IST, Lisbon, Portugal

Keywords: Optimal control, stochastic control, LQG, stochastic regulation, stochastic servo control, stochastic linear plants, ARMAX, ARIMAX, separation principle, dual effect, quadratic performance index, dynamic programming application, Riccati difference equation, Riccati differential equation, algebraic Riccati equation, Kalman–Bucy filter application, matrix fraction description, polynomial control, reduced complexity regulator, Karush–Kuhn–Tucker theorem, 2-DOF control, LQG/LTR, minimax problems, entropy approach to optimal control.

Contents

Summary

This chapter addresses the problem in which a linear plant affected by stochastic disturbances and noise is to be controlled so as to minimize a quadratic cost. This is the subject of LQ stochastic control. The class of problems considered comprises different subclasses according to the type of plant used (state–space, input–output), the information available to the controller (direct measure of the plant state, available or not), control objectives (regulation, servo), or time domain (discrete, continuous). The LQG problem and the separation principle are presented. The chapter also considers the relationship of LQ stochastic control and H_∞ mixed sensitivity compensation,

LQG/LTR, and the maximum entropy formulation of optimal stochastic control, which provides an interpretation of the separation principle.

1. Introduction

LQ-stochastic control refers to a problem in which a linear plant affected by stochastic disturbances and noise is to be controlled so as to minimize a quadratic cost.

Example 1.1

As an engineering example, consider the problem of regulating the superheated steam temperature in a thermoelectric power plant. Among other factors, plant economic performance is directly proportional to the average superheated steam temperature. However, due to safety and life extending considerations, there is a maximum bound, T_{max}, which cannot be exceeded "too often." In a probabilistic framework, this means that the risk, measured by the area under the probability density function (pdf) of the steam temperature T to the right of T_{max}, must be below a prescribed value R_{max}.

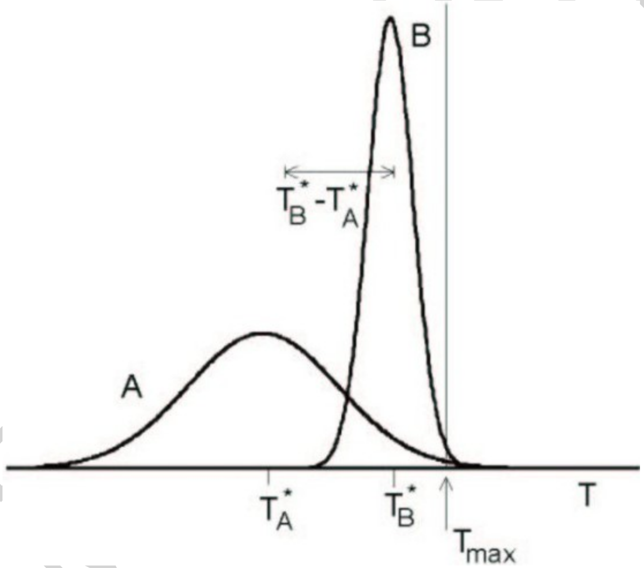

Figure 1. Motivating example to LQ-stochastic control

Figure 1 shows, for motivating purposes, two situations labeled A and B, corresponding to two different controllers operating in stationary situations. Steam temperature pdfs are shown in both cases. In situation A, the set-point T_A^* is adjusted to a value low enough to meet the risk specification, while yielding the highest possible economic performance. In situation B, the controller is tuned so as to reduce steam temperature fluctuations around the set-point.

The variance of these fluctuations is therefore smaller, and the set-point T_B^* may consequently be made higher while still meeting the risk specification. This results in an increase in plant economic performance proportional to $T_B^* - T_A^*$.

The above considerations suggest that the controller should be designed such as to minimize the steady-state value of the power of the fluctuations around the set-point, *i. e.* $E\left[\left(T(t)-T^*\right)^2\right]$ where $E[\cdot]$ denotes the statistical average, T^* is the desired temperature set-point and t stands for time. However, since this may require an excessive actuator action (in this case valve movement), a penalty on the control signal $u(t)$ is to be imposed. This results in the controller being designed to minimize the steady-state value of a cost functional given by $J = E\left[\left(T(t)-T^*\right)^2 + \rho\left(u(t)-u^*\right)^2\right]$, where u^* and ρ are constants. The parameter ρ establishes a penalty on the control action, being used as a "design knob" for which an appropriate trade-off should be found: when ρ is small, more importance is given to reducing temperature fluctuations; when ρ increases, the power of control fluctuations around u^* decreases while temperature regulation degrades.

LQ-stochastic (LQS) control extends the above example by considering cost functions which involve the plant state and non-stationary situations as well as the tracking of time-varying references.

The origin of the LQS control problem and the methods associated with its solution can be traced back to the decade of 1950, in relation to economic problems. Major impetus came however from aerospace problems in the following decade, together with the associated development of state–space theory.

Shortly afterwards, mainly process industry problems propelled an approach relying on plant input–output models and polynomial (or polynomial matrix) techniques. Today, a fairly complete overall theory of LQS control is available, including the relation among the various approaches, as well as with other control theories, such as H_∞ mixed sensitivity compensation or the entropy based formulation.

As can be anticipated, LQS control has a close relation with optimal linear quadratic control and the Kalman–Bucy filter, when formulated in a state–space setting, and with controller design using polynomial matrix descriptions when using input–output models. Actually, a remarkable fact holds for LQS control, viz. that the optimal controller is designed using a separation principle. According to this principle, the problem is split in the separate design of an LQ controller and a Kalman–Bucy filter yielding a plant state estimate, one not affecting the other.

In continuous time, understanding the design of LQ-stochastic controllers requires the machinery of stochastic differential equations, and is thus mathematically more involved then if a discrete time setting is assumed. Therefore, for the sake of understanding by the non-specialist, this article starts by considering the problems in discrete time, only later presenting their counterpart results in continuous time. Furthermore, in the same vein, basic regulation problems are first considered before their extension to the stochastic servo problem.

2. LQ Regulation for Discrete Time Plants

This section considers the basic theory of the LQ regulation problem for stochastic linear dynamics in discrete time. The plant is initially modeled by a state–space model. Another class of models will later be considered. The solution is first provided for the case in which a perfect state measurement (i.e. yielding a measure of all state components unaffected by noise, a situation referred in the literature as *complete state information* or *complete state observations*) is available. This is then extended to the situation, often found in applications, in which the state is not directly available for measurement but has to be estimated from input–output observations.

2.1. Complete State Information

Let the plant to be regulated be modeled by the state representation

$$x(k+1) = \Phi(k)x(k) + \Gamma(k)u(k) + v(k) \tag{1}$$

in which k is an integer index denoting discrete time, $x(k) \in \Re^n$ is the plant state, $u(k) \in \Re^m$ is the manipulated variable and $v(k) \in \Re^n$ is a stochastic disturbance, inaccessible for direct measurement. For each k, $\Phi(k)$ and $\Gamma(k)$ are $n \times n$ and $n \times m$ real matrices.

The sequence $\{v\}$ is assumed to be a white, zero mean sequence, verifying

$$\left.\begin{array}{r} E\left[v(k)' v(k-i)\right] = 0_{n\times n} \quad i \neq 0 \\ E\left[v(k) v'(k)\right] = P_v(k) < \infty \end{array}\right\} \tag{2}$$

and

$$E\left[v(k_0) x'(k_0)\right] = 0_{n\times n} \tag{3}$$

where prime denotes transpose and 0_n and $0_{n\times n}$ are matrices of zero entries with the indicated dimensions. Although no assumption is made on the Gaussianity of $\{v\}$, this case is not ruled out.

For formulating a regulation problem in relation to plant (1), the following quadratic performance index is considered:

$$J_{dr} = E\left[\sum_{k=k_0}^{T-1} L_d\left(k, x(k), u(k)\right)\right] + \|x(T)\|_{Q(T)}^2 \tag{4}$$

in which:

$$L_d\left(k, x(k), u(k)\right) := \|x(k)\|^2_{Q(k)} + \|u(k)\|^2_{R(k)} \tag{5}$$

for $k < T$, $T > k_0$ being the terminal time. Hereafter $\|z\|^2_M$ is defined for a vector z and a matrix M as:

$$\|z\|^2_M := z'Mz \tag{6}$$

For all $k \in \{k_0, \ldots, T\}$ the weight matrices $Q(k), R(k)$ are assumed to satisfy:

$$Q(k) = Q'(k) \geq 0 \tag{7}$$

$$R(k) = R'(k) > 0 \tag{8}$$

The scalar quantity $L_d(k, x(k), u(k))$ is referred to as the instantaneous loss at time k. With the assumptions (7, 8) it is non-negative. There is no loss of generality in assuming the weights $Q(k)$ and $R(k)$ symmetric and no cross-product terms between $x(k)$ and $u(k)$ in the loss.

The admissible regulation strategies are defined by:

$$u(k) \in I^k \tag{9}$$

meaning that $u(k)$ can be computed as a function of I^k, the information available at time t, which consists of the past realizations of u and present and past realizations of x (assumed available for direct observation). Therefore, admissible regulation strategies as defined by (9) are non-anticipative or causal, since the control decision to be taken at a generic time k depends only on the variable samples available.

In relation to this plant, performance index, and admissible regulation strategies, the following problem is considered:

LQ-stochastic (LQS) regulation problem with complete state information (discrete time)

Consider the plant described by the stochastic linear model (1) and the quadratic performance index J_{dr} given by (4). Assuming complete state information available, find, among all the admissible regulation strategies defined by (9), an input sequence to the plant, $\{u(k), k_0 \leq k < T\}$, minimizing J_{dr}.

The solution to this problem is given by the following theorem

Theorem 1.1: LQS regulator with complete state information (discrete time)

The solution to the LQS regulation problem with perfect state observations is given by the linear state feedback law:

$$u(k) = F_d(k)x(k) \qquad k_0 \le k < T \tag{10}$$

where the feedback gain matrix $F_d(k)$ is computed by:

$$F_d(k) = -\left[R(k) + \Gamma'(k)P(k+1)\Gamma(k)\right]^{-1} \cdot \Gamma'(k)P(k+1)\Phi(k) \tag{11}$$

and the matrix $P(k)$ is the symmetric nonnegative definite matrix resulting from the solution of the following backward Riccati difference equation (RDDE):

$$P(k) = \Phi'(k)P(k+1)\Phi(k) -$$

$$-\Phi'(k)P(k+1)\Gamma(k)\left[R(k) + \Gamma'(k)P(k+1)\Gamma(k)\right]^{-1}\Gamma'(k)P(k+1)\Phi(k) + Q(k) \tag{12}$$

Furthermore, the minimum cost achievable in $\{k, \ldots, T-1\}$ is given by:

$$\left\|E\left[x(k)\right]\right\|^2_{P(k)} + \mathrm{tr}\left(P(k)\,\mathrm{cov}(x(k))\right) + \sum_{i=k}^{T-1} \mathrm{tr}\left[P(i+1)P_v(i)\right] \tag{13}$$

This result is obtained by a stochastic version of dynamic programming. For $k \in \{k_0, \ldots, T-1\}$ consider the Bellman function:

$$V\left(k, x(k)\right) := \min_{u_k^{T-1}} E\left\{\sum_{i=k}^{T} L_d\left(i, x(i), u(i) \mid x(k)\right)\right\} \tag{14}$$

This function satisfies the stochastic Bellman equation:

$$V\left(k, x(k)\right) = \min\left\{L_d\left(k, x(k), u(k)\right) + E\left[V(k+1, x(k+1)) \mid x(k)\right]\right\} \tag{15}$$

with terminal condition:

$$V\left(T, x(T)\right) = \left\|x(T)\right\|^2_{Q(T)} \tag{16}$$

Using an induction argument, the solution of the above equation is shown to be:

$$V(k, x) = x'P(k)x + \sum_{i=k}^{T-1} \mathrm{tr}\left[P(i+1)P_v(i)\right] \tag{17}$$

with $P(k)$ satisfying the RDDE (12).

The solution of the LQS regulation problem with perfect state observations is the same as for the LQ control regulation problem obtained by letting $v(k)=0_n$. As expected, the minimum achieved cost achievable in the LQS problem is increased with respect to the one of the corresponding LQ problems by the stochastic disturbance v.

2.2. Partial State Information: The LQG Regulator in Discrete Time

In many practical cases the full plant state is not available for measurement and, furthermore, the measures made are corrupted by noise. Therefore, the plant model (1) has to be complemented with the sensor model, resulting in:

$$\left.\begin{aligned} x(k+1) &= \Phi(k)x(k)+\Gamma(k)u(k)+v(k) \\ y(k) &= H(k)x(k)+w(k) \end{aligned}\right\} \tag{18}$$

Here, $y \in R^p$ is the vector of observations and $w \in R^p$ models observation noise. The real matrix $H(k)$ has dimension $p \times n$. Let $\xi(k) := \left[v'(k)\; w'(k)\right]'$ and now Σ_d^k be the σ-field generated by $\left\{x(k_0), \xi(k_0), \ldots, \xi(k)\right\}$ and assume that the joint process $\{\xi\}$ verifies

$$\left.\begin{aligned} E\left[\xi(k)\xi(k-i)'\right] &= 0_{n+p\times n+p} \quad i \neq 0 \\ E\left[\xi(k)\xi'(k)\right] &= P_\xi(k) = \begin{bmatrix} P_v(k) & 0_{n\times p} \\ 0_{p\times n} & P_w(k) \end{bmatrix} \end{aligned}\right\} \tag{19}$$

Furthermore, assume that

$$x(k_0) \text{ and } \left\{\xi(k), k_0 \le k < T\right\} \text{ are jointly Gaussian distributed} \tag{20}$$

This hypothesis is necessary because the state is to be estimated from observations using a Kalman–Bucy filter.

The admissible regulation strategies are in this case defined by:

$$u(k) \in \sigma\left\{y^k\right\} \tag{21}$$

where $u(k)$ is a function of plant observations y up to time k. With these elements, the following version of the LQS regulation problem is now defined:

LQ Gaussian (LQG) regulation problem (discrete time)

Consider the linear stochastic plant (18), where observation noise and disturbances have a Gaussian distribution, and the quadratic performance index J_{dr} defined by (4). Find an input sequence to the plant, $\left\{u(k), k_0 \le k < T\right\}$, minimizing J_{dr} among all the admissible regulation strategies defined by (21).

The solution to this problem is given by the following theorem:

Theorem 1.2: LQG regulator (discrete time)

The solution of the LQG regulation problem is given by the feedback law of the filtered state estimate

$$u(k) = F_d(k)\hat{x}(k|k) \qquad k_0 \le k < T \tag{22}$$

where $F_d(k)$ is the optimal feedback gain matrix, which is the same as the one given in theorem 1.1, computed by (11, 12). The filtered state estimate is given by the discrete Kalman–Bucy filter, defined by the difference equations:

$$\hat{x}(k|k) = \hat{x}(k|k-1) + \tilde{K}_d(k)e(k) \tag{23}$$

$$\hat{x}(k+1|k) = \Phi(k)\hat{x}(k|k) + \Gamma(k)u(k) \tag{24}$$

with:

$$e(k) = y(k) - H(k)\hat{x}(k|k-1) \tag{25}$$

$$\tilde{K}_d(k) = \Pi(k)H'(k)\left[H(k)\Pi(k)H'(k) + P_w(k)\right]^{-1} \tag{26}$$

and $\Pi(k)$, the sate prediction error covariance, given by the forward Riccati difference equation:

$$\Pi(k+1) = \Phi(k)\Pi(k)\Phi'(k) -$$

$$-\Phi(k)\Pi(k)H'(k)\left[H(k)\Pi(k)H'(k) + P_w(k)\right]^{-1} H(k)\Pi(k)\Phi'(k) + P_v(k) \tag{27}$$

initialized by $\Pi(k_0)$, the corresponding *a priori* value. Furthermore, the minimum cost yielded by the optimal control sequence starting at time k, $\{u(i), k \le i < T\}$ is given by:

$$\left\|E\left[x(k)\right]\right\|^2_{P(k)} + \mathrm{tr}\left(P(k)\Pi(k|k) + \sum_{i=k}^{T-1} \mathrm{tr}\left[P(i+1)P_v(i)\right] + \sum_{k=1}^{T} \mathrm{tr}\left[Q(k)\Pi(i|i)\right]\right. \tag{28}$$

with $\Pi(k|k) := \mathrm{cov}\left(x(k)|y^k\right)$ satisfying:

$$\Pi(k|k) = \Pi(k) - \Pi(k)H'(k)\left[H(k)\Pi(k)H'(k) + P_w(k)\right]^{-1} H(k)\Pi(k) \tag{29}$$

The LQG regulator consists (Figure 2) of a Kalman–Bucy filter together with a LQS regulator acting on its output: that is, with the state replaced by its filter estimate.

The main fact about theorem 1.2 is that the LQG regulator is designed according to a *separation principle*. This principle states that the regulator and the filter (state estimator) are designed independently of each other and then fitted together.

Furthermore, the feedback is computed from the state estimate as if this is the actual state, a fact referred to as the *certainty equivalence principle*.

This is an important result which does not extend to nonlinear systems. Indeed, in general the control variable affects both the plant output and the posterior pdf of the state given observations, something called “the dual effect.”

In LQG the control is such that there is no dual effect and the certainty equivalence principle holds.

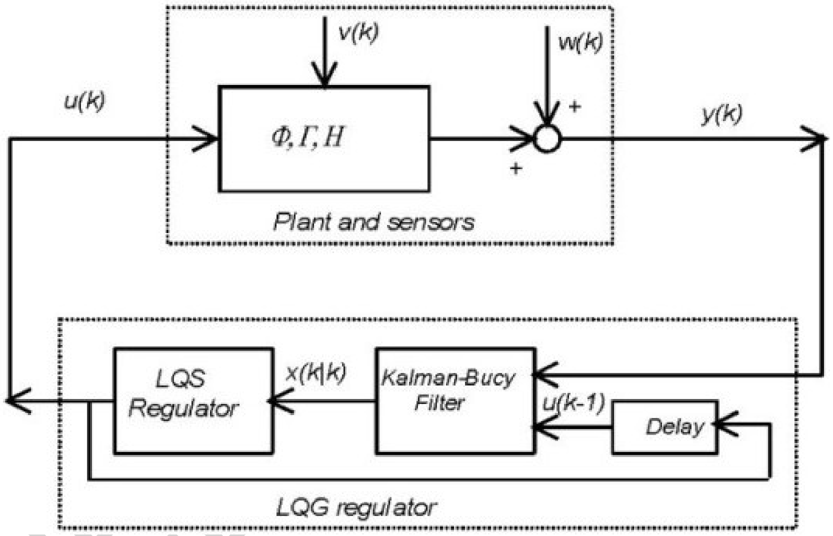

Figure 2. Structure of the LQG regulator

As a final point, compare the minimum value of the cost for the LQS regulator with perfect state observations (given by (13)) and the one yielded by the LQG regulator (given by Eq. (28)).

The latter is increased with respect to the former by the last term of (28), which depends on the posterior covariance matrices $\Pi(k \mid k) = \operatorname{cov}\left(x(k) \mid y^k\right)$ and reflects the fact that in LQG a perfect state measurement is not being used.

-
-
-

TO ACCESS ALL THE **35 PAGES** OF THIS CHAPTER,
Click here

Bibliography

Åström K.J. (1970). *Introduction to Stochastic Control Theory*. New York, San Francisco, London: Academic Press, 299 pages. [A textbook introduction to LQ stochastic control in both discrete and continuous time. Presents and proves an algorithm based on a modified Jury table for computing the stochastic steady-state variance of the output of a linear system driven by white noise.]

Åström K.J. and Wittenmark B. (1990). *Computer Controlled Systems – Theory and Design*. London: Prentice-Hall (UK) Limited. 2^{nd} edn, 544 pages. [A textbook on discrete LQG control for discrete systems. Considers state–space and polynomial models for SISO plants.]

Blackburn T.R. and Vaughn D.R. (1971). Application of linear optimal control and filtering theory to the Saturn V launch vehicle. *IEEE Transactionsof Autoatic . Control*, AC-16(6):799–806. [An early application of LQ stochastic control to a problem of great significance for humankind.]

Burl J.B. (1999). *Linear Optimal Control*. Reading. Massachusetts: Addison Wesley, 300 pages. [A text book covering LQG control and its relation to other methods.]

Doyle J.C. (1978). Guaranteed margins for LQG regulators. *IEEE Transactions on. Automatic. Control*, AC-23(4):756–757. [The paper shows that, if proper care is not taken, the insertion of a Kalman–Bucy filter may yield a non-robust controller.]

Doyle J.C. and Stein G. (1979). Robustness with observers. *IEEE Transactions.on Automatic. Control*, AC-24(4):607–611. [The early reference on the LQG/LTR design procedure.]

Fleming W.H. and Rishel R.W. (1975). *Deterministic and Stochastic Optimal Control.* New York, Heidelberg, Berlin: Springer Verlag, 270 pages [Provides a mathematical treatment of the LQG regulator for continuous time systems.]

Goodwin G.C. and Sin K.S. (1984). *Adaptive Filtering Prediction and Control*. Englewood Cliffs, New Jersey: Prentice Hall. , 540 pages [An advanced textbook covering LQ stochastic control.]

Green M. and Limebeer D. (1995). *Linear Robust Control*. Englewood Cliffs, New Jersey: Prentice Hall, 538 pages. [A modern, advanced text book providing an account of LQG control and its relations with H_∞ control.]

Grimble M.J. (1986). Optimal H_∞ robustness and the relationship to LQG design problems. *International Journal of. Control*, 43(2):351-372. [Solution of the H_∞ optimal control problem using a LQG polynomial based solution.]

Hunt K.J. (1989). *Stochastic Optimal Control Theory with Application in Self-Tuning Control*. Berlin, Heidelberg, New York: Springer-Verlag, 308 pages. [A monograph on the polynomial approach to stochastic optimal control. Also covers the use of LQG in the self-tuning control framework, including an application on the steam pressure control loop of a power station.]

Kalman R.E. (1960). A new approach to linear filtering and prediction problems. *journal of. Basic Engineering* 82 (March):34–45. [This and the following reference are the original derivations of the Kalman–Bucy filter.]

Kalman R.E. and Bucy R.S. (1961). New results in linear filtering and prediction theory. *Transactions of*

ASME, seie. D, Journal of. Basic Engineering, 83(December): 95–107.

Kucera V. (1979). *Discrete Linear Control: The Polynomial Equation Approach*. Chichester: Wiley, 280 pages. [A monograph where the LQ stochastic control problem for discrete time systems is dealt by the polynomial approach.]

Kwakernaak H. and Sivan R. (1972). *Linear Optimal Control Systems*. New York, London, Sydney, Toronto: Wiley-Interscience, 575 pages. [A classical textbook. Presents the basic LQ stochastic theory based on state–space plant models for both the regulation and the servo problems.]

Lehtomaki N.A., Sandell N.S., Jr., and Athans M. (1981). Robustness results in linear-quadratic Gaussian based multivariable control designs. *IEEE Transactionson Automomatic Control*, AC-26(1):75–92. [Robustness results for MIMO systems with LQG are presented.]

Mäkilä P.M., Westerlund T., and Toivonen H.T. (1984). Constrained linear quadratic Gaussian control with process applications. *Automatica* 20(1):15–29. [Formulation and numerical algorithms for constrained LQG control problems. Industrial applications are described.]

Mendel J.M. and Gieseking D.L. (1971). Bibliography on the linear-quadratic-Gaussian problem. *IEEE Transactions on . Automomatic Control*, 16(6):847–869. [A systematic recollection at the time of publication containing over 900 references.]

Mitter S.K. (1996). Filtering and stochastic control: a historical perspective. *IEEE Control Systems*, 16(3):67–76. [Provides a broad tutorial view of advanced topics on stochastic control for continuous time systems. Explicitly covers the LQG problem. See also here references to the original papers in the area.]

Modén P.E. and Söderström T. (1982). Stationary performance of linear stochastic systems under single step optimal control. *IEEE Transactions on Automomatic Control*, AC-27(1):214–216. [The original version of example 3.1.]

Mosca E. (1995). *Optimal, Predictive, and Adaptive Control*. Englewood Cliffs, New Jersey: Prentice Hall, 477 pages. [Includes a complete exposition of LQ stochastic control of discrete plants, including some original material.]

Mosca, E., Casavola A., and Giarré L. (1990). Minimax LQ Stochastic tracking and servo problems. *IEEE Transactions on Automatic Control* 35(1):95–97. [Solution of a 2-DOF minimax LQ stochastic problem.]

Peressini A.L., Sullivan F.E., and Uhl J.J., Jr. (1988). *The Mathematics of Nonlinear Programming*. New York, Berlin, Heidelberg: Springer, 273 pages. [An undergraduate level textbook on nonlinear programming presenting the Karush–Kuhn–Tucker conditions.]

Sage A. and White C.C., III (1980). *Optimum Systems Control*. Englewood Cliffs, New Jersey: Prentice Hall, 310 pages. [A textbook covering LQG control.]

Saridis G.N. (1988). Entropy formulation for optimal and adaptive control. *IEEE Transactions on Automatic Control*, AC-33(8): 713–721. [Includes the entropy formulation of LQ stochastic control for continuous time systems.]

Saridis G.N. (1995). *Stochastic Processes, Estimation and Control – The Entropy Approach*. New York, Chichester, Brisbane, Toronto, Singapore: John Wiley, 230 pages. [An advanced textbook, covering LQ stochastic control. A comprehensive review of the entropy approach to optimal control is provided.]

Soderstrom T. and Stoica P. (1994). *Discrete-Time Stochastic Systems*. Prentice Hall, 350 pages. [A text book giving a comprehensive introduction to the field of stochastic dynamic systems, including their estimation and control.]

Stein G. and Athans M. (1987). The LQG/LTR Procedure for multivariable feedback control design. *IEEE Transactions on Automatic Control*, AC-32(2):105–114. [A tutorial overview of the LQG/LTR design procedure for SISO and MIMO plants.]

Sternad M. (1987). *Optimal and Adaptive Feedforward Regulators*. Uppsala: Uppsala University, S-751 21, 215 pages. [A monograph on feedforward LQG regulator design for plants with accessible disturbances.]

Tsai Y.A., Casiello F.A., and Loparo K.A. (1992). Discrete-time entropy formulation of optimal and adaptive control problems. *IEEE Transactions on Automatic Control*, AC-37(7):1083–1088. [Includes the entropy formulation of optimal control problems for discrete-time systems.]

Tse E. (1971). On the optimal control of stochastic linear systems. *IEEE Transactions on Automatic Control*, AC-16(6): 776–785. [Presents the theory of LQ stochastic regulation for continuous plants. Discusses the importance of the different assumptions. Includes an appendix with the interpretation of some probabilistic concepts underlying estimation and control.]

Witsenhausen H.S. (1971). Separation of estimation and control for discrete time systems. *IEEE Procedings* 59(11):1557–1566. [A broad view of the results relating to separation of estimation and control in discrete time stochastic control theory.]

Biographical Sketch

João Miranda Lemos is Professor of Automatic Control at IST, the engineering faculty of the Technical University of Lisbon, Portugal, where he is currently the Coordinator of the Post-Graduation Programme of the Electrical Engineering Department. His research interests encompass computer control, adaptive control, control based on multiple models and modeling and control of industrial continuous processes. He obtained a Ph.D. at IST in 1989 after extensive periods of work at the University of Florence, Italy. He has published over 100 research papers in journals, peer-reviewed symposia, and as book chapters.