CONTROLLABILITY AND OBSERVABILITY OF 2-D SYSTEMS

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Contents

1. Introduction
2. Unconstrained controllability
2.1. Mathematical model
2.2. General response formula
2.3. Controllability conditions
3. Singular systems
3.1. Mathematical model
3.2. Controllability conditions
4. Constrained controllability
4.1. Admissible controls
4.2. Controllability conditions
5. Positive systems
5.1. The concept of positive systems
5.2. Controllability conditions
6. Continuous-discrete systems
6.1. Mathematical model
6.2. Controllability conditions
7. Nonlinear systems
7.1. Mathematical model
7.2. Controllability conditions
8. Observability
8.1. Mathematical model
8.2. Observability conditions
Glossary
Bibliography
Biographical Sketch

Summary

This article contains fundamental theorems concerning unconstrained and constrained controllability problems both for linear and nonlinear 2-D systems with constant coefficients. In the literature, there are many other controllability and observability results, derived for more general 2-D dynamical systems. For example, controllability and observability of the following multidimensional discrete systems have recently been considered:

- linear 2-D systems with variable coefficients,
- linear 2-D systems defined in infinite-dimensional linear spaces e.g., Hilbert spaces or Banach spaces,

- linear 2-D systems with delays ,
- linear M-D systems i.e. discrete systems with M independent variables,
- nonlinear 2-D systems with variable coefficients.

For different classes of the above multidimensional discrete systems, it is necessary to introduce different types of controllability and observability. For example, for infinite-dimensional systems, it is necessary to introduce two fundamental notions of controllability and observability, namely, approximate (weak) controllability, exact (strong) controllability and approximate (weak) observability and exact (strong) observability.

Controllability of dynamical systems is strongly connected with the so-called minimum energy control problem, which has been considered in many publications for various kinds of linear 2-D systems. Moreover, it should be pointed out that similarly to classical linear dynamical systems, there are relationships between controllability and spectrum assignability for 2-D systems.

1. Introduction

Controllability and observability are two fundamental concepts in modern mathematical control theory. Many dynamical systems are organized such that the control does not affect the complete state of the dynamical system, but only a part of it. On the other hand, very often in real industrial processes, it is possible to observe only a certain part of the complete state of the dynamical system. Therefore, it is very important to determine whether or not control and observation of the complete state of the dynamical system are possible. Roughly speaking, controllability generally means that it is possible to steer dynamical system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. On the other hand, observability means that it is possible to recover uniquely the initial state of the dynamical system from a knowledge of the input and output. Controllability and observability play an essential role in the development of the modern mathematical control theory. Moreover, it should be pointed out that there exists a formal duality between the concepts of controllability and observability.

In the literature, there are many different definitions of controllability and observability that depend on the type of dynamical system. A growing interest has been developed over the past few years in problems involving signals and systems that depend on more than one of the independent variables. The motivations for studying 2-D systems have been well justified in several papers and monographs. Most of the major results concerning the multidimensional signals and systems are developed for two-dimensional cases. Discrete dynamical systems with two independent variables, so called 2-D systems, are important mainly in image processing, multivariable network realizability and in multidimensional digital filters. During last two decades controllability of 2-D systems has been considered in many papers and books. The main purpose of this article is to present a compact review over the existing controllability and observability results mainly for linear discrete 2-D systems. The majority of the results in this area concern linear 2-D systems with constant coefficients. It should be pointed out that for linear systems, controllability and observability conditions have pure algebraic forms and are easily computable. These conditions require verification of the rank conditions for suitably defined constant controllability and observability matrices.

The article is organized as follows. Section 2 contains systems descriptions and fundamental results concerning unconstrained controllability for the most popular linear 2-D models with constant coefficients. In Section 3, unconstrained controllability of linear singular 2-D systems with constant coefficients is discussed. The next Section, 4, is devoted to a study of constrained controllability of linear regular 2-D systems. The special attention is paid for the so-called positive controllability. Section 5 presents results on positive controllability for linear positive 2-D systems. In Section 6 controllability of the so-called continuous-discrete linear systems is investigated. Local controllability of nonlinear 2-D systems with constrained controls is considered in Section 7. Section 8 contains fundamental definition of observability and necessary and sufficient conditions for observability of linear discrete 2-D systems with constant coefficients. Finally, in Section 9, concluding remarks and comments concerning possible extensions are presented. Since the article should be limited to a reasonable size, it is impossible to give a full survey of the subject. In consequence, only selected fundamental results, without proofs, are presented.

2. Unconstrained controllability

2.1. Mathematical model

In the theory of 2-D systems, several different models are considered. The most popular and the most frequently used are based on the Fornasini-Marchesini model, given by the following linear difference equation

$$x(i+1,j+1) = A_0x(i,j) + A_1x(i+1,j) + A_2x(i,j+1) + Bu(i,j) \qquad (1)$$

where $i,j \in Z_+=\{0,1,2,3,...\}$ is a set of nonnegative integers, $x(i,j) \in R^n$ is a local state vector, $u(i,j) \in U \subset R^m$ is an input vector, U is a given set, A_0, A_1, A_2, and B are real matrices of appropriate dimensions. Boundary conditions for the equation (2.1) are given by the following equalities

$$x(i,0) = x_{i0} \in R^n \quad \text{for } i \in Z_+ \qquad x(0,j) = x_{0j} \in R^n \quad \text{for } j \in Z_+ \qquad (2)$$

2.2. General response formula

In order to present the general response formula for equation (1) in a convenient compact form, it is necessary to introduce (n×n)-dimensional so called state transition matrix $A^{i,j}$ defined as follows:

$A^{0,0} = I$, (n×n) identity matrix,

$A^{-i,j} = A^{i,-j} = A^{-i,-j} = 0$ for $i,j > 0$,

$$A^{i,j} = A_0A^{i-1,j-1} + A_1A^{i,j-1} + A_2A^{i-1,j} = A^{i-1,j-1}A_0 + A^{i,j-1}A_1 + A^{i-1,j}A_2 \qquad \text{for } i,j>0$$

Therefore, general response formula for equation (1) with boundary conditions (2) and given admissible controls sequence has the following compact form

$$\begin{aligned} x(i,j) &= A^{i-1,j-1}A_0x_{00} + \sum_{p=1}^{p=i}(A^{i-p,j-1}A_1 + A^{i-p-1,j-1}A_0)x_{p0} + \\ &+ \sum_{q=1}^{q=j}(A^{i-1,j-q}A_2 + A^{i-1,j-q-1}A_0)x_{0q} + \sum_{p=0}^{p=i-1}\sum_{q=0}^{q=j-1}A^{i-p-1,j-q-1}Bu(p,q) \end{aligned} \tag{3}$$

It is well known that for 2-D systems, it is possible to introduce several different notions of controllability. For example, we may consider global controllability of 2-D systems, or the so- called straight line controllability of 2-D systems.

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Biographical Sketch

Jerzy KLAMKA was born in Poland in 1944. He received the M.S. and Ph.D. degrees in control engineering from the Silesian Technical University in Gliwice, Poland, in 1968 and 1974, respectively. He also received the M.S. and Ph.D. degrees in mathematics from the Silesian University in Katowice, Poland, in 1971 and 1978, respectively. In 1981, he received habilitation in control engineering, and in 1990, became titular Professor in control engineering from the Silesian Technical University in Gliwice, Poland. Since 1968, he has been working for the Institute of Control Engineering of the Silesian technical University in Gliwice, where he is now a full Professor. In 1973 and 1980, he taught semester courses in mathematical control theory at the Stefan Banach International Mathematical Center in Warsaw. He has been a member of the American Mathematical Society (AMS) from 1976, and Polish Mathematical Society (PTM) since 1982. He is also a permanent reviewer for Mathematical Reviews (from 1976) and for Zentralblatt für Mathematik (from 1982). In 1981 and 1991, he was awarded the Polish Academy of

Sciences awards. In 1978, 1982 and 1990, he received awards from the Ministry of Education. Moreover, in 1994 he was awarded the Polish Mathematical Society award. In 1991, he published the monograph "Controllability of Dynamical Systems", Kluwer Academic Publishers, Dordrecht, The Netherlands. In the last thirty years, he has published more than 100 papers in international journals, e.g., in IEEE Transactions on Automatic Control, Automatica, International Journal of Control, Journal of Mathematical Analysis and Applications, Systems and Control Letters, Foundations of Control Engineering, Systems Science, Kybernetika, IMA Journal on Mathematical Control and Information, Nonlinear Analysis, Theory, Methods and Applications, Systems Analysis, Modelling, Simulation, Archives of Control Science, Applied Mathematics and Computer Science, Advances in Systems Science and Applications, Bulletin of the Polish Academy of Sciences, Mathematical Population Dynamics, Lecture Notes in Control and Information Sciences, Analele Universitati din Timisoara, Acta Mathematicae Silesiane. He took part in many international Congresses, Conferences and Symposiums. His major current interest is controllability theory for linear and nonlinear dynamical systems, and in particular controllability of distributed parameter systems, dynamical systems with delays, and multidimensional discrete systems.