PASSIVITY BASED CONTROL

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Keywords: Passivity, automatic control, mechanical systems, robot manipulators, Euler-Lagrange systems, nonlinear systems, input-output stability.

Contents

Summary

Passivity is a fundamental property of many physical systems which may be roughly defined in terms of *energy* dissipation and transformation. It is an inherent *Input-Output* property in the sense that it quantifies and qualifies the energy balance of a system when stimulated by external inputs to generate some output. Passivity is therefore related to the property of stability in an *input-output* sense, that is, we say that the system is *stable* if bounded "input energy" supplied to the system, yields bounded output energy. This is in contrast to *Lyapunov stability* which concerns the *internal* stability of a system, that is, how "far" the *state* of a system is from a desired value. In other words, how differently a system behaves with respect to a desired performance.

Passivity based control is a methodology which consists in controlling a system with the aim at making the closed loop system, passive. The field constitutes an active research direction and therefore in this chapter we give only a basic overlook of the most important concepts involved. A section is also devoted to a wide class of physical

passive systems: the Euler-Lagrange (EL) systems and their passivity-based control.

The reader should rather consider this presentation as very concise image of the material cited in the Bibliography. Therefore, we invite the reader who wishes to obtain a deeper knowledge in the subject, to see those references.

1. Introduction

To better understand the passivity concept and passivity-based control (PBC), we need to leave behind the notion of *state* of a system and think of the latter as a device which interacts with its environment by *transforming inputs* into *outputs.* From an energetic viewpoint we can define a passive system as a system which cannot store more energy than is supplied by some "source", with the difference between stored energy and supplied energy, being the dissipated energy.

Hence, it shall be clear that passivity is closely related to the *stability* of a system, in the input-output sense evoked in the Summary. In PBC achieving stability from this viewpoint is the first goal.

A fundamental property of passive systems is that, regarding a feedback interconnection of (other physical) passive systems, passivity is *invariant* under negative feedback interconnection. In other words, the feedback interconnection of two passive systems yields a passive system.

Thus, if the overall energy balance is positive, in the sense that the energy generated by one subsystem, is dissipated by the other one, the closed loop will be *stable in an input-output sense* (see Proposition 2). This property constitutes the basis of passivity-based control (PBC).

The term PBC was coined in 1989 in the context of adaptive control of robot manipulators to define a controller methodology whose aim is to render the closed-loop system passive, seen as a map from an external *new* input. This objective seemed very natural within that context, since the robot dynamics defines a passive map from *input torques* to *output link velocities.* As a matter of fact this passivity property is inherent to many other physical systems such as electrical and electromechanical. (See section 4).

Since the aim in PBC is to render the closed loop system passive, the main property used in PBC is the fact that the interconnection of passive systems is passive. Conversely, passive systems can be decomposed in passive "subsystems". Thus, in this philosophy the controller may be designed as a *passive* system.

In terms of energy dissipation, the PBC approach may be viewed as an extension of the so-called *energy-shaping plus damping injection* technique introduced to solve state-feedback set (operating) point regulation problems in fully actuated robotic systems back in 1981. For this particular problem we can concentrate our attention on the potential energy and the dissipation functions to proceed along two basic stages: firstly, as *energy shaping* stage which consists on modifying the potential energy of the system in such a way that the "new" potential energy function has a global and unique minimum at the desired equilibrium. This is motivated by the well known fact (stated by Joseph Lagrange in 1788 and proved 50 years later by Dirichlet) that the stable equilibria of mechanical systems correspond to the minima of the potential energy function. Secondly, a *damping injection* stage which consists in modifying the dissipation properties of the system, to render it strictly passive.

Viewed from the PBC perspective the energy shaping stage accomplishes the objective of rendering the closed loop system passive with a desired *storage function* that consists of the original kinetic energy and the new desired potential energy. The damping injection reinforces this property to *output strict passivity*. Finally, Lyapunov asymptotic stability follows from the input-output stability of the output strictly passive map provided some dissipation propagation (i.e., detectability) conditions are met. That is, the system evolves in a way that it reaches the desired set point asymptotically. See Sections 2.1 and 3.2.

The generality of the PBC allows us to deal with different problems such as output feedback and tracking control in a unified way. Moreover, even though here we will only illustrate the PBC methodology with simple examples of control of EL mechanical systems, the reader must keep present that, having its origins in electrical circuits, it is natural that PBC is most suitable for electrical and electromechanical systems such as power converters, electrical machines, etc. This will be illustrated through a time-varying reference tracking control problem in Section 4.4.

2. Passivity: Mathematically Speaking

In this section we will introduce the precise definitions of passivity and some important theorems on passivity.

2.1. In a General Input-Output Framework

As it may be clear from the discussion above, when talking about a passive system (operator) one aims at measuring the energy (storage) transformation performed in the system. The concept of passivity in dynamical systems has its roots in the same concept used by electrical engineers to characterize elements which *consume* energy but do not supply it. In this context, the input and output signals have a direct physical meaning, i.e., current and voltage hence, the (electrical) energy measure of these is evident: it simply corresponds to the integral of the power over time. However, if we would like to talk about the passivity property of physical systems of different nature (electrical, mechanical, chemical, etc), we need a more general concept of measure.

To that end, we must keep in mind that passivity is a property of the system, seen as an operator which maps inputs into outputs. In this respect, we will find characterizations and sufficient conditions for passivity, which apply to systems that can be modeled by rational transfer functions as well as to systems modeled by nonlinear (possibly time-varying) differential equations.

In this section we will introduce the precise definitions of passivity, which reflect the fact that passivity is an energy transformation property. We will also extend to the case of nonlinear systems, some of the arguments made before, to sustain the fundamental properties of passive systems.

Definition 1 ($\mathcal{L}_2^n$ and $\mathcal{L}_2^n$ norms): The $\mathcal{L}_2^n$ norm of a signal $f:\mathbb{R}_{\geq 0}^n \to \mathbb{R}^n$, $f(t)$ is denoted $\| f(t)\|_{2T}$ and defined by

$$\left(\int_0^T \| f(t)\|^2 \, dt\right)^{\frac{1}{2}} \tag{1}$$

and the $\mathfrak{L}_2^n$ norm denoted $\| f(t)\|_2$ is defined as by $\lim_{T\to\infty} \| f(t)\|_{2T}$.

With this metric we can then define the normed $\mathcal{L}_{2e}$-space:

Definition 2 ($\mathcal{L}_{2e}$-space): We say that $f:\mathbb{R}_{\geq 0} \to \mathbb{R}$ belongs to $\mathcal{L}_{2e}$ if and only if $\| f(t)\|_{2T} < \infty$

The definitions above makes sense from a practical viewpoint if we consider the case when $f(t)$ corresponds to power and therefore, the $\mathcal{L}_2^n$ borrows the interpretation of energy amount over a time interval.

Now in order to properly define the passivity concept for $\mathcal{L}_2^n$ signals we introduce the following product, which generalizes the concept of *supplied* energy discussed above.

Definition 3 (Inner product): Let $u, y \in \mathcal{L}_2^n$ and $T > 0$, then the inner product is defined $\forall T > 0$ by

$$\langle u \mid y \rangle_T := \int_0^T u(t) y(t) dt \, . \tag{2}$$

For illustration we may consider u to be an *input voltage* supplied to an electrical load in which case i may be interpreted as the *output current*. Therefore, inner product gives a measure on the energy stored in the resistor.

With these tools we can now properly define the following

Definition 4 (Passivity): An operator $H: u \mapsto y$ is passive if there exists a $\beta \in \mathbb{R}$ such that

$$\langle u \mid y \rangle_T \geq \beta . \tag{3}$$

The number β depends on the *initial conditions* of the signals. Often, it quantifies the initial energy stored in the system. This will become clearer when dealing with passivity of mechanical systems, in section 4.1.

Definition 5 (Output Strict Passivity): An operator $H : u \mapsto y$ is output strictly passive if there exists $\beta \in \mathbb{R}$ and $\delta_o > 0$ such that

$$\langle u \mid y \rangle_T \geq \delta_o \|y\|_{2T}^2 + \beta . \tag{4}$$

Definition 6 (Input Strict Passivity): An operator $H : u \mapsto y$ is input strictly passive if there exists $\beta \in \mathbb{R}$ and $\delta_i > 0$ such that

$$\langle u \mid y \rangle_T \geq \delta_i \|u\|_{2T}^2 + \beta . \tag{5}$$

Passive interconnected systems

The following theorems formalize the fact that passivity is sustained for the interconnection of passive systems.

Theorem 1: Consider the input-output system depicted in Figure 1. Let $e := (e_1, e_2), u := (u_1, u_2)$ and $y := (y_1, y_2)$ be in $\mathcal{L}_{2e}^{2n}$. If $\sum_1$ and $\sum_2$ are both passive then $\sum : u \mapsto y$ is also passive. If $\sum_1$ and $\sum_2$ are OSP then $\sum : u \mapsto y$ is also OSP.

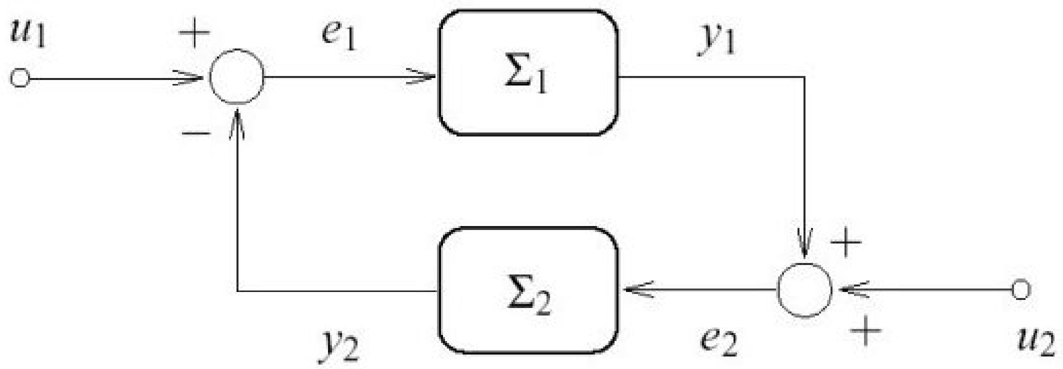

Figure 1: Feedback interconnection of passive systems

The theorem below regards a special case of the feedback interconnection depicted in Figure 1, when $u_2 \equiv 0$. This structure is particularly important since it is the typical case of a plant $(\sum_1)$ in closed loop with a controller $(\sum_2)$. In this case the input

u_1 plays the role of an external signal to the closed loop. Notice that this input can be in its turn the output of another passive block. In this way one can build a *new* passive system upon a core passive block. Therefore these theorems are fundamental to passivity-based control.

Theorem 2: Consider the closed loop system of Figure 1, with $u_2 \equiv 0$. Assume that $\sum_i : \mathcal{L}_{2e}^n \mapsto \mathcal{L}_{2e}^n$, $i = 1, 2$. Then $e_2 = y_1 \in \mathcal{L}_{2e}^n$ if either of the following statements is true:

- If $\sum_1 : e_1 \mapsto y_1$ is passive and $\sum_2 : y_1 \mapsto y_2$ is ISP or
- If $\sum_1 : e_1 \mapsto y_1$ is OSP and $\sum_2 : y_1 \mapsto y_2$ is passive.

So far we have stated formally and in fair generality, under which conditions a feedback interconnection of passive systems yields a passive system. However, it will be also useful to know that interconnections not only preserve the passivity properties of the subsystems but, in certain cases, passivity can be *strengthened*. To illustrate this idea, we briefly discuss next, a technique called *loop transformation*.

Consider the interconnected system of Figure 1 with only one input, i.e., let $u_2 \equiv 0$. Assume that $\sum_2$ is ISP and $\sum_1$ is passive. The loop transformation technique will make evident that since the system $\sum_2$ is "more dissipative" than $\sum_1$ (some readers will know that the term "dissipative" and "passive" are mathematically different.

With an abuse of notation we use here the term dissipative to denote a system which dissipates energy in a non recoverable manner, e.g., heat.), by performing the interconnection 'some' of the "dissipation of $\sum_2$ is propagated to $\sum_1$".

To show this we will use Figure 2, which represents a system *equivalent* to that of Figure1 with $u_2 \equiv 0$, and the following

Fact 1: Assume that the system $\sum_2$ is ISP and has finite $\mathcal{L}_2$ gain, i.e., there exists $\infty > c > 0$ such that $\|y_2\|_{2T} \leq c\|e_1\|_{2T}$. Then, the map $\sum_2$ is also OSP.

Let us perform a few simple calculations to exhibit the new passivity properties of the feedback interconnected system of Figure 2. For $\sum_1' : e_1 \mapsto y_1$, using the passivity property of $\sum_1$, we have that

$$\langle e_1 \mid y_1 \rangle_{2T} = \langle u_1 - y_2 + ky_1 \mid y_1 \rangle_{2T} = \langle u_1 - y_2 \mid y_1 \rangle_{2T} + \langle ky_1 \mid y_1 \rangle_{2T}$$
$$\geq \beta_1 + k\|y_1\|_{2T}.$$

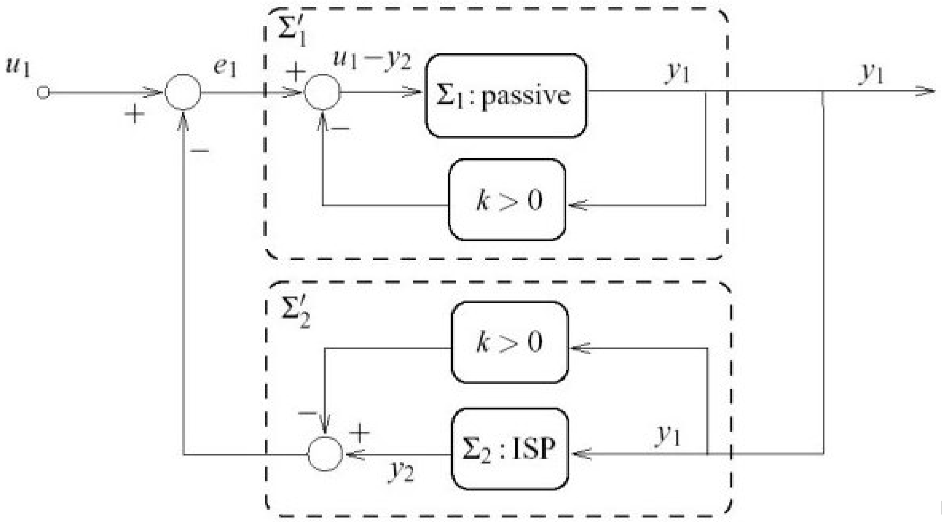

Figure 2: Loop transformed feedback interconnected system

That is, the loop transformation has rendered the map $\sum_1'$, OSP. The price paid for this is that the ISP of $\sum_2$ has been "weakened" more precisely,

$$\begin{aligned}\langle y_1 \mid y_2 - ky_1 \rangle_{2T} &= \langle y_1 \mid y_2 \rangle_{2T} - \langle ky_1 \mid y_1 \rangle_{2T} \\ &\geq \beta_2 + \delta_{i2} \|y_1\|_{2T} - k\|y_1\|_{2T}\end{aligned}$$

hence, we must impose $k < \delta_{i2}$.

It is important to remark at this point that the coefficient k is used only for analysis hence, there is no loss of generality in restricting it to be $k < \delta_{i2}$. Notice also that the physical system has not changed with the loop transformation but only the way we look at it!

Using the Fact 1 we obtain that the system of Figure 1 with $\sum_1$ passive and $\sum_2$ ISP and finite $\mathcal{L}_2$ gain is equivalent to the interconnection of an OSP with an OSP and ISP system.

This observation is sometimes fundamental in the stability analysis of passive systems, and consequently in passivity –based control, as we will see in section 3.2 and 4.2.

3. Stability of Passive Systems

Before discussing PBC we need to discuss about stability. In particular, the type of stability which one pursues in PBC, is, in an input-output sense.

3.1. $\mathcal{L}_2$-Stability

A "relaxed" definition of $\mathcal{L}_2$ stability is that an operator is $\mathcal{L}_2$ stable if it maps $\mathcal{L}_2$ inputs into $\mathcal{L}_2$ outputs. However, in a more strict sense one may also be interested in *measuring* the "amount of stability". This is given by the so-called $\mathcal{L}_2$ gain.

Definition 7 ($\mathcal{L}_2$ **-stability):** The state space system $\sum$ is said to be $\mathcal{L}_2$ stable with finite $\mathcal{L}_2$ gain if there exists a positive constant γ such that for every initial condition $x_0 = x(0)$, there exists a finite constant $\beta(x_0)$ such that

$$\|y(t)\|_{2T} \leq \gamma \|u(t)\|_{2T} + \beta(x_0).$$

Proposition 1: If $\sum : u \mapsto y$ is OSP then it has finite $\mathcal{L}_2$-gain.

Proof. The proof follows straight forward observing that OSP implies the existence of $\delta > 0$ and $\beta \in \mathbb{R}$ such that $\delta_o \|y\|_{2T}^2 \leq \langle u \mid y \rangle_T - \beta + \frac{1}{2}\left\| \frac{1}{\sqrt{\delta_o}} u - \sqrt{\delta_o} y \right\|_{2T}^2$, therefore $\frac{\delta_o}{2}\|y\|_{2T}^2 \leq \frac{1}{2\delta_o}\|u\|_{2T}^2 - \beta$. Thus the $\mathcal{L}_2$ gain $\gamma \leq \frac{1}{\delta_o}$.

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Biographical Sketches

Antonio Loria was born in Mexico in 1969. He got the BSc degree in Electronic Engineering from the

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Henk Nijmeijer (1955) obtained his MSc-degree and PhD-degree in Mathematics from the University of Groningen, Groningen, the Netherlands, in 1979 and 1983, respectively. From 1983 until 2000 he was affiliated with the Department of Applied Mathematics of the University of Twente, Enschede, the Netherlands. Since, 1997 he was also part-time affiliated with the Department of Mechanical Engineering of the Eindhoven University of Technology, Eindhoven, the Netherlands. Since 2000, he is full-time working in Eindhoven, and chairs the Dynamics and Control section. He has published a large number of journal and conference papers, and several books, including the 'classical' Nonlinear Dynamical Control Systems (Springer Verlag, 1990, co-author A.J.van der Schaft). Henk Nijmeijer is editor in chief of the Journal of Applied Mathematics, corresponding editor of the SIAM Journal on Control and Optimization, and board member of the International Journal of Control, Automatica, European Journal of Control, Journal of Dynamical Control Systems, SACTA, International Journal of Robust and Nonlinear Control, and the Journal of Applied Mathematics and Computer Science. He is a fellow of the IEEE and was awarded in 1987 the IEE Heaviside premium.