OPTIMIZATION OF FUZZY CONTROLLERS

J. Wernstedt,
Ilmenau University of Technology, Germany

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Summary

This chapter presents methods of design of optimal control strategies based on fuzzy logic. Fuzzy Control strategies use the expert knowledge and/or the static/dynamic behavior of the controlled plant. The main objective of this chapter is model-based design of optimal robust fuzzy controllers and not the optimal imitation of the control behavior of human experts.

After a short introduction in Section 1, the types of design of optimal control strategies are described in Section 2. The design steps, the modification of optimality criteria, the design of a parametric fuzzy concept and the optimal design strategy are presented in section 3. The integration of the design strategy into the Matlab® Tools as Fuzzy-Control-Toolbox FCD® was discussed (Section 4). Simulation and real-time control results of an unstable mechatronic system demonstrated the performance of the control strategies (Section 5). Finally, some unsolved problems are discussed in Section 6.

1. Introduction
The prime objective of the earliest applications of fuzzy logic in control was the imitation by a machine of the control that a human expert can exert. The only indication for the application of fuzzy control was that classic control methods had failed. The use of fuzzy logic for the control of plant which could be managed by conventional methods was unthinkable, because industrial computers were too expensive at that time. When microcontrollers arrived, they provided cheap and powerful hardware for the implementation of fuzzy control. Fuzzy logic is now as simple to implement as are classic control algorithms. In most cases, conventional methods could equally well be applied. Fuzzy logic has been brought in for the sake of higher quality, greater robustness and a simpler control design. Imitation of the human expert having ceased to be the main goal, there was the chance of formulating and evaluating complete new rule-based control strategies. Unfortunately the design strategy did not much change from what it was in the beginning, during the 70s. Even now, pure heurism in the setting up of rules and the use of “Trial and Error” for the fine adjustment of the membership functions are the most frequent ways of designing fuzzy control. This procedure is not only subjective, but also dangerous.

In recent years, two objective methods for the design of fuzzy control have been developed. The first of these is a method for the primary goal, the best possible imitation of the static and / or dynamic control behavior of a human expert. This is not the main objective method dealt with in this paper. The second method is the model-based design of optimal, relatively robust, fuzzy controllers. For the design of fuzzy systems to such a “best possible” level of control, it is necessary to modify the criteria of non-linear control theory, design a parametric fuzzy concept, and select efficient search strategies. This model-based design of fuzzy controllers is best achieved with Control Design Tools, which permit exports to programmable logic controllers and the sending of C source code to industrial PC’s.

2. Basic Principles of Optimization

To be optimal a feature should be defined by a criterion which is mathematically formulated. Thus, the term optimal control only applies if the criterion for optimization covers all the technical requirements of the control problem. Moreover, neither the models of the plant in question nor the exemplary control activities of the human experts will limit the optimum obtainable. When a classic optimum state controller is designed, the state space model of the real plant plays the central role. If this model is inexact, the controller is only the optimum for the inexact model, but useless in practice. Equally the human expert as an example of control may be a criterion of whether the fuzzy (mode) control is at its optimum. No mode of control designed can improve on this example.

Thus it is important to decide which of the two design strategies should be used as demanded by given control design task. Where fuzzy mode control is the title given to the first strategy here mentioned and fuzzy control the title given to that based on control theory. The opportunity of observing and recording the activities of a human expert who is in control of very complex plant can be the basis of training a fuzzy mode controller to imitate expert’s behavior. This procedure is appropriate if the plant is too complex for a structured control design. But if the plant can be modeled and simulated
in any way, the control design which relies on the criteria of control theory is preferable. The conditions for and goals of both strategies are summarized in the following table, Table 1, where fuzzy mode control is the title given to the first strategy here mentioned, and fuzzy control the title given to that based on control theory.

<table>
<thead>
<tr>
<th>Method:</th>
<th>Fuzzy mode control</th>
<th>Fuzzy control</th>
</tr>
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| Conditions for use: | Comprehensive training data, obtained from  
|  | interviews  
|  | recorded data  
|  | A priori knowledge about the plant and the appropriate control structure  
|  | static behaviour  
|  | dynamic behaviour  
|  | details obtained from simulations  
|  | a control strategy  
|  | criteria of relevance to the functioning |
| Goal:         | optimal imitation of a human expert  
|  | optimal quality of control, to achieve the predefined criteria |

Table 1: Conditions for and goals of optimal control strategies

3. Optimal Design of Fuzzy Controllers

3.1. Design Steps

Model-based design as a strategy for optimum fuzzy control allows the controller design to be performed on the plant or model directly, as it is based on analytic models and / or neural networks and / or linguistic (fuzzy) models. For the optimization, non-linear mathematical criteria are used.

The main advantage of this strategy is that it does not depend on there being any data, of whatever quality; moreover, it is possible to take account of the available linguistic knowledge in the fuzzy rules. One disadvantage of the strategy is that the plant should not be in a critical state, because unstable states during the optimization might destroy it. To counter the disadvantage, a sufficiently exact model of the process is necessary. The strategy combines three main ideas, firstly, the modification of known criteria of quality of control for the non-linear case, secondly, the parameterization of the fuzzy logic, i.e., reduction of the large number of degrees of freedom to get an optimizable structure, and, last but not the least, the selection of appropriate search strategies for parameter optimization.

The overall goal is the design of an optimum fuzzy controller with the behavior
\[ u(kT) = f(s(kT), m_1(kT), m_2(kT), u[(k-i)T]) \]  

The structure of the fuzzy controller is shown in Figure 1.

For the design of the optimal fuzzy controllers as **non-linear sampled multivariable controllers** three steps are necessary:

- **Step 1:** Design of static behavior of the linear or non-linear fuzzy system
- **Step 2:** Design of dynamic behavior of fuzzy controllers
  [P-, I-, D-fuzzy controllers and combinations]
- **Step 3:** Optimal design of the fuzzy controllers, achieved by using
  - multiple quality criteria
  - sequences of set-point.

![Figure 1: Structure of fuzzy controller](image)

### 3.2. Design of Fuzzy Systems and Fuzzy Controllers

In this section, steps 1 and 2, the design of the static and the dynamic behavior of fuzzy systems, are described. As mentioned above, the problem of parameterization of the fuzzy concept demands that the large number of degrees of freedom of the common fuzzy logic concept be reduced. This will be achieved if:

- the rule set is treated as invariant
- non-linear, parametric membership functions for the fuzzy inputs are defined (Figure 2).

Each membership function has a maximum of four gridpoints \( (p_1, p_2, p_3, p_4) \). For the values \( \mu_i(x) \) of membership functions the following is true:
* $0 \leq \mu(x) \leq 1$
* $\sum_{i=1}^{n} \mu_{A_i}(A) = 1 \quad (2)$

The same design is possible using the membership functions [MBF] of Zadeh and Bocklisch (Figure 3).

- MIN or PROD is used as AND operator and MAX and OR as aggregation operator
- singletons are used as membership functions for the fuzzy output terms.

The design of typical static behavior for a SISO-fuzzy system is shown in Figure 4.

Figure 2: Linear, parametric membership functions shown segment by segment

Figure 3: Non-linear parametric membership functions of Zadeh and Bocklisch
When it comes to the design of the dynamic non-linear behavior of fuzzy controllers, the investigation of basic types of linear dynamic fuzzy controllers is necessary, mainly so that the sampled period $T$ in the linguistic variables can be included.

The basic types are:

**Figure 4: Design of static behavior of a SISO-fuzzy system**

The characteristic static behavior for a MISO-fuzzy system (2 inputs, 1 output) is shown in Figure 5.

**Figure 5: Design of static behavior of a MISO-fuzzy system**
- **Fuzzy-P-Controller**

  Control law:
  \[ u(k) = K_p \cdot e(k) \rightarrow u(k) = K_p \cdot e(k) \]

  **Feature:** \[ E = e(k) \]

  **Structure:**
  \[
  \begin{array}{c}
  F \\
  \text{Fuzzy} \\
  \text{P-Controller} \\
  U \\
  \end{array}
  \]

  **Rule set:**
  - IF \( E = N \) THEN \( U := N \)
  - IF \( E = Z \) THEN \( U := Z \)
  - IF \( E = P \) THEN \( U := P \)

  **MBF’s:**

  Characteristic/step response:

  ![Figure 6: Behavior of the Fuzzy-P-Controller](image)

- **Fuzzy-I-Controller**

  Control law:
  \[ u(k) = \frac{1}{T_i} \int_0^k e(\tau) d\tau \rightarrow u(k) = \frac{T}{T_i} \sum_{i=0}^{k} e(i) \]

  **Feature:** \[ \text{SEK} = \sum_{i=0}^{k} e(i) \]

  **Structure:**
  \[
  \begin{array}{c}
  \text{SEK} \\
  \text{Fuzzy} \\
  \text{I-Controller} \\
  U \\
  \end{array}
  \]

  **Rule set:**
  - IF \( \text{SEK} = N \) THEN \( U := N \)
  - IF \( \text{SEK} = Z \) THEN \( U := Z \)
  - IF \( \text{SEK} = P \) THEN \( U := P \)

  **MBF’s:**

  **Step response:**

  ![Figure 7: Behavior of the Fuzzy-I-Controller](image)

- **Fuzzy-D-Controller**
An example of a combined type is

- **Fuzzy-PID-Controller**

![Diagram of Fuzzy-PID-Controller](image-url)

Figure 9: Behavior of the Fuzzy-PID-Controller
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Biographical Sketch

Jürgen Wernstedt was born in Hall/Saale, Germany, in 1940. Since 1985 he is Professor for System Analysis (1985- ), Ilmenau Technical University, Department of Informatics and Automatic Control, Germany.

He teaches courses in automatic control, modeling, system analysis, fuzzy control, decision systems. The research areas are modeling, knowledge based system, fuzzy control. He has been responsible for 32 Ph. D. Thesis, published more than 120 technical papers in journals and technical meetings and is author / co-author of two internationally published books to modeling and fuzzy control.
Since 1995 he is the leader of the Application Center System Technology Ilmenau of the Fraunhofer Society. The activities are the planning and management of large-scale energy and water-supply system.