FUZZY CONTROL SYSTEMS

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Contents

Summary

This chapter presents a perspective of fuzzy control systems. Fuzzy control is a form of intelligent control characterized by the use of expert knowledge on the control strategy and/or the behavior of the controlled plant. This expert knowledge is represented by means of IF-THEN rules and linguistic variables. Attributes or values of these linguistic variables are linguistic terms associated with fuzzy sets, a generalization of ordinary ("crisp") sets. Fuzzy set theory is the theoretical basis underlying information processing in fuzzy control systems. From the systems theory's view, a fuzzy controller is a static nonlinear transfer element incorporated into a control loop. This gives rise to

methods for analysis and systematic design. Fuzzy systems may perform different tasks within an automatic control system leading to different structural schemes. Nowadays, fuzzy control systems are successfully applied in many technical and non-technical fields. The application of fuzzy control systems is supported by numerous hardware and software solutions.

1. Introduction

Fuzzy control has been a new paradigm of automatic control since the introduction of fuzzy sets by L. A. Zadeh in 1965. Its rationale can be summarized by the statement of Zadeh "As complexity rises, precise statements lose meaning and meaningful statements lose precision." Thus, fuzzy control is an attempt to meet the challenges of increasing complexity of the processes to be controlled and of the tasks to be solved by automatic control systems.

To be more concrete, fuzzy control may be an advantageous alternative to conventional control techniques if

- the process to be controlled exhibits a pronounced nonlinear behavior,
- no mathematical model of the process is available because the modeling effort is unacceptably high or the process is not well understood,
- expert knowledge plays a key role in controlling the process and should be acquired and used for automatic control, or
- a multidimensional nonlinear relationship (e.g. a control law) should be represented such that it can be understood and modified easily.

Fuzzy control systems may be considered under various aspects: A fuzzy controller may be seen as a nonlinear controller described by linguistic rules rather than differential equations. Or a fuzzy control system may be seen as the implementation of the control strategy of a human expert. Understanding the functioning of fuzzy control systems, i.e. the information processing taking place within the fuzzy control system and its interaction with the plant and other components of the automatic control system requires knowledge of fuzzy logic and control theory.

The aim of this chapter, therefore, is

- to introduce the basic ideas of fuzzy control by means of a simple example (Section 2),
- to provide the essential theoretical bases of fuzzy systems (Section 3), and
- to discuss the control issues of fuzzy control (Section 4).

2. Fuzzy Control - A Simple Example

2.1. Example

In the following section, a simple and illustrative example will be used to explain information processing in fuzzy systems:

Example 1. (**Control of room temperature)** The temperature of a room equipped with a hot water heating should be controlled by adjusting the position of the valve at the radiator (see Fig. 1). A human being would use meta-rules, such as
If things are not OK but change in the right direction then maintain present settings
or more specifically

If the temperature is too warm but decreases, then leave valve position unchanged or
If the temperature is too cold and decreases, then increase the valve opening significantly.

Starting from these meta-rules, an experienced user would develop a set of control rules which are more specific regarding the linguistic description of the values of temperature, temperature change, and change of valve position.

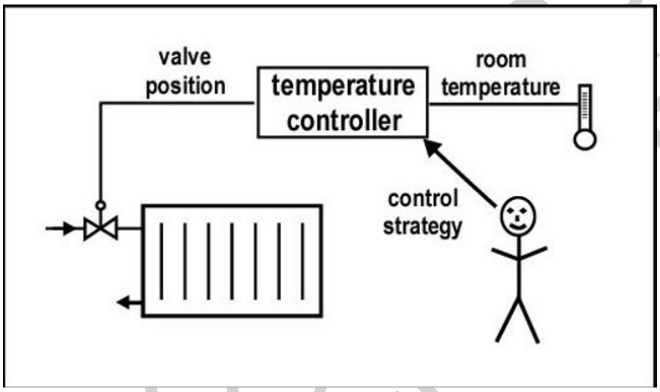

Figure 1: Schematic representation of the control for Example 1

Fuzzy systems provide a means to represent and process expert knowledge as stated in the example above. By treating them as knowledge-based systems, the separation of knowledge representation and information processing is realized. Knowledge is represented in form of rules and the meaning of the expressions or symbols appearing in them. In Example 1, the knowledge consists of the control rules and the meaning of the linguistic labels describing the values of temperature, temperature change, and change of valve position (valve change for short). The formal concepts for knowledge representation in fuzzy systems are *linguistic variables* and *fuzzy IF-THEN rules* presented in the next subsection.

The *inference engine* forms the core of the information processing components. As part of a control system, a fuzzy system/controller usually processes numerical inputs to numerical outputs. Therefore, *fuzzification* and *defuzzification* supplement inference:

- Fuzzification: Transformation of numerical values, e.g. measurements, into a fuzzy representation of the input situation,
- Inference: Transformation of the fuzzy input representation into a fuzzy deision, and
- Defuzzification: Transformation of the fuzzy decision into a real decision, e.g. a real value of a manipulated variable.

Fig. 2 shows a schematic representation of a fuzzy system whose components will be described in the following using Example 1 as a basis.

2.2. Fuzzy Sets, Linguistic Variables and Fuzzy IF-THEN Rules

By means of Example 1, it will be shown first how the formal concepts of a linguistic variable with their linguistic terms and membership functions and of a fuzzy rule are used to represent the available knowledge. The notion of a *linguistic variable* formalizes the practices of many domains to describe the values of certain variables in terms of natural language. For Example 1, a linguistic variable is the *temperature deviation* expressing the discrepancy between the desired and the actual room temperature, denoted as $T_{Dev} = T_{desired} - T_{room}$, with the linguistic terms *TOO WARM*, *OK* and *TOO COLD*. A second one is the *temperature change* ΔT with the terms {*INCR*, *EQUAL*, *DECR*} where *INCR* stands for INCREASING and *DECR* for DECREASING.

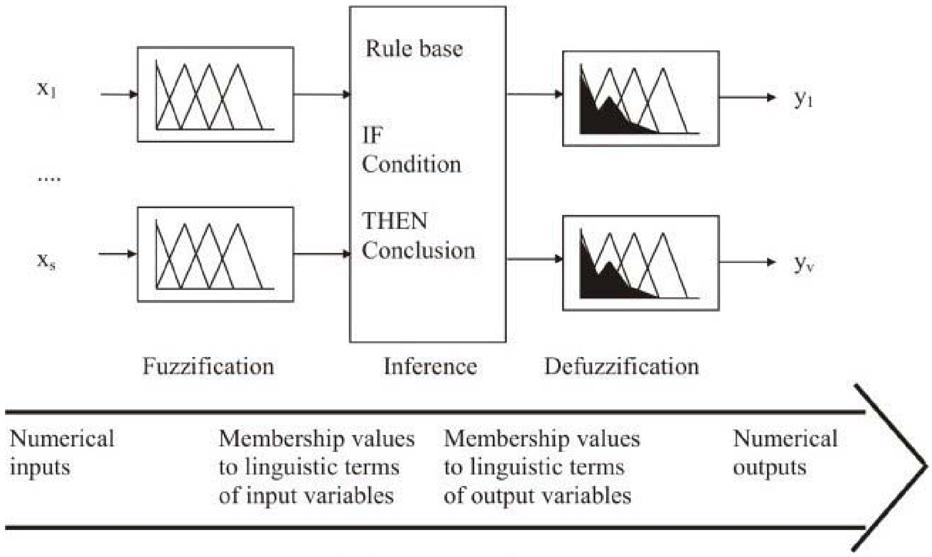

Figure 2: Structure of a fuzzy system with numerical inputs $x_1,\ldots,x_s$ and numerical outputs $y_1,\ldots,y_v$

Temperature change is the difference between the current room temperature and the room temperature at the last time instant. Assuming a constant sampling time the temperature difference is proportional to the temperature trend. It should be noted that the expression of temperature in the example has two different 'meanings'. Specification is done by using different linguistic terms: *TOO WARM* specifies a difference between the desired and the room temperature and *INCR* a temperature change. In contrast to this, the term *WARM* could characterize the room temperature itself.

A third linguistic variable is the *valve change* y of the radiator, the output of the fuzzy controller. Valve change is an incremental variable, such that the actual valve opening results from the previous opening plus valve change both expressed in percent. The term set $\{NB, NS, ZE, PS, PB\}$ uses standardized names with the typical abbreviations *B* (big), *M* (medium), *N* (negative), *P* (positive), *S* (small), *ZE* (zero). These abbreviations will be combined in names as *NB* (negative big) and so on. In practical applications, a linguistic variable usually has between two and seven linguistic terms. This corresponds to the result of psychological investigations stating that human beings differentiate a maximum of five to seven objects at the same time.

The next problem is to define the 'meaning' of each linguistic term. In many real-world problems, the decision, whether a given x (e.g. a temperature deviation) satisfies a certain property *A* (e.g. *TOO COLD*) or not is impossible or not reasonable. In the example above, a human being would not consider a small temperature deviation (e.g. $T_{Dev} = 0.01\,K$) as *TOO COLD*, whereas deviations of 2 K or 5 K probably would be felt as *TOO COLD*, to a certain extent at least. In other words, the membership of x in the subset A should be a matter of degree as x satisfies the property up to a certain degree.

According to set theory, each ordinary subset A of the universe of discourse X is determined by its characteristic function $\mu_A : X \to \{0,1\}$. This means that $\mu_A(x) = 1$ when x is an element of A and zero when it is not. The value of $\mu_A(x)$ can be interpreted as the truth value of a proposition 'x is an element of A' relating set theory with logic.

Zadeh proposed to introduce a *fuzzy set* as a generalization of ordinary ("crisp") sets. This means that the proposition of 'x is an element of A' is no longer true or false, but may be true with a certain degree (*fuzzy truth value*). The characteristic function $\mu_A(x)$ or *membership function* of the fuzzy (sub-) set A is allowed to assume real values between 0 and 1:

$$\mu_A : X \to [0,1]. \qquad (1)$$

Such fuzzy sets associated with the linguistic terms in Example 1 are depicted in Fig. 3 and Fig. 4, respectively.

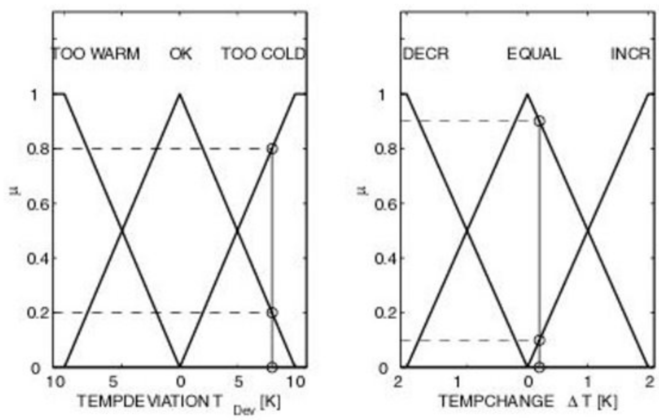

Figure 3: Membership functions and fuzzification for *temperature deviation* (left) and *temperature change* (right)

Statements as, for example, '$\mathrm{T_{Dev}}$ is TOO COLD' are called *fuzzy propositions* as the truth value of such a statement is a matter of degree. It is determined by the membership degree of $\mathrm{T_{Dev}}$ in the fuzzy set labeled *TOO COLD*. Using connectives such as AND and OR, compound fuzzy propositions can be formed.

A *fuzzy IF-THEN rule* or fuzzy conditional statement is expressed as

IF < fuzzy proposition > THEN < fuzzy proposition >

where <fuzzy proposition> is a simple or compound fuzzy proposition. For Example 1 a fuzzy rule of a controller might be 'IF *temperature deviation* is *TOO COLD* AND *temperature change* is *INCR* THEN *valve change* is *ZE*'. The IF part is called premise, condition or antecedent, the THEN part conclusion or consequence.

In Example 1, the rule premises contain the two linguistic input variables *temperature deviation* $\mathrm{T_{Dev}}$ and *temperature change* $\Delta\mathrm{T}$, while the conclusions contain the linguistic output variable *valve change* y. The complete rule base is shown in Table 1.

			T_{Dev}	
y		*TOO WARM*	*OK*	*TOO COLD*
	INCR	R_1: *NB*	R_2: *NS*	R_3: *ZE*
ΔT	*EQUAL*	R_4: *NS*	R_5: *ZE*	R_6: *PS*
	DECR	R_7: *ZE*	R_8: *PS*	R_9: *PB*

Table 1. Rule base of Example 1

2.3. Fuzzification - From Measurements to a Fuzzy Representation of the Input Situation

The inputs of a fuzzy system, especially a fuzzy controller, are (crisp) values of some variables, e.g. measurement signals. The vector of these values characterizes an input situation which may be the system state, for example. Likewise, rule premises specify such input situations, but this specification uses linguistic terms for the values of the input variables. In order to determine the degree of fulfillment of the (compound) fuzzy proposition in the premise the truth values of the simple propositions have to be known. It is the task of fuzzification to provide these values.

If the input variables assume the values of $\mathrm{T_{Dev}} = 8\ \mathrm{K}$ and $\Delta\mathrm{T} = 0.2\ \mathrm{K}$, fuzzification in Example 1 results in

$$\text{temperature deviation } \mathrm{T_{Dev}}: \quad \mu_{\mathrm{TOO\ WARM}}(8) = 0.0, \quad \mu_{\mathrm{OK}}(8) = 0.2, \quad \mu_{\mathrm{TOO\ COLD}}(8) = 0.8,$$
$$\text{temperature change } \Delta\mathrm{T}: \quad \mu_{\mathrm{INCR}}(0.2) = 0.1, \quad \mu_{\mathrm{EQUAL}}(0.2) = 0.9, \quad \mu_{\mathrm{DECR}}(0.2) = 0.0.$$

A graphic explanation is depicted in Fig. 3.

This transformation is unique, but not one to one in general. In the example, only $\Delta T \geq 2$ can be deduced for a given $\mu_{\mathrm{INCR}}(\Delta T) = 1$, $\mu_{\mathrm{EQUAL}}(\Delta T) = 0$.

2.4. Inference - From a Fuzzy Input Representation to a Fuzzy Decision

Inference essentially consists of three steps, namely,

- aggregation,
- activation, and
- accumulation.

Aggregation A rule premise in general is a compound fuzzy proposition (e.g. an AND connection of two propositions with T_{Dev} and ΔT). Its degree of fulfillment $\mu_{\mathrm{P_k}}$ results from the aggregation of the truth values of the simple propositions given by fuzzification. The operations have to be chosen in accordance with the connectives (AND, OR) between simple propositions. The connective AND is related to the *intersection*, OR to the *union* of two (fuzzy) sets.

	$\mu_B(x)=0$	$\mu_B(x)=1$
$\mu_A(x)=0$	$\mu_{A\cap B}(x)=0$	$\mu_{A\cap B}(x)=0$
$\mu_A(x)=1$	$\mu_{A\cap B}(x)=0$	$\mu_{A\cap B}(x)=1$

	$\mu_B(x)=0$	$\mu_B(x)=1$
$\mu_A(x)=0$	$\mu_{A\cup B}(x)=0$	$\mu_{A\cup B}(x)=1$
$\mu_A(x)=1$	$\mu_{A\cup B}(x)=1$	$\mu_{A\cup B}(x)=1$

Table 2. Intersection (left) and union (right) of crisp sets defined by their characteristic functions

The definitions in Table 2, valid for crisp sets, have to be generalized for fuzzy sets. According to the original proposal of Zadeh, the *intersection* $A \cap B$ and the *union* $A \cup B$ can be defined *point-wise* using the respective membership degrees

$$\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \tag{2}$$

$$\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}. \tag{3}$$

For Example 1, the results of aggregation using a minimum for the AND connective are given in Table 3.

			T_{Dev}	
		$\mu_{TOO\ WARM}(T_{Dev})=0.0$	$\mu_{OK}(T_{Dev})=0.2$	$\mu_{TOO\ COLD}(T_{Dev})=0.8$
	$\mu_{INCR}(\Delta T)=0.1$	$\mu_{P1}=0.0$	$\mu_{P2}=0.1$	$\mu_{P3}=0.1$

ΔT	$\mu_{EQUAL}(\Delta T)$=0.9	μ_{P4}=0.0	μ_{P5}=0.2	μ_{P6}=0.8
	$\mu_{DECR}(\Delta T)$=0.0	μ_{P7}=0.0	μ_{P8}=0.0	μ_{P9}=0.0

Table 3. Aggregation for Example 1 with the minimum operation ($T_{Dev} = 8$ K, $\Delta T = 0.2$ K)

Activation A fuzzy IF-THEN rule is a connection of two (compound) fuzzy propositions. Hence, this connective has to be interpreted within the framework of set theoretic or logical operators. The simplest interpretation is that of the conjunction of premise and conclusion, such that the appropriate operation is the minimum. Thus, the result of activation μ_{C_k} of a rule k is the minimum of the degree of fulfillment μ_{P_k} and the fuzzy set in the conclusion. In other words, the fuzzy set in the conclusion is clipped to μ_{P_k}.

In Example 1, activation leads to nonempty fuzzy sets as depicted in Fig. 4a for rules 2,3,5, and 6, only.

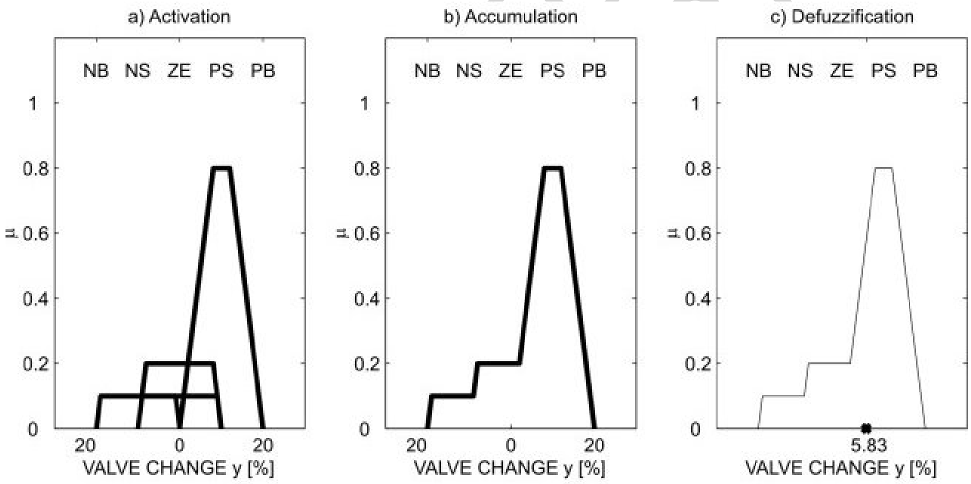

Figure 4: Results of a) activation (only positive for rules R_2, R_3, R_5, R_6), b) accumulation, and c) defuzzification for Example 1

Accumulation Usually, a rule base is interpreted as a disjunction of rules i.e. rules are seen as independent "experts". Accumulation has the task to combine the individual "expert statements", which actually are fuzzy sets of recommended output values. Consequently, an appropriate accumulation operation is the maximum. The maximum of the activated rule conclusions $\max_Y\{\mu_{C_k}(y)\}$ is a fuzzy set $\mu(y)$ over the domain Y of the output variable. The membership degree $\mu(y)$ can be interpreted as the degree to which the value y is suggested by the "expert committee" to be the real output value.

In Example 1, accumulation results in a fuzzy set $\mu(\mathrm{y})$ as shown in Fig. 4b.

To summarize, inference yields the output fuzzy set as the maximum of all clipped fuzzy sets of the linguistic terms of the output variable. The clipping results from applying the minimum operation between each output fuzzy set referred to in the rule conclusion and the degree of fulfillment of the rule premise. This degree of fulfillment is calculated from the results of fuzzification by applying the minimum for the AND and maximum for the OR connective. This inference scheme, using minimum and maximum operations is called max-min inference.

2.5. Defuzzification - From a Fuzzy Decision to a Real Decision

As inference results in a fuzzy set, the task of defuzzification is to find the numerical value which "best" comprehends the information contained in this fuzzy set. A frequently used method is the so-called Center-of-Gravity defuzzification (CoG; also called Center-of-Area defuzzification COA):

$$\mathrm{y} = \frac{\int_{\mathrm{Y}} \mu(\mathrm{y})\,\mathrm{y}\,\mathrm{dy}}{\int_{\mathrm{Y}} \mu(\mathrm{y})\mathrm{dy}}, \tag{4}$$

which chooses the y-coordinate of the center of gravity of the area below the graph $\mu(\mathrm{y})$. This defuzzification can be interpreted as a weighted mean, i.e. each value y is weighted with $\mu(\mathrm{y})$ and the integral in the denominator serves for normalization. In Example 1, defuzzification of $\mu(\mathrm{y})$ using Center-of-Gravity defuzzification yields a value of 5.83 %, as shown in Fig. 4c.

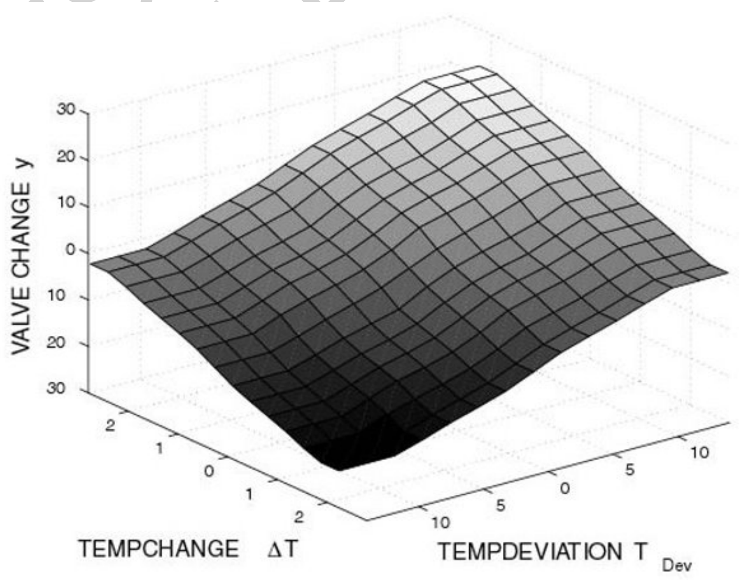

Figure 5: Control surface of the fuzzy controller in Example 1

The characteristic surface of the fuzzy controller or control surface, that is the graphic representation of the function $y(T_{Dev}, \Delta T)$, is depicted in Fig. 5. Here, the tasks of fuzzification, inference, and defuzzification have been performed for all possible combinations of $T_{Dev}, \Delta T$ in the universe of discourse (with some reasonable discretization).

3. Fuzzy Logic-related Issues in Fuzzy Control

In the previous section, only the simplest possible type of a fuzzy system has been discussed. In this section the theoretical fundamentals of fuzzy systems are introduced. This introduction goes as far as necessary for the understanding of key concepts of knowledge representation and information processing in fuzzy systems. Furthermore, it is intended to reveal alternatives to choices made in the previous section and again uses Example 1 for illustration.

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