INTELLIGENT CONTROL OF ROAD VEHICLES FOR AUTOMATED DRIVING: PATH ARCHITECTURE FOR AUTOMATED HIGHWAY SYSTEMS AND LATERAL GUIDANCE

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Summary

The California Partners of Transit and Highway (PATH) Program has conducted research on intelligent transportation systems (ITS) since 1987. Automated highway systems (AHS) represent one of the most sophisticated ITS. In this paper, after stating the history of the AHS and the PATH AHS architecture, the development of lateral controllers for AHS by PATH researchers will be reviewed. Three kinds of lateral controllers, each based on frequency shaped linear quadratic (FSLQ) control, sliding mode control and loop shaping control, will be presented. An evolutionary aspect of the three different controllers will be mentioned. There has been a natural shift of emphasis...
from passenger vehicles to heavy duty vehicles (HDV) in the PATH program. Some recent works on the lateral control of HDV will be described in the paper. PATH's accomplishments on lateral controls are of value for not only AHS applications but also almost any automated driving including driver assistance.

1. Introduction

The California Partners for Advanced Transit and Highway (PATH) Program was established in 1986 to study and promote the application of advanced technology to help meet California’s growing need for increased highway capacity to relieve congestion. The idea of Automated Highway Systems (AHS) was identified to be an attractive option for solving the congestion problem while improving the operation of highways in many regards including safety, fuel economy and pollution. The research on AHS has been a major part of the PATH program. The concept of AHS is not a new one. An exhibit at the GM Pavilion of the 1939 World’s Fair in New York aroused a good deal of interest. However, the major motivation for AHS, i.e., congestion, was not severe enough and the required technologies were not mature enough to arouse more than a passing interest. Another surge of interest in AHS occurred in the early 1970's when the newly created U.S. Department of Transportation tried to focus recent advances in aerospace technology on the ground transportation problems. Again, the seriousness of the congestion problem and the state of the art control/communication/computer technology did not justify any widespread development programs. The PATH’s efforts can be regarded as the third wave of interest. PATH was a core member of the National Automated Highway Systems Consortium (NAHSC), which was formed under the auspice of the U.S. Department of Transportation. NAHSC demonstrated key technologies of AHS in 1997. While the demonstration was a technical success, NAHSC was dissolved in 1998. PATH continues to conduct research on AHS and related technologies.

This paper describes the research and development of the lateral control systems for AHS conducted at PATH. In order to place the PATH’s lateral control work in a proper perspective, the PATH AHS architecture must be understood (see PATH AHS architecture). In this architecture, the AHS system is structured as a hierarchical control system, and its implementation requires the use of advanced system theory concepts such as discrete event systems as well as communications among vehicles and between vehicles and roadside computers. The vehicle lateral control problem is a fundamental vehicle level control problem, which makes PATH’s effort valuable not only for AHS but also for other applications such as automated steering systems for driving assistance and safety enhancement. The vehicle lateral control system for AHS has two major functions: 1) lane following and 2) lane changing. Primary focus in this paper is on the lane following problem for vehicles with front wheel steering. In this case, technical challenges include: 1) the lateral controller must assure good riding comfort in addition to lane following capability; 2) despite the terminology, "vehicle lateral control", the steering input affects both the lateral motion and the yaw motion, which must be simultaneously controlled to properly address the first challenge; 3) the vehicle dynamics depend on variables such as the vehicle speed and the tire pressures as well as parameters such as tire-road adhesion coefficients. The second challenge implies that the vehicle lateral control system is an under-actuated control system. Further
challenges are coming from the road reference systems, which will be addressed later in this paper.

The remainder of this paper is organized as follows. In the next section, the PATH AHS architecture is explained. Section 3 presents vehicle models for lateral control and some fundamental issues arising from the vehicle dynamics coupled with the location of sensors and the selection of control algorithms. Section 4 introduces a magnetic road reference and sensing system, an important element of the vehicle lateral control system. In Section 5, the lane following control algorithms for single-unite vehicles are presented. Section 6 is on a recent PATH’s effort on the lateral control of heavy duty vehicles. Concluding remarks are given in Section 7.

2. AHS Architecture

A major objective of the AHS is to increase the capacity and safety of highway systems. To achieve this objective, PATH adopted the idea of platooning. A platoon is one or more vehicles traveling together as a group with relatively small spacing. For example, with an average platoon size of 15, intra-platoon distance of 2 m, inter-platoon distance of 60 m, vehicle length of 5 m, and speed of 72 km/h, the maximum flow or capacity through an automated lane is over 6,000 vehicles/lane/hour. This capacity is about three times as large as the maximum capacity under manual driving. The optimal speed and size of platoons may depend on traffic conditions, and the maneuvers of platoon formation and dissolution require coordination among nearby platoons. The small intra-platoon distance requires tight longitudinal control, which involves communication among the vehicles in a platoon. On automated highways, each vehicle is under lateral control. The on-board vehicle controller must determine the command signal to the steering actuator to either follow a road reference to stay in a designated lane or change from one lane to another. The lane change maneuver requires coordination among nearby platoons. Based on these considerations, Varaiya proposed the AHS architecture in Figure 1.

The proposed AHS control architecture defines a hierarchical control system. In this structure, tasks are distributed among four control layers: the regulation layer, the coordination layer, the link layer and the network layer. Control algorithms for the regulation and coordination layers are implemented on vehicle computers, and those for the link layer are implemented on roadside computers. Thus, intelligence is distributed between the roadside systems and the vehicle systems.

The link layer broadcasts, for each 1 or 2 km stretch of highway, or link, targeted values for speed and platoon size for vehicles on that link, based on information about the aggregate traffic state (speed, density, flow). Using, in addition, estimates of the proportion of the traffic flow destined for the various exits, the link layer also advises vehicles where to begin changing lanes in order to reach their exits. The link layer also receives from the network layer information about incidents or congestion in downstream links, and based on that information, it may reassign vehicle paths. A description of a link layer controller consistent with the PATH AHS architecture has been found. Lyapunov based control laws may be developed to stabilize the actual traffic state defined by a pair of density and velocity profile to the desired values.
Each vehicle's coordination layer determines which maneuver to initiate at any time so that it will conform to its assigned path; coordinates that maneuver with neighboring vehicles so that the maneuver can be undertaken safely; and then commands the regulation layer to execute the control law that implements that maneuver. After some time, the regulation layer reports to the coordination layer that the maneuver is completed or aborted for reasons not anticipated by the coordination layer. Three basic maneuvers are proposed (see maneuver protocols): join, which permits a platoon to join the platoon ahead of it; split, which separates a platoon into two platoons; and lane change, which permits a one-car platoon to change lane. Each maneuver is coordinated by a structured exchange of messages - protocol – among relevant neighboring vehicles.

The plant, "the vehicle dynamics," is directly controlled by the regulation layer. The regulation layer provides feedback control laws to execute the maneuvers and more basic longitudinal and lateral control laws such as spacing control and steering control. The longitudinal control and lateral control for AHS provide a number of challenges, and they have been extensively studied by PATH. The lateral lane guidance control problems will be described in the following sections. Longitudinal control laws must assure that a short intra-platoon distance can be robustly maintained. For this purpose, sliding mode control has been found to be appropriate (see sliding mode control for platooning). An important issue in longitudinal control is string stability; namely, a platoon must be string stable in the sense that the effect of disturbances should be attenuated from one vehicle to the next as it propagates to upstream. The propagation takes place because the primary input to the longitudinal controller for each vehicle in a
platooon is the distance between the vehicle and its preceding vehicle. String stability is now well understood (see string stability of platooning).

The regulation layer has been extensively tested by experiments. In fact, the success of the eight-car-platoon demonstration at the 1997 NAHSC Demonstration was largely due to nearly ten years of research on longitudinal and lateral control conducted by PATH. The coordination layer has been partially tested by experiments. The study of coordination and link layer controllers, however, has mostly been based on simulations. PATH has developed several new simulation tools for this purpose (see PATH simulation tools).

3. Vehicle Models for Lateral Control

The lateral dynamics of vehicles have been studied since the late 1950's. A three-degree-of-freedom vehicle model was developed to describe the vehicle directional responses, which include the yaw, lateral and roll motions (see three degree-of-freedom vehicle model). The vehicle forward speed was considered to be the major stability parameter and was assumed to be constant. Most of previous works on vehicle lateral control have relied on a simplified dynamic model that retains the lateral and yaw motions. This model is usually referred to as the bicycle model (see vehicle lateral control based on the bicycle model). Vehicle models with more degree-of-freedoms were also developed (see vehicle models with higher degree-of-freedoms). Since all the external forces for vehicle motion come from the ground via tire-ground interaction, a tire model is a very important element of any vehicle lateral dynamic model. The major input for the lateral control system is the front steering angle. Secondary inputs include the rear steering angle for four wheel steering vehicles and the brake forces. In this paper, we will consider bicycle models for a lane following situation as depicted in Figure 2. In the figure, \( y_{CG} \) and \( \varepsilon \) denote the lateral deviation of the vehicle from the road center line at the vehicle’s center of gravity (CG) and the vehicle yaw angle relative to the road, respectively, \( \delta \) is the front steering angle, \( \rho \) is the curvature of the road and \( y_s \) is the lateral tracking error measured by a sensor located ahead of the vehicle’s CG by a distance \( d_s \) in the road coordinate system. The Newton-Euler equations for the vehicle lateral control system are given by (see the Newton-Euler equations for vehicle lateral control)

\[
\begin{align*}
    m \ddot{y} &= -m \dot{\varepsilon} \dot{V}_x + C_{atf} \left( V_y - \dot{\varepsilon} l_2 \right) \frac{C_{atl}}{V_x} \frac{V_y + \dot{\varepsilon} l_1}{V_x} + C_{atf} \delta \\
    I_z \ddot{\varepsilon} &= l_2 \frac{C_{atf} \left( V_y - \dot{\varepsilon} l_2 \right)}{V_x} - l_1 \frac{C_{atf} \left( V_y + \dot{\varepsilon} l_1 \right)}{V_x} + l_1 C_{atf} \delta
\end{align*}
\]

where \( V_x \) and \( V_y \) are the components of the vehicle velocity along the longitudinal and the lateral principle axis of the vehicle body at the center of gravity and \( \dot{\varepsilon} \) is the yaw rate. \( m \) and \( I_z \) are the mass and the yaw moment of inertia, respectively. \( l_1 \) and \( l_2 \) are distances of the front and rear axle from the vehicle’s CG, and \( C_{atf} \) and \( C_{atl} \) are the front and lateral factors.
and rear tire cornering stiffness, respectively.

![Bicycle Model for Lateral Control](image)

Figure 2: Bicycle Model for Lateral Control

Transformation to the \( y_s \) and \( \varepsilon_r \) coordinates is given by

\[
\dot{y}_s = V_y \cos(\varepsilon_r) + V_x \sin(\varepsilon_r) + d_s \dot{\varepsilon}_r
\]

where \( d_s \) is the distance of the measurement point from the vehicle’s CG (see Figure 2). Linearization of the resulting model is

\[
\begin{align*}
\dot{y}_s &= f_1 + b_1 \delta \\
\dot{\varepsilon}_r &= -\frac{l_1 C_{at} - l_2 C_{at}}{l_x V_x} \dot{y}_s + \frac{l_1 C_{at} - l_2 C_{at}}{l_z} \dot{\varepsilon}_r + \frac{C_{at} l_1}{l_z} \delta \\
&= -\frac{C_{at} (l_1^2 - l_1 d_s)}{l_x V_x} \dot{y}_s + \frac{C_{at} (l_2^2 + l_2 d_s)}{l_z V_x} \dot{\varepsilon}_r - \frac{C_{at} l_1^2 + C_{at} l_2^2}{l_x V_x} \dot{\varepsilon}_d
\end{align*}
\]

(2)

where \( \dot{\varepsilon}_d = V_x \rho \) is the desired yaw rate,

\[
\begin{align*}
f_1 &= -\frac{\phi_1 + \phi_2}{V_x} \dot{y}_s + (\phi_1 + \phi_2) \varepsilon_r + \frac{\phi_1 (d_s - \frac{l_1}{2}) + \phi_2 (d_s + \frac{l_2}{2})}{V_x} \dot{\varepsilon}_r \\
&= \frac{\phi_2 l_1 - \phi_1}{V_x} \dot{\varepsilon}_d \\
b_1 &= \phi_1 \\
\phi_1 &= C_{at} \left( \frac{1}{m} + \frac{l_1 d_s}{l_z} \right), \quad \phi_2 = C_{at} \left( \frac{1}{m} - \frac{l_2 d_s}{l_z} \right)
\end{align*}
\]

Note that this equation involves certain approximations (see the Newton-Euler equations for vehicle lateral control). Another way of writing these equations is in the
standard state-space form

\[ \dot{\xi} = A \xi + B \delta + W \rho \]  

where \( \xi = \begin{bmatrix} y_s & j_s & \varepsilon_r & \dot{\varepsilon}_r \end{bmatrix}^T \)

\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{a_{22}}{V_x} & \frac{a_{11}}{V_x} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{a_{42}}{V_x} & \frac{a_{44}}{V_x} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ b_{21} \\ 0 \\ b_{41} \end{bmatrix}, \quad W = \begin{bmatrix} 0 \\ w_{21} \\ 0 \\ w_{41} \end{bmatrix} \]

\[ a_{22} = (\phi_1 + \phi_2), \quad a_{24} = \phi_1 (d_s - l_1) + \phi_2 (d_s + l_2) \]

\[ a_{42} = l_1 C_{ar} - l_2 C_{ar}, \quad a_{44} = l_1 C_{ar} (d_s - l_1) + l_2 C_{ar} (d_s + l_2) \]

\[ b_{21} = \phi_1, \quad b_{24} = \frac{l_1 C_{ar}}{I_z} \]

\[ w_{21} = \phi_2 l_2 - \phi_1 l_1 - V_x^2, \quad w_{41} = -\frac{l_1^2 C_{ar} + l_2^2 C_{ar}}{I_z} \]

The transfer domain expression for the second derivative of the sensor output, \( \ddot{y}_s \), is given by

\[ \ddot{y}_s(s) = s^2 y_s(s) = V_s(s) \delta(s) - V_x^2 \rho \]  

where

\[ V_s(s) = \left( C_{ar} V_x^2 \left( ml_1 d_s + I_z \right) s^2 + C_{ar} C_{ar} V_x (l_1 + l_2) (d_s + l_2) s \right) / D(s) \]

\[ D(s) = I_z m V_x^2 s^2 + V_x \left( I_z (C_{ar} + C_{ar}) + m \left( C_{ar} l_1^2 + C_{ar} l_2^2 \right) \right) s \]

\[ + m V_x^2 \left( C_{ar} l_2 - C_{ar} l_1 \right) + C_{ar} C_{ar} (l_1 + l_2)^2 \]

For \( d_s = 0 \), we get \( \ddot{y}_s = \ddot{y}_{CG} \). The block diagram of the vehicle lateral dynamics is shown in Figure 3.
Notice that \( V_S(s) \) depends on both the vehicle speed and the distance between the vehicle's CG and the lateral position sensor. Figure 4 shows frequency responses from the wheel steering angle to the second derivative of the sensor signal for a typical passenger vehicle. Figure 5 shows the pole/zero configuration of the corresponding transfer function.

In Figure 5, notice that the open loop system has a pair of weakly damped zeros, in particular at high speeds when \( d_i \) is short. If a high gain controller is applied to this problem, two closed loop poles are attracted to these zeros. As a result, the closed loop system has a weakly damped oscillatory mode, which the sensor does not see but the passengers feel. Such controllers are not acceptable. The same problem arises when the input-output linearization is applied to the nonlinear model in (1). In this case, the so called "zero dynamics" will contain such a weakly damped mode. Similar to the linear case, zero dynamics is not seen by the sensor but is felt by the passengers.
Notice that the plant dynamics for feedback control is the combination of the frequency response in the figure and that of the double integrator dynamics; the latter exhibits -40 db/decade gain and 180 degree phase lag characteristics. Figure 4 indicates that when the distance between the vehicle's CG and sensor is relatively short (2m is a typical number when the magnetometer is placed at the front bumper), the phase lag increases as the vehicle speed is increased. This implies that automation for high speed driving is more challenging than that for low speed driving. The figure also shows that at high speeds, the input/output dynamics have smaller phase lags when the sensor is placed further ahead of the vehicle's CG. This nature was utilized in the design of lateral controller utilized in the 1997 NAHSC Demonstration by the PATH lateral control team (see the PATH lateral controller for the 1997 NAHSC Demonstration).

4. Road Reference System

In the vehicle lateral control problem for AHS, it is critically important how the vehicle's position and orientation relative to the road are obtained. Various road reference/sensing systems have been proposed in the past. Perhaps, the oldest among them are wire reference systems (see lateral control based on the wire reference system). Other schemes include the optical marker system, the optical line following system, various radar- and vision-based systems, the side-looking radar system with a reference wall, GPS systems, and the magnetic marker (nail) system with on-board magnetometers.

Guidance by permanently magnetized markers was studied in the early 1970's. This scheme was revived in the late 1980's by PATH (see the magnetic reference system for vehicle lateral control). PATH researchers developed robust signal processing schemes for obtaining the lateral error, as well as, encoding schemes to embed other information such as preview road curvature information in binary form by alternating the polarity of the magnets. Furthermore, the magnetic marker scheme compares favorably with other schemes in terms of evaluation criteria such as accuracy, reliability, maintainability and cost. Therefore, it was adopted as the primary reference system in PATH at the early stage, and it has remained as a key element in PATH-AHS.

The road reference-sensing system based on magnetic markers is a look down system.
If the magnetometer is placed only under the front bumper, the system allows only a small amount of "look ahead," and $d_s$ in the model equations of the previous section is typically about 2 m. On the other hand, the vision camera defines a look-ahead system with an ample amount of look ahead, i.e. a large $d_s$. Recall that the open loop dynamics from the steering input to the sensor output defines dynamics easier for feedback control for larger $d_s$, which explains why the vision based lateral controllers have been reported to work well at high speeds (see vision based lateral guidance). As we describe in the next section, the output of the magnetometer at the front bumper has been verified to work fine in early PATH experiments, in which the vehicle speed was less than 60 km/hour. For the 1997 NAHSC Demonstration for which the vehicle speed was higher than 100 km/h, the PATH lateral team installed magnetometers under both the front and rear bumpers to allow for a large $d_s$ (see Figure 6).

![Figure 6: Two Sensor Scheme](image)

Under the assumption that the road is straight, the lateral error at the vehicle’s CG, $y_{CG}$, and the relative yaw error, $\varepsilon_r$, can be obtained from the lateral error at the front bumper, $y_f$, and one at the rear bumper, $y_r$, by

$$y_{CG} = \frac{l_2 y_f + l_1 y_r}{l_1 + l_2}$$

$$\varepsilon_r = \tan^{-1} \left( \frac{y_f - y_r}{l_1 + l_2} \right) = \frac{y_f - y_r}{l_1 + l_2}$$

(5)

where the last approximation holds for small $\varepsilon_r$. These two quantities can be combined to synthesize the output of a virtual sensor located at any distance ahead of the vehicle’s CG. This makes it possible to let the look down system based on magnetic markers behave as a look-ahead system.

5. Lateral Controllers for AHS

In this section, we review several lateral controllers in the order as they were developed by PATH.
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Biographical Sketches

Masayoshi Tomizuka received his B.S. and M.S. degrees in Mechanical Engineering from Keio University, Tokyo, Japan and Ph. D. degree in Mechanical Engineering from the Massachusetts Institute of Technology in February 1974. In 1974, he joined the faculty of the Department of Mechanical Engineering at the University of California at Berkeley, where he currently holds the Cheryl and John Neerhout, Jr., Distinguished Professorship Chair. At UC Berkeley, he teaches courses in dynamic systems and controls. His current research interests are optimal and adaptive control, digital control, signal processing, motion control, and control problems related to robotics, machining, manufacturing, information storage devices and vehicles. He has served as a consultant to various organizations, including Lawrence Berkeley Laboratory, General Electric, General Motors and United Technologies.

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