# MATHEMATICAL MODELS OF TRANSPORTATION AND NETWORKS

**Anna Nagurney**
*University of Massachusetts, Amherst, Massachusetts USA*

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## Contents



## Summary

In this chapter, we provide the foundations of the rigorous formulation, analysis, and solution of transportation network problems. We discuss user-optimization, which corresponds to decentralized decision-making, and system-optimization, which corresponds to centralized decision-making where the central controller can route the traffic in an optimal manner. We describe a spectrum of increasingly sophisticated models and also relate transportation networks to other network application domains in which flows (and associated decision-making) are essential, such as the Internet, supply chains, electric power distribution and generation networks, as well as financial networks. Finally, we demonstrate how the importance of transportation network components, that is, nodes and links can be identified (and ranked) through a recently proposed transportation network efficiency measure and accompanying component importance definition. Examples are included throughout the chapter for illustrative purposes.

## 1. Introduction

Transportation networks are complex, large-scale systems, and come in a variety of forms, such as road, rail, air, and waterway networks. Transportation networks provide the foundation for the functioning of our economies and societies through the movement of people, goods, and services. From an economic perspective, the supply in such network systems is represented by the underlying network topology and the cost characteristics whereas the demand is represented by the users of the transportation system. An equilibrium occurs when the number of trips between an origin (e.g., residence/place of employment) and destination (place of employment/residence) equals the travel demand given by the market price, typically, represented by the travel time for the trips (Nagurney (2004)).

The study of transportation networks and their efficient management dates to ancient times. It is known, for example, that Romans imposed controls over chariot traffic during different times of day in order to deal with the congestion (see Banister and Button (1993)). From an economic perspective, some of the earliest contributions to the subject date to Kohl (1841) and to Pigou (1920), who considered a two-node, two-link transportation network, identified congestion as a problem, and recognized that distinct behavioral concepts regarding route selection may prevail (see also Knight (1924)).

The formal study of transportation networks has challenged transportation scientists, economists, operations researchers, engineers, and physicists for reasons, including: the size and scope of the systems involved; the behavior of the users of the network which may vary according to the application setting, thereby leading to different optimality/equilibrium concepts; distinct classes of users may perceive the cost of utilizing the network in an individual fashion, and congestion, which is playing an increasing role in numerous transportation networks.

For example, to help one fix the size and scope of modern-day transportation networks, we point out that the topology of the Chicago Regional Transportation Network consists of 12,982 nodes, 39,018 links, and 2,297,945 origin/destination pairs of nodes between which travelers choose their routes (cf. Bar-Gera (2002)), whereas in the Southern California Association of Governments' model there are 25,428 nodes, 99,240 links, 3,217 origin/destination pairs, and 6 distinct classes of users (Wu, Florian, and He (2000)).

Road congestion results, according to estimates, in approximately $100 billion in lost productivity in the United States alone with the figure being about $150 billion in Europe with the number of cars expected to increase by 50 percent by 2010 and to double by 2030 (see Nagurney (2000) and the references therein). In particular, the growth in the usage of motorized vehicles, especially, cars, in the developing world, is transforming such countries as China and India. Moreover, in many of today's transportation networks, the "noncooperative" behavior of users aggravates the congestion problem. For example, in the case of urban transportation networks, travelers select their routes from an origin to a destination so as to minimize their own travel cost or travel time, which although optimal from a user's perspective (user-optimization) may not be optimal from a societal one (system-optimization) where a decision-maker or central controller has control of the flows on the network and seeks to allocate the flows so as to minimize the total cost in the network. Coupled with road congestion is increasing pollution, another negative externality, which is further impacting the world that we live in (see Nagurney (2000)).

The famous Braess (1968) paradox example, illustrates the distinction between noncooperative (or user-optimized) behavior versus system-optimized behavior, in a concrete, vivid way. It that example, it is assumed that the underlying behavioral principle is that of user-optimization and travelers select their routes accordingly. In the Braess network, the addition of a new road with no change in travel demand results in all travelers in the network incurring a higher travel cost. Hence, they are all worse off after the addition of the new road! Actual practical instances of such a phenomenon have been identified in New York City and in Stuttgart, Germany. In 1990, 42nd Street in New York was closed for Earth Day, and the traffic flow in the area improved (see Kolata (1990)). In Stuttgart, in turn, a new road was added to the downtown, but the traffic flow worsened and, following complaints, the new road was torn down (cf. Bass (1992)). Similar experiences have been found recently in Seoul, Korea (Vidal (2006)). Interestingly, this phenomenon is also relevant to telecommunications networks (see Korilis, Lazar, and Orda (1999)) and, specifically, to the Internet (cf. Cohen and Kelly (1990) and Nagurney, Parkes, and Daniele (2006)). Such a result does not occur in system-optimized networks where the addition of a new road/link, if used, would lower the total network cost. Today, congestion pricing is an active topic of research, and tolls have had success in ameliorating traffic by altering people's behavior in various cities around the world, including the much-publicized London, United Kingdom experience (see, e.g., Lawphongpanich, Hearn, and Smith (2006)).

In this chapter, we recall the foundations of the rigorous study of transportation networks and we trace the evolution of modeling frameworks for their study. The exposition is meant to be accessible to practitioners and to students, as well as to researchers and policy makers and to those interested in related network topics. Technical derivations and further supporting documentation are referred to in the citations. Further useful material and a supplementary chronological perspective of developments on this topic can be found in the review articles of Florian (1986), Boyce, LeBlanc, and Chon (1988), and Florian and Hearn (1995); in the books by Beckmann, McGuire, and Winsten (1956), Sheffi (1985), Patriksson (1994), Ran and Boyce (1996), Nagurney (1999, 2000), Nagurney and Dong (2002a), and in the volumes edited by Florian (1976, 1984), Volmuller and Hamerslag (1984), Lesort (1996), Marcotte and Nguyen (1998), Gendreau and Marcotte (2002), Taylor (2002), Mahmassani (2005), Bar-Gera and Boyce (2005), and Nagurney (2006a, b).

This chapter, specifically, overviews some of the methodologies, whose very development, has been motivated by the need to formulate, analyze, and solve transportation network problems. It also relates the contributions of transportation modeling and algorithmic advances to other network application domains. Finally, given the importance of transportation networks and the closely related telecommunication, electric power generation and distribution networks, supply chain, as well as financial networks (cf. Nagurney (2006a,b)), we also, for completeness, discuss a network efficiency measure, which enables the identification of the critical nodes and links. This measure was proposed by Nagurney and Qiang (2007) and can aid policy makers, planners, engineers, as well as network designers in identifying which network components need to be protected, since their absence due, for example, to destruction by natural disasters, structural failures, terrorist attacks, etc., has the greatest impact. Hence, the transportation network is most vulnerable when such nodes/links are removed from the system.

## 2. Fundamental Decision-Making Concepts and Models

Over half a century ago, Wardrop (1952) explicitly considered alternative possible behaviors of users of transportation networks, notably, urban transportation networks and stated two principles, which are commonly named after him:

**First Principle:** The journey times of all routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.

**Second Principle:** The average journey time is minimal.

The first principle corresponds to the behavioral principle in which travelers seek to (unilaterally) determine their minimal costs of travel whereas the second principle corresponds to the behavioral principle in which the total cost in the network is minimal.

Beckmann, McGuire, and Winsten (1956) were the first to rigorously formulate these conditions mathematically. Specifically, Beckmann, McGuire, and Winsten (1956) established the equivalence between the *transportation network equilibrium* conditions, which state that all used paths connecting an origin/destination (O/D) pair will have equal and minimal travel times (or costs) (corresponding to Wardrop's first principle), and the Kuhn-Tucker (1951) conditions of an appropriately constructed optimization problem, under a symmetry assumption on the underlying functions. Hence, in this case, the equilibrium link and path flows could be obtained as the solution of a mathematical programming problem. Their approach made the formulation, analysis, and subsequent computation of solutions to transportation network problems based on actual transportation networks realizable.

Dafermos and Sparrow (1969) coined the terms *user-optimized* (U-O) and *system-optimized* (S-O) transportation networks to distinguish between two distinct situations in which, respectively, users act unilaterally, in their own self-interest, in selecting their routes, and in which users select routes according to what is optimal from a societal point of view, in that the total cost in the network system is minimized. In the latter problem, marginal total costs rather than average costs are equilibrated. The former problem coincides with Wardrop's first principle, and the latter with Wardrop's second principle. See Table 1 for the two distinct behavioral principles underlying transportation networks.

| **User-Optimization** | | **System-Optimization** |
|---|---|---|
| $\Downarrow$ | | $\Downarrow$ |
| **User Equilibrium Principle:** | | **System Optimality Principle:** |

| | |
|---|---|
| User travel costs on used paths for each O/D pair are equalized and minimal. | Marginals of the total travel cost on used paths for each O/D pair are equalized and minimal. |

Table 1. Distinct Behavior on Transportation Networks

The concept of "system-optimization" is also relevant to other types of "routing models" in transportation, as well as in communications (cf. Bertsekas and Gallager (1992)), including those concerned with the routing of freight and computer messages, respectively. Dafermos and Sparrow (1969) also provided explicit computational procedures, that is, *algorithms*, to compute the solutions to such network problems in the case where the user travel cost on a link was an increasing (in order to handle congestion) function of the flow on the particular link, and linear. Today, the concepts of user-optimization versus system-optimization also capture, respectively, decentralized versus centralized decision-making on networks, including, the Internet (cf. Roughgarden (2005) and Boyce, Mahmassani, and Nagurney (2005)).

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**Biographical Sketch**

**Anna Nagurney** is the John F. Smith Memorial Professor in the Department of Finance and Operations Management in the Isenberg School of Management at the University of Massachusetts at Amherst. She is also an Affiliated Faculty Member of the Department of Mechanical and Industrial Engineering and the Department of Civil and Environmental Engineering at the University of Massachusetts at Amherst. She is the Founding Director of the Virtual Center for Supernetworks and the Supernetworks Laboratory for Computation and Visualization at the University of Massachusetts at Amherst. She received an AB degree in Russian Language and Literature in 1977, the ScB degree in 1977, the ScM in 1980, and the PhD degree in 1983, all in Applied Mathematics, and all from Brown University in Providence, Rhode Island. She devotes her career to education and research that combines operations research/management science, economics, and engineering. Her focus is the applied and theoretical aspects of decision-making on network systems, particularly in the areas of transportation and logistics, energy and the environment, and economics and finance. Her most recent book is Supply Chain Network Economics: Dynamics of Prices, Flows, and Profits published in 2006. She has authored or co-authored 8 other books including Supernetworks: Decision-Making for the Information Age and Network Economics: A Variational Inequality Approach, and more than 120 refereed journal articles.

She has given invited talks in Sweden, New Zealand, China, Germany, Italy, Canada, Australia, Cyprus, Iceland, the US, and other countries and her research has garnered funding from many foundations, including the National Science Foundation. Among the honors she has received are: the University of Massachusetts Award for Outstanding Accomplishments in Research and Creative Activity, an INFORMS Moving Spirit Award, a Science Fellowship at the Radcliffe Institute for Advanced Study at Harvard University, a Rockefeller Foundation Bellagio Center Research Team Fellowship, a Distinguished Fulbright Chair at the University of Innsbruck, Austria, two AT&T Foundation Industrial Ecology Fellowships, the Chancellor’s Medal from the University of Massachusetts, an Eisenhower Faculty Fellowship, a National Science Foundation Faculty Award for Women, a Faculty Fellowship from the University of Massachusetts, and the Kempe Prize from the University of Umea, Sweden. She has been a visiting professor at the University of Innsbruck, Austria, the Royal Institute of Technology in Stockholm, Sweden, the Massachusetts Institute of Technology, and Brown University.

She has served on numerous prize committees, including the Fudan Premium Fund of Management Prize committee in Shanghai, China, the INFORMS Transportation Science Section Robert Herman Lifetime Achievement Prize in Transportation Science committee, the INFORMS Computer Science Technical Section prize committee, and the Discipline Advisory Committee for Fulbright Scholar Awards. Professor Nagurney is on the Advisory Board of the European Union funded COMISEF project, which is a multi-year, multi-national project and regularly serves on government panels and international conference organizing committees. She is the editor of the book series, New Dimensions in Networks (Edward Elgar Publishing), and the co-editor of the book series, Advances in Computational Economics (Springer). She is on the editorial boards of the journals: *Networks, Journal of Economic Dynamics and Control, Computational Economics, Computational Management Science, Annals of Regional Science, International Journal of High Performance Computing Applications, The Journal of Financial Decision Making, Netnomics: Economic Research and Electronic Networking, Optimization Letters, and the International Journal of Sustainable Transportation.*