NUCLEAR REACTIONS

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Contents

Summary

Nuclear reactions mean the interaction of atomic nuclei with electromagnetic radiation, electrons and other leptons, nucleons, pions and other hadrons, and with nuclei. In a typical reaction, a projectile is incident on a target nucleus and after the collision a nucleus and an outgoing particle are detected. Whether a reaction is exoergic or endoergic depends on the Q-value which is the difference between the kinetic energies in the outgoing and incident channels. The cross section expresses the number of emitted particles per unit time divided by the incident flux of projectile particles and is given by the S-matrix. One distinguishes two extreme cases, namely, compound nucleus and direct reactions. The compound nucleus model factorizes the cross section in the formation cross section of the compound nucleus and the probabilities for the decay in the various outgoing channels. In its simplest version compound nucleus reactions are describable with the one-level dispersion formula of Breit and Wigner, which is based on the R-matrix theory. Averaging over closely spaces resonances, typical for a compound nucleus formation, one arrives at the optical model which describes the absorption of probability out of the elastic channel by an imaginary potential. Direct

reactions occur in relatively short time scales of about 10^{-22}s and only involve a few degrees of freedom of the system. They can be treated with the coupled channels method or if possible with the distorted Born approximation. Nuclear reactions with heavy ions are distinguished according to their kinetic energy. At low kinetic energies near the Coulomb barrier cluster effects may occur which show up in quasi-molecular resonances and in the fusion to super-heavy nuclei. At relativistic kinetic energies signatures are searched for a phase transition of hadronic matter to deconfined matter, called the quark-gluon plasma.

1. Introduction

Nuclear reactions mean the interaction of atomic nuclei with electromagnetic radiation, electrons and other leptons, nucleons and other baryons, pions and other mesons, and with nuclei. In a wider sense one also denotes the alpha-particle decay, the β-decay, the γ-decay and fission of nuclei as nuclear reactions. This article does not include the latter processes under the class of nuclear reactions and restricts the discussion to the first mentioned reactions.

Nuclear reactions stand at the threshold of nuclear physics. In 1911 Lord Rutherford measured the scattering of low energy α-particles by the nuclei of a thin gold foil and interpreted his measured data as caused by the finite extension of the charge distribution of gold nuclei. In contrast to the historical evolution of atomic physics where spectra of atoms were first studied by experimental and theoretical methods, nuclear physics was mainly explored by nuclear reactions up to the years 1936 and 1949 when the compound nucleus model and the nuclear shell model were introduced, respectively. Since then nuclear spectroscopy and nuclear reaction physics were considered with equal importance in nuclear physics.

Nuclear reactions can be distinguished according to the leading force between the constituents of the reaction. Electron- and γ-scattering on nuclei are governed by the electromagnetic fields of the scattered particles, whereas in nucleon and pion scattering the particles interact via the strong force. In high energetic collisions when hadronic particles are produced, it is necessary to consider the constituents of the hadrons, namely the quarks and gluons, and to describe the reaction with the theory of quantum chromodynamics (QCD)

In a typical nuclear reaction, a projectile *a* is incident on a target nucleus *A* and after the collision a nucleus *B* and an outgoing particle *b* are observed. This is written as

$$A + a \to b + B \qquad (1)$$

or

$$A(a,b)B.$$

Particles *a* and *b* can be photons, electrons, mesons, nucleons or nuclei. If *a* and *b* are nuclei, one denotes the collision as a heavy ion reaction. The basic theory for the description of nuclear reactions is the quantum theory. When the quantum numbers of the relative motion between the particles, the kind of particles and their intrinsic quantum numbers like their individual angular momenta (nuclear spins) are specified, one speaks about a reaction channel. A nuclear reaction proceeds from the entrance channel to various exit channels.

In order to study the structure of nuclear matter, nuclei, and nucleons, nuclear reactions have to be carried out. The elastic scattering of electrons and nucleons gives information on the charge and mass distributions of the nuclei. Nuclear states are excited by inelastic scattering of particles, by transfer and knock-out reactions which are named direct reactions. If an excited intermediate nuclear system is formed which lives up to 10^{-15}s and decays with particle and γ - ray emissions, one denotes the reaction as a compound nucleus reaction. New nuclei, as exotic nuclei far outside the line of highest stability, e.g. ^{16}C, or superheavy nuclei with charge numbers $Z \geq 100$ can be produced with heavy ion reactions. The transition of hadronic matter into a quark-gluon plasma is presently investigated with reactions applying highly relativistic heavy ions. This brief overview of various types of reactions demonstrates the close connection of the nuclear reaction theory with the theory of nuclear structure and with nuclear models.

2. Cross Section and Collision (S-) Matrix

2.1. Energy Relations

When the incident particle a and the target nucleus A are far separated in the initial channel before the collision, their interaction energy is zero and the total energy can be written (channel $\alpha = (A, a)$

$$E_\alpha = T_{rel}^\alpha + E_A + E_a, \tag{2}$$

where T_{rel}^α is the relative kinetic energy of the particles in the ingoing channel and E_A and E_a their total energies which consist of the energies of the rest masses and of the negative values of the binding energies. In the outgoing channel $\beta = (B, b)$ the corresponding equation is true:

$$E_\beta = T_{rel}^\beta + E_B + E_b, \tag{3}$$

Since the total energy is conserved in the reaction $(E_\alpha = E_\beta)$, the relative kinetic energies of the ingoing and outgoing channels can be related with the Q-value of the reaction from channel α to channel β:

$$T_{rel}^\beta = T_{rel}^\alpha + Q_{\alpha\beta}, \tag{4}$$

$$Q_{\alpha\beta} = E_A + E_a - E_B - E_b \tag{5}$$

If $Q > 0$, the reaction is exoergic, for $Q < 0$ endoergic. Exoergic reactions proceed even with zero initial kinetic energy. In contrast, endoergic reactions occur only if the initial kinetic energy exceeds the threshold energy $-Q_{\alpha\beta}$. For example, the reaction $^{7}\mathrm{Li}(\alpha,\mathrm{n})^{10}\mathrm{B}$ has a Q-value of -2.79MeV.

2.2. Differential Cross Section

The cross section is a measurable quantity of a nuclear reaction. It is defined as

$$\sigma(\alpha,\beta) = \frac{\text{Number of emissions in channel } \beta \text{ per time unit and scattering center}}{\text{Number of particles incident in channel } \alpha \text{ per unit of time and area}}$$

$$= \int j_r^{\beta} r^2 d\Omega / j^{\alpha} \tag{6}$$

Here, j^{α} is the current density in the direction of motion of the incident particles, j_r^{β} the radial current density of emitted particles and $r^2 d\Omega$ the infinitesimal area element on a sphere with radius r and solid angle $d\Omega$. The cross section differential in the emission angle is given by

$$\frac{d\sigma(\alpha,\beta)}{d\Omega} = j_r^{\beta} r^2 / j^{\alpha}. \tag{7}$$

The cross sections can be calculated by means of the wave function at a large distance from the reaction center.

2.3. Hamiltonian and Asymptotic Wave Function

In order to simplify the following presentation let us assume that the particles in equation (1) are nuclei, which are subjected only to inelastic excitation. Then $a = b$ and $A = B$. This two-body fragmentation $\alpha = (A, a)$ has the following Hamiltonian

$$H = H_{\alpha}$$

$$= T_{\alpha} + h_A + h_a + W(\mathbf{r}_{\alpha}, \{A\}, \{a\}), \tag{8}$$

where $\mathbf{r}_{\alpha}$ is the relative coordinate between the centers of nuclei A and a: $\mathbf{r}_{\alpha} = \mathbf{R}_{\mathrm{a}} - \mathbf{R}_{\mathrm{A}}, T_{\alpha} = T_{rel}^{\alpha}$ is the operator of the relative kinetic energy, h_A and h_a are the Hamiltonians of the nuclei A and a, and W is the interaction energy between these nuclei described by the sets of coordinates $\{A\}$ and $\{a\}$. The interaction vanishes for large separations r_{α} up to the Coulomb interaction $W \to Z_A Z_a e^2 / (4\pi\epsilon_0 r_{\alpha})$ where Z_A and Z_a are the charge numbers of the nuclei. The wave function of the stationary

Schrödinger equation $H\Psi = E\Psi$ can be expanded as follows ($\mathbf{r}_\alpha = \mathbf{r} = (r,\vartheta,\varphi)$, antisymmetrization neglected):

$$\Psi = \sum_c R_c(r)\varphi_c \tag{9}$$

with the channel functions

$$\varphi_c = \left[i^\ell Y_\ell(\vartheta,\varphi) \otimes [\varphi^{J_A}_{\lambda_A}(\{A\}) \otimes \varphi^{J_a}_{\lambda_a}(\{a\})]^J \right]^I_M \tag{10}$$

with $c = \left(\ell, \lambda_A, J_A,, \lambda_a, J_a, J, I, M\right)$.

Here, the channel quantum number *c* includes the quantum number ℓ of orbital angular momentum, the quantum numbers λ_A and λ_a of the eigenstates of h_A and h_a , the quantum numbers *J* and *(I, M)* resulting from the coupling of J_A and J_a to *J* and ℓ and *J* to *I*, respectively. The functions $Y_{\ell m}(\vartheta,\varphi)$ are spherical harmonics. For large separations the radial functions are given by

$$R_c = \sum_{c'} C_{c'}(\delta_{cc'} I_c(r) - S_{cc'} O_c(r))/(r v_c^{1/2}). \tag{11}$$

In this formula enter the incoming and outgoing Coulomb waves $I_c = \exp(i\sigma_\ell)(G_\ell - iF_\ell)$ and $O_c = \exp(-i\sigma_\ell)(G_\ell + iF_\ell)$, respectively, with the regular and irregular Coulomb functions F_ℓ and G_ℓ . The collision or S-matrix connects the incoming and outgoing waves. The coefficients $C_{c'}$ define the incoming wave specified by the reaction, and v_c is the asymptotic relative velocity in channel *c*.

2.4. The S – Matrix, Reciprocity Relation and Principle of Detailed Balance.

The S- matrix is of fundamental importance for the theory of nuclear reactions. The element $S_{cc'}$ is the amplitude of the transition from an ingoing channel c' to an outgoing channel *c*. Cross sections are proportional to the absolute squares of the S-matrix elements. On the basis of the conservation of the probability in quantum mechanics, one can show that the S-matrix is an unitary matrix with the property

$$\sum_c S^*_{cc'} S_{cc''} = \delta_{c'c''}. \tag{12}$$

This relation is not fulfilled if the Hamiltonian violates the Hermitian adjoint property. This is the case when complex potentials are introduced in the optical model to simulate the absorption which means the flux of probability into not explicitly considered channels.

Another symmetry of the S-matrix arises from the concept of time reversal. The invariance of a theory under time reversal is connected with the invariance of the Hamiltonian under complex conjugation. That means $\mathbf{H} = \mathbf{T}^{-1}\mathbf{H}^{*}\mathbf{T}$ where the operator $\mathbf{T}$ reverses the signs of the spins. Since the definition of the channel c depends on the magnetic quantum number M of the total angular momentum, we introduce the channel $-c$ where M is replaced by $-M$. Then the invariance of the Hamiltonian under time reversal yields the relation

$$S_{cc'} = S_{-c',-c}. \tag{13}$$

This relation is named reciprocity relation and states that the probability for the process $c' \to c$ is the same as the probability for the time reversed process $c \to c'$. If spin-zero particles are involved, this relation has the consequence that the S-matrix is symmetric.

The reciprocity relation leads to the important principle of detailed balance. This principle relates the cross section for the reaction $\alpha \to \beta$ with that of the time-reversed reaction $\beta \to \alpha$:

$$\sigma(\alpha,\beta) = \frac{(2J_B+1)(2J_b+1)k_\beta^2}{(2J_A+1)(2J_a+1)k_\alpha^2}\sigma(\beta,\alpha), \tag{14}$$

where k_α and k_β are the asymptotic wave numbers in the channel α and β, respectively.

2.5. Cross Sections for Neutron – induced Reactions

The cross sections can be obtained from a given S- matrix. We illustrate the general procedure for reactions with neutral particles in order to circumvent the more complicated formulas with Coulomb wave functions for charged particles. Introducing the uncoupled channel functions (see equation (10))

$$\varphi_{c\ell mJ\mu} = i^{\ell} Y_{\ell m}(\vartheta,\varphi)\left[\varphi_{\lambda_A}^{J_A}(\{A\}) \otimes \varphi_{\lambda_A}^{J_A}(\{a\})\right]_\mu^J \tag{15}$$

with $c = (\lambda_A, J_A, \lambda_a, J_a)$

and defining the asymptotic forms of the ingoing and outgoing radial wave functions as $I_{c\ell} \to \exp(-i(k_c r - \ell\pi/2))$ and $O_{c\ell} \to \exp(i(k_c r - \ell\pi/2))$, we can write the asymptotic wave function as

$$\Psi = \frac{i\pi^{1/2}}{k_c}\sum_{\ell}(2\ell+1)^{1/2}\Big\{(I_{c\ell} - O_{c\ell})\varphi_{c\ell 0J\mu}/(rv_c^{1/2})$$

$$+\sum_{c'\ell'm'J'\mu'}(\delta_{c'\ell'm'J'\mu',c\ell 0J\mu} - S_{c'\ell'm'J'\mu',c\ell 0J\mu})\,O_{c'\ell'}\varphi_{c'\ell'm'J'\mu'}/(rv_{c'}^{1/2})\Big\} \tag{16}$$

The first part of this equation describes the incident plane wave in the channel $cJ\mu$, the second part the radially outgoing wave. The differential cross section for the reaction from channel $cJ\mu$ to $c'J'\mu'$ is given by

$$\frac{d\sigma_{cJ\mu\to c'J'\mu'}}{d\Omega_{c'}} =$$

$$\pi\lambda\!\!\!^{-}{}_c^2 \left| \sum_{\ell,\ell'} (2\ell+1)^{1/2} (\delta_{c'\ell'm'J'\mu',c\ell 0J\mu} - S_{c'\ell'm'J'\mu',c\ell 0J\mu}) Y_{\ell'm'}(\vartheta_{c'},\varphi_{c'}) \right|^2 \qquad (17)$$

For the special case that the channel spins J and J' are zero, the orbital angular momenta of the outgoing channels are equal to the ones of the ingoing channels: $\ell' = \ell$. If it is further assumed that the energy is so low that only the s-wave of the ingoing channel contributes, the reaction is isotropic and the total cross section is $(\{0\} = (\ell = 0, m = 0, J = 0, \mu = 0))$

$$\sigma_{c\{0\}\to c'\{0\}} = \pi\lambda\!\!\!^{-}{}_c^2 \left| \delta_{cc'} - S_{c'\{0\},c\{0\}} \right|^2. \qquad (18)$$

The total cross section for unpolarized beams can be obtained by integrating equation (17) over the angles and summing over μ' and averaging over μ. Since the total angular momentum $I = \ell + J$ is conserved in the reaction, it is convenient to introduce a S-matrix depending on the indices $c\ell JI$ and defined as (Here, $(j_1 m_1 j_2 m_2 | j_3 m_3)$ are Clebsch-Gordan coefficients.):

$$S^I_{c\ell J,c'\ell'J'} = \sum_{m,m',\mu,\mu'} (\ell m J\mu | IM)(\ell' m' J'\mu' | IM) S_{c\ell mJ\mu,c'\ell'm'J'\mu'}. \qquad (19)$$

Then the total cross section for unpolarized beams and the transition cJ to $c'J'$ is

$$\sigma_{cJ\to c'J'} = \frac{\pi\lambda\!\!\!^{-}{}_c^2}{2J+1} \sum_{\ell,\ell',I} (2I+1) \left| \delta_{c\ell J,c'\ell'J'} - S^I_{c\ell J,c'\ell'J'} \right|^2. \qquad (20)$$

If initially the two particles have internal states λ_A and λ_a and nuclear spins J_A and J_a, respectively, the total cross section including elastic and inelastic channels is obtained as $(c = (\lambda_A, J_A, \lambda_a, J_a))$::

$$\sigma_c = \frac{\pi\lambda\!\!\!^{-}{}_c^2}{(2J_A+1)(2J_a+1)} \sum_{\ell,J,I} 2(2I+1)(1 - Re(S^I_{c\ell,J,c\ell J})) \qquad (21)$$

When the particles are charged the in- and out-going asymptotic wave functions are Coulomb functions. In this case the reaction amplitude consists of a sum of two parts:

The Rutherford amplitude and the amplitude containing the nuclear effects. In general the electrostatic Rutherford amplitude contributes to small scattering angles whereas the nuclear effects arise at larger angles.

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Biographical sketch

Werner Scheid, was born 28 June 1938 in Offenbach/ Main (Germany), studied physics at the Technische Hochschule Darmstadt from 1958 up to 1964, received Diploma in Physics in 1964 (adviser

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