MODELING FOR GEOTECHNICAL ENGINEERING APPLICATIONS

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Contents

Summary

Modeling which is essentially a simplification of real world problems is an integral part of all geotechnical engineering analysis and design process. Modeling in geotechnical engineering could range from constitutive soil models that describe material behavior to experimental and intelligent models that simulate geotechnical systems under various loading and environmental conditions. Complex problems may be analyzed using numerical techniques such as finite difference, finite element, and discrete element methods. This paper describes the fundamental principles of soil modeling in geotechnical engineering.

1. Introduction

1.1. Geotechnical Engineering and Modeling

Geotechnical engineering is the branch of civil engineering that is concerned with the characterization of the subsurface, determination of engineering soil properties, and the analysis, design and construction of geotechnical systems. Almost all civil engineering structures are built on soils, or below the surface. Soils are also used as construction material for building earth structures such as pavements, embankments, slopes, and dams.

Modeling is primarily concerned with finding solutions to real-world problems and involves approximations which are essentially a simplification of reality. Geotechnical modeling is often more complex than modeling structural systems using engineered materials such as steel, concrete, plastics or composites. The properties of soils and rocks are uncertain, since the strata might change drastically within a short distance.

Modeling could include making predictions of stresses induced by the interactions of civil engineering systems with the soil; displacements as a result of imposed loads; development of pore water pressures and their effects on the stability; analyzing the stability of slope and bearing capacity of shallow and deep foundation. Dynamics behavior of geotechnical systems, earthquake phenomena and soil liquefaction can also be modeled. Since modeling involves simplifications and assumptions, it is very important to be aware of the simplifications and assumptions made, and the nature and consequences of those assumptions. Detailed discussion of geotechnical modeling is beyond the scope of this chapter. Only the basic principles and a general overview of this vast field are described in this chapter. Several important references are given in the bibliography section for those who may want to conduct an in depth study of various topics.

1.2. The Characteristics and Behavior of Soils

Traditional methods used in geotechnical analysis and design make use of the behavior and properties of soils (strength, stiffness, and flow characteristics) and its interactions with geotechnical systems. Soils are formed by the mechanical and or chemical disintegration of rocks. Soils are porous media and in its most general form may be considered to exist as a three phase system consisting of soil solids, water, and air. Based on grain size distribution, soils may be broadly classified as coarse grained soils (such as sand and gravel) if more than 50 percent by weight is retained on a 0.075 mm sieve, and fine grained soils (such as silts and clays) if 50 percent or more passes a 0.075 mm sieve.

The behavior of soils can be very different. Dry cohesionless sand can be poured like water however it will pile in the shape of a cone (unlike water). Wet cohesive clays may behave as a plastic material and can be easily molded with our fingers. If the clay has high water content it loses its strength and can flow like slurry. If wet clay is slowly dried it loses its plasticity and becomes brittle and hard. The stress-strain behavior is non-linear and can behave in a very complex manner (elasto-visco-plastic behavior). The stress-strain-strength behavior depends on the type of soil, confining pressure, initial void ratio and stress history. Stresses, stress-strain behavior, and strength of soils influence compressibility, and stability of geotechnical systems. The mechanical behavior of soils may be modeled using various constitutive (stress-strain) relationships. There are several constitutive models to describe various aspects of soil behavior and some of these are described in Section 2.

1.3. Bernoulli's Equation and Darcy's Law

Since soils are porous, the voids may be partially saturated or completely saturated with water below the ground water table. When there is no flow of water, hydrostatic pore water pressure (u_s) exists in the saturated soil below the ground water table (Figure 1a). The hydrostatic pore water pressure at a depth z_w below the ground water table (phreatic surface) can be determined as follows:

$$u_s = z_w \gamma_w \tag{1}$$

where γ_w is the unit weight of water

If there is a hydraulic gradient (difference in total head) water can seep (flow) through the interconnected voids. In coarse grained soils the water seeps through easily because of the high hydraulic conductivity (due to the large voids between the large soil grains). For the same hydraulic gradient (difference in total head per unit length of soil column in the direction of flow) the rate of seepage in silts and clays are relatively small (compared to sand) due to the low hydraulic conductivity (due to the small void size). The pore water pressure during seepage, which is different from the hydrostatic pore water pressure, can be determined by the application of the reduced Bernoulli's equation below (Figure 1b):

$$h_T = h_z + h_p \tag{2}$$

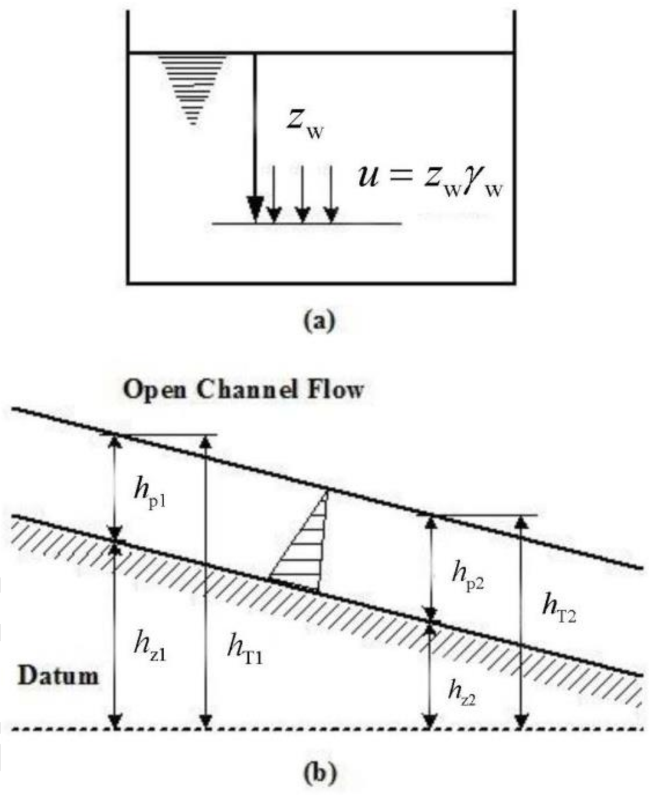

Figure 1. Illustration of (a) Hydrostatic pore water pressure (b) Bernoulli's principle

In the above equation the velocity head (h_v) is ignored because it is insignificant compared to the total head (h_T), elevation head (h_z), and the pressure head (h_p). The velocity of flow of water through soils (v) is described by Darcy's law as follows:

$$v = ki \tag{3}$$

where k is the hydraulic conductivity which depends on the size of the pores, and ($i = \Delta h / L$) is the hydraulic gradient of the total head causing flow (i.e., the ratio of the total head lost Δh, over the length of soil, L in which the head is lost). The steady state pore water pressure (u_{ss}) during steady state seepage is given by ($u_{ss} = h_p \gamma_w$).

1.4. Effective Stress Principle, Seepage and Consolidation

The pore water pressure through its influence on effective stress plays an important role in the strength and compressibility of soils. In a saturated soil the total stress (σ) is partially carried by the soil skeleton as effective stress (σ') and the remainder is carried by the pore water as pore water pressure (u). Thus Terzahi's effective stress principle may be expressed as follows:

$$\sigma' - \sigma - u \tag{4}$$

According to the effective stress principle, it is the effective stress that governs shear strength, comressibility and distortion. When saturated fine-grained soils are subjected to isotropic stress or when laterally confined soils are subjected to one-dimensional normal stress (with no lateral strains), the applied total stress is initially carried by the pore water as excess pore water pressure (i.e., water pressure in excess of the hydrostatic or steady state pore pressure). If drainage is permitted the excess pore pressure dissipates with time depending on the hydraulic conductivity as water is squeezed out of the specimen. The soil consolidates (i.e., undergoes a volume decrease) corresponding to the volume of water expelled. The dissipated excess pore water pressure is transferred to the soil skeleton as effective stress according to the Terzaghi's consolidation theory and effective stress principle.

Geotechnical engineers often need to calculate the pore pressure distribution within a soil region.

The basic continuity equation that must be satisfied at all points within the soil is obtained by considering the flow of water into and out of an infinitesimal soil element (Figure 2):

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{5}$$

In an anisotropic soil the permeability may be different in different directions (x, y, z), and the general form of Darcy's Law may be expressed as:

$$v_x = -\frac{k_x}{\gamma_w}\frac{\partial u}{\partial x} \tag{6}$$

$$v_y = -\frac{k_y}{\gamma_w}\frac{\partial u}{\partial y} \tag{7}$$

$$v_z = -\frac{k_z}{\gamma_w}\frac{\partial u}{\partial z} \tag{8}$$

Substituting these Eqs. (6-8) in the continuity Eq. (5) gives the differential equation governing 3-dimensional flow (for steady state seepage):

$$\frac{k_x}{\gamma_w}\frac{\partial^2 u}{\partial x^2}+\frac{k_y}{\gamma_w}\frac{\partial^2 u}{\partial y^2}+\frac{k_z}{\gamma_w}\frac{\partial^2 u}{\partial z^2}=0 \tag{9}$$

These equations may be derived / developed for transient flow (consolidation) where the last term in the equation is equal rate of volumetric strain of a soil element and may be expressed as:

$$\frac{k_x}{\gamma_w}\frac{\partial^2 u}{\partial x^2}+\frac{k_y}{\gamma_w}\frac{\partial^2 u}{\partial y^2}+\frac{k_z}{\gamma_w}\frac{\partial^2 u}{\partial z^2}+\frac{\partial v}{\partial t}=0 \tag{10}$$

The above equation combined with the equilibrium equations, the effective stress principle, and the effective stress-strain relation gives the general Biot's equation of consolidation. The one-dimensional form of the Biot's equation is equivalent to the Terzaghi's one dimension consolidation equation:

$$\frac{\partial \bar{u}_e}{\partial t}=c_v\frac{\partial^2 \bar{u}_e}{\partial z^2} \tag{11}$$

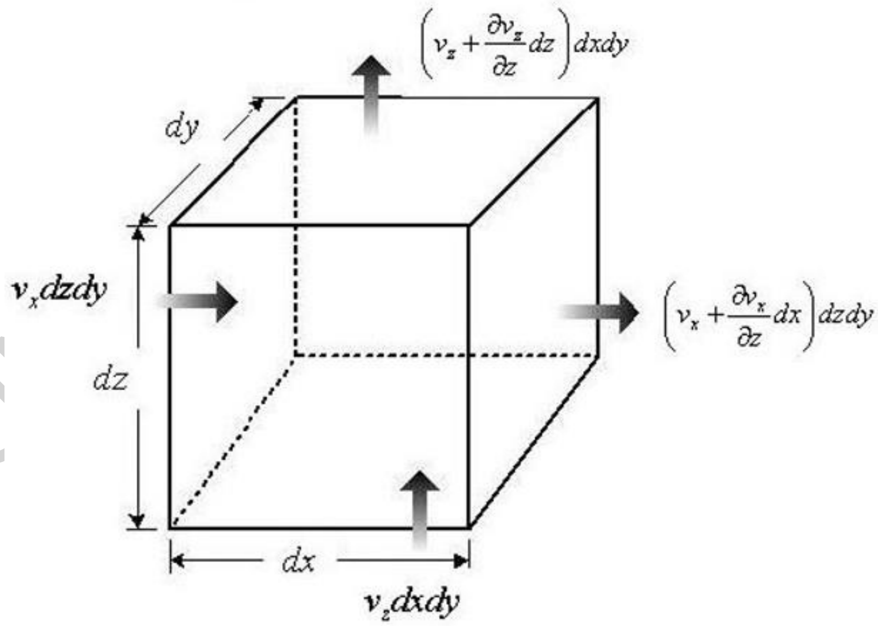

Figure 2. Flow of water through an infinitesimal soil element

2. Constitutive Models

A large number of constitutive models have been proposed by several researchers and practitioners to describe various aspects of soil behavior. The simplest available stress-strain relationship is the Hooke's law of linear, isotropic elasticity. There are four material parameters for an elastic model, the elastic modulus E, Poisson's ratio ν, bulk modulus K and shear modulus G; and only two are required to fully specify the material (say E and ν). The linear elastic model is generally too crude to capture essential features of soil behavior using such a simple model. It is well known that the behavior of soil is non-linear, and anisotropic. It is often modeled as an elasto-plastic material even though the actual behavior may be even more complex. In order to model the elasto-plastic behavior of soils it is first necessary to select an idealization for soil plasticity. Figure 3.0 (a) shows the idealization for an elastic-perfectly plastic material. The initial part of the stress-strain curve is linear and elastic until the material yields. The material then continues to deform at constant yield stress. Figure 3.0 (b) shows the idealization for a linear-elastic strain-hardening-plastic material. After yield point the stress-strain curve is still linear but at a slope (yield stress is not constant). Figure 3.0 (c) shows the idealization for a rigid perfectly-plastic material in which the material does not strain until the yield stress.

Soils usually exhibit a more complex behavior than that described above. A realistic description of the elasto-plastic stress-strain behavior requires the four relationships:
(i) Yield function to describe the concept of yield stress in two or three dimensions.
(ii) Relationship between the directions of principal plastic strain increment and principal stresses.
(iii) Flow rule that describes the relative magnitudes of the incremental plastic strains during yield.
(iv) Hardening law that relates the amount a material hardens and the plastic strain it undergoes.

Some of the common yield functions are discussed in the sections below.

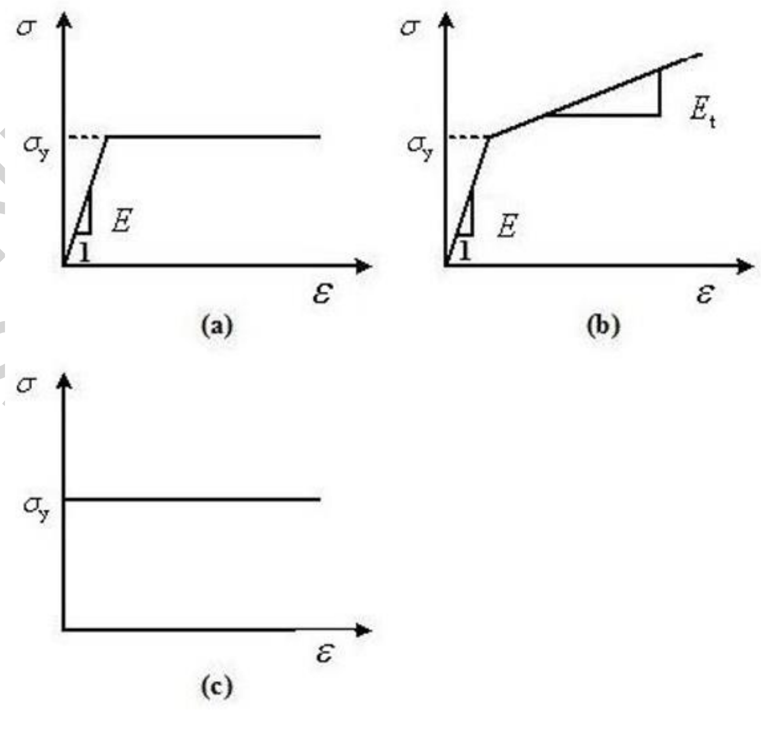

Figure 3. (a) Elastic-perfectly plastic material (b) Linear-elastic strain-hardening-plastic material (c) Rigid perfectly-plastic material

2.1. Von-Mises Model

The Von Mises yield criterion applies best to ductile materials, such as metals as the yielding is independent of the first stress invariant (hydrostatic component of the stress tensor). Figure 4 shows the Von Mises yield surface in the three-dimensional space of principal stresses. It is a circular cylinder of infinite length with its axis inclined at equal angles to the three principal stresses. According to the Von-Mises model, yielding will initiate when the second invariant of the deviatoric stress tensor reaches a certain critical value. Deformation prior to yielding is assumed to be linear elastic, governed by the elastic parameters E and ν.

The Von-Mises yield function f can be expressed as:

$$f = \sqrt{J_2} - k = 0 \tag{12}$$

where $k = \frac{\sigma_y}{\sqrt{3}}$ is the yield stress of the material in pure shear and σ_y is the yield stress in uniaxial compression; and J_2 is the second deviatoric stress invariant. Substituting J_2 in terms of the principal stresses $(\sigma_1, \sigma_2, \sigma_3)$ into the Von Mises criterion equation we have

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 6k^2 = 2\sigma_y^{\,2} \tag{13}$$

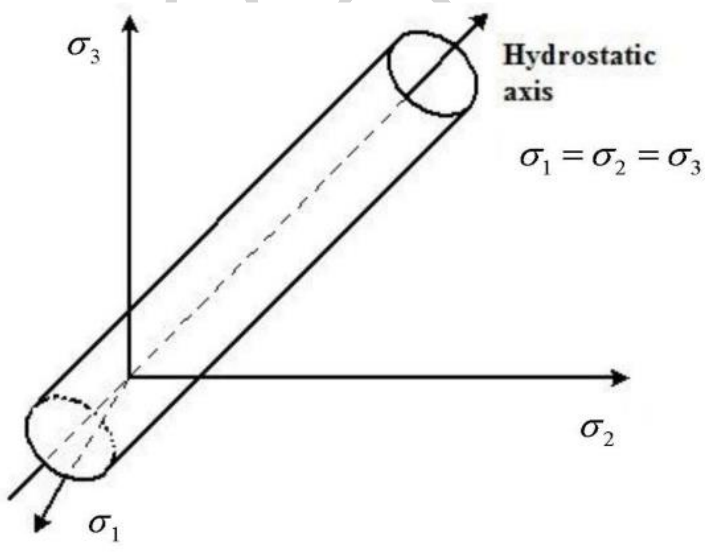

Figure 4. Von Mises yield surface in 3-D

2.2. Tresca Model

Tresca's criterion, states that plastic yielding will initiate when the maximum shear stress reaches a critical shear stress value. The Tresca criterion can be expressed in terms of principal stresses as follows:

$$\max\left(\left|\sigma_1-\sigma_2\right|\right),\left(\left|\sigma_2-\sigma_3\right|\right),\left(\left|\sigma_3-\sigma_1\right|\right)=2k \tag{14}$$

where $2k=\pm\sigma_y$

The above equation can be represented in the principal stress space as a hexagonal cross-section (Figure 5), centered on the hydrostatic axis $(\sigma_1=\sigma_2=\sigma_3)$. Any state of stress located inside the yield surface is considered to be under elastic state. The yield strength in tension and compression is assumed to be equal.

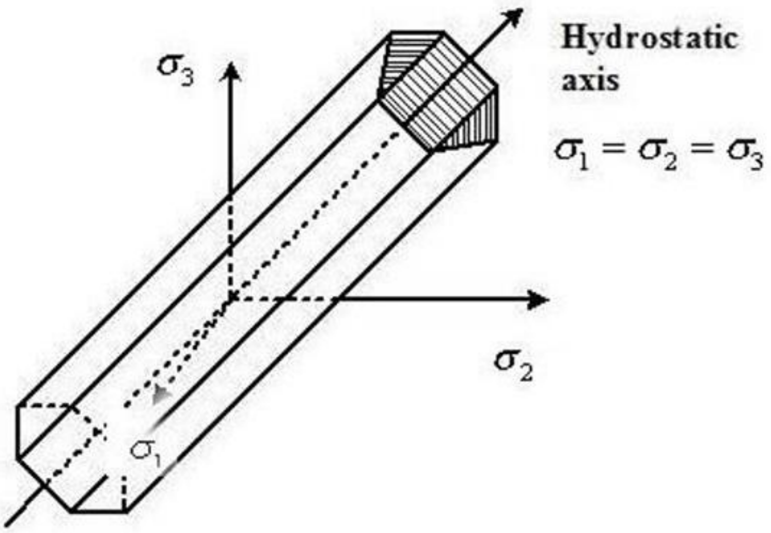

Figure 5. Tresca yield surface in 3-D

The yield criterion can also be expressed as:

$$\sigma_1-\sigma_2=\pm\sigma_y \tag{15}$$

$$\sigma_2-\sigma_3=\pm\sigma_y \tag{16}$$

$$\sigma_3-\sigma_1=\pm\sigma_y \tag{17}$$

A cross section of the von Mises cylinder on the plane of σ_1,σ_2 produces the elliptical shape of the yield surface. Figure 6 shows the Von Mises yield surface in two-dimensional space compared with the Tresca criterion.

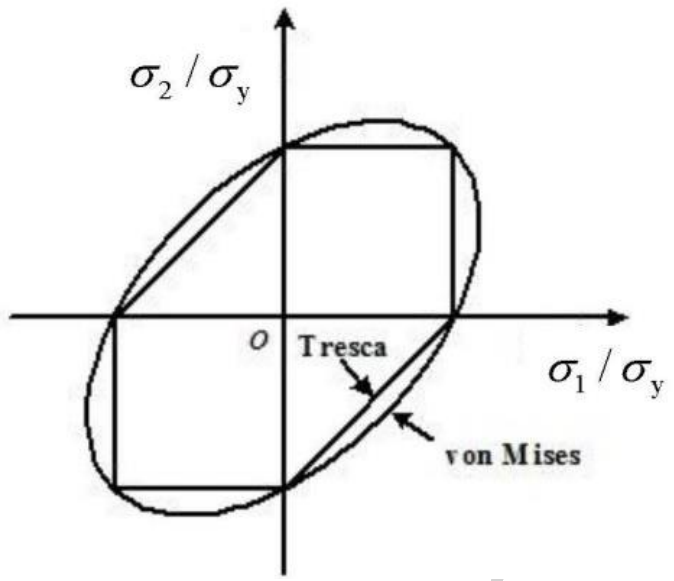

Figure 6. Von Mises and Tresca criteria on the $\sigma_1 - \sigma_2$ plane

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Biographical Sketch

Pradeep U. Kurup is a Professor in the Department of Civil and Environmental Engineering at the University of Massachusetts Lowell, USA. He received his B.Tech. in Civil Engineering in 1985 from the University of Kerala and obtained his M.Tech. from the Indian Institute of Technology Madras (1987). He holds a Ph.D. in Civil Engineering (1993) from Louisiana State University (LSU). Dr. Kurup has conducted extensive research in multi-sensor data fusion, artificial olfaction, and geotechnical & geo-environmental site characterization. Dr. Kurup has done extensive research in the areas of multi-sensor data fusion, site characterization & monitoring, application of novel sensing technology to geotechnical & geo-environmental engineering, finite element modeling, artificial olfaction, neural networks, calibration chamber testing, and "seeing-ahead-techniques" for trenchless technologies. Dr. Kurup is the recipient of the prestigious CAREER Award from the U.S. National Science Foundation, and the “Civil Engineering Research Foundation Award” from the American Society of Civil Engineers. Dr. Kurup’s research has been supported by several agencies including the National Science Foundation, Army Research Laboratory, Environmental Protection Agency, Federal Highway Administration, and the Louisiana Department of Transportation. Dr. Kurup is a member of several professional societies, and is a registered Professional Engineer in the state of Louisiana.