EXPERIMENTAL METHODS AND PHYSICAL MODELING

R. J. Keller
Department of Civil Engineering, Monash University, Clayton, Victoria, Australia

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Contents

Summary

A review is given of the historical development of experimentation in the modeling of hydraulic flow, form and surface resistance and sediment transport. The basic theory of dimensional analysis, and the use of nondimensional flow parameters, to represent flow phenomena, are described. With a sound theoretical basis, such as the use of process functions, hydraulic modeling is employed to provide information regarding open channel flow, and these techniques are extended to analyze flow behavior in movable bed models of sediment transport. Precautions against excessive scale reduction are advised due to scale effects that may invalidate the results. The use of vertically distorted models is also discussed, as well as the prediction of erosion and scour depths by means of model studies. Sediment transport, grain-size modeling and river training are also discussed. Future issues in the direction of combining physical models with theoretical analysis, so-called hybrid modeling, are suggested. Some considerations on the near-field and far-field mixing processes in effluent disposal are also given. The future scenario is that the best answers would come from experimenters and theoreticians combining their efforts towards developing better understanding of the underlying processes, in analyzing flow behavior.

1. Introduction

In most cases of fluid motion in hydraulics, the complexity is such that the strict application of basic equations is only possible in relatively simple geometry. Analytical treatment requires the situation to be idealized to some extent, and the effect of the consequent simplifications can only be tested by experiment. As a result, the science of hydraulics has been marked by intense development of experimental methods. Experimental observation and measurements, and consequent conceptual deductions, have been at the heart of many of the great discoveries in fluid mechanics and hydraulics. Along with the experimental study of basic fluid phenomena, the science and art of physical hydraulic modeling have developed.

The experiment offers a classical way to study real flows. Experiments at the proper scale have the advantage that they do not use simplifying assumptions, and can precisely predict the properties and characteristics of a real flow situation. However, with high-level computing power now routinely and cheaply available, the emphasis on numerical solutions of fluid flow problems is ever growing. Nevertheless, experimental hydraulics continues to flourish because it is based on the behavior of real fluids. The experiment should be seen as a full partner with numerical methods in developing understanding and predictive ability for hydraulic problems.

Model flows at small scale have similar properties to those of the prototype, thereby permitting an exploration of natural flows. In addition, laboratory flows in pipes, channels and tanks are studied in order to gain insight into the motions and forces in basic flow situations. However, this similarity is never complete, and needs to be carefully evaluated by the research engineer. In some situations where scale effects may prevent adequate simulation of prototype behavior, the necessary understanding may only be obtained by the study of prototype situations (see chapter *Fluid Mechanics*).

This article is concerned with the science and art of determining the behavior of prototype hydraulic situations using experiments. The emphasis herein is on the application of scale models since they normally represent the most convenient way of predicting the performance of a specific hydraulic situation under a large range of flows and other variable conditions. The flow situations encountered in practice are often notoriously difficult to measure with instruments. However, new acoustic and optical measurement methods are capable of providing accurate data without adversely affecting the flow field in the experiment.

The subject of instrumentation in fluid flows is not dealt with in this article. New instruments and new capabilities are rapidly evolving in parallel with the evolution of high-technology electronics, optics and computer technology. Information on these developments is best obtained from the many companies working in this area and from such specialized periodicals as the *Journal of Scientific Instrumentation*.

This article examines the development of physical modeling and identifies outstanding issues which remain to be solved. The field of physical modeling is vast, both with regard to the range of problems tackled and the breadth of literature in the field. Excellent reviews and texts include those of Martens, Kobus, and Novak and Cábelka. Due to the vastness of the field, it is only possible to cover selected topics, and inevitably, these are largely confined to the author's range of interest and experience.

To set the scene, a brief history of hydraulic modeling is first presented. Modeling criteria are then reviewed. In particular, the need to supplement pure dimensional analysis with process functions, based on sound analytical concepts, is emphasized. Attention is then focused on scale effects and their management. Some outstanding issues for further research are then identified and conclusions drawn.

2. Brief History of Hydraulic Modeling

The history begins with Leonardo da Vinci (1452–1519) whose accomplishments in many fields dwarfed those of his contemporaries. He is well known for his contributions to Italian drawing, painting and sculpture, and showed genius in the areas of music, natural philosophy, anatomy, botany, geology, mechanics, architecture, and engineering. He left over five thousand pages of sketches and comments, moving without system from subject to subject. The task of sorting and interpreting his records was made difficult by his habit of writing left-handed, in mirror image, and often from back to front as described by Rouse and Ince.

Da Vinci advocated the study of hydraulics in an experimental manner. His basic premise is quoted as “Remember when discoursing on the flow of water to adduce first experience and then reason.” Among phenomena which he observed and described are the velocity distribution in a vortex; the profiles of free jets; the formation of eddies at abrupt expansions and in wakes; the propagation, reflection, and interference of waves; and the hydraulic jump.

Isaac Newton (1642–1727) is generally credited with the first theoretical treatment of similarity criteria of mechanical processes under different scales. He also formulated the rule of corresponding velocities representing a statement of the ratio of velocities created by the action of gravity in similar motions but at different scales, as stated by Ivicsics.

John Smeaton (1724–1792), the first of the great English engineers, conducted the first known scale model experiments. He carried out tests aimed at determining the performance of water wheels and windmills. His introductory remarks to a classic paper presented to the Royal Society in 1759 are worth quoting in full:

What I have to communicate on this subject was originally deduced from experiments made on working models, which I look upon as the best means of obtaining the outlines in mechanical inquiries. But in this case it is very necessary to distinguish the circumstances in which a model differs from a machine at large; otherwise a model is more apt to lead us from the truth rather than towards it. Hence the common observation, that a thing may do very well in a model that will not answer in large. And, indeed, though the utmost circumspection be used in this way, the best structure of machines cannot be fully ascertained, but by making trials with them, when made of their proper size.

The sentiments expressed of the need to be aware of scale effects and to verify model performance against prototype behavior are as valid 240 years later. In 1852, Ferdinand Reech postulated that, in the study of the dynamic resistance of vessels, use could be made of small-scale models, multiplying observed model velocities by the square root of the length scale to predict the corresponding prototype values. The same deduction was drawn by Bourne, also in 1852, applied to the study of screw propellers. This hypothesis is the heart of the Froude criterion of similarity, although predating Froude’s work by some twenty years.

Froude worked largely in the development of towing tank techniques for the testing of model ships. He used Reech's criterion for scaling up model velocities to prototype values, without, however, acknowledging Reech's work. Froude did, however, correctly identify a scale effect due to surface resistance, explaining it in terms very similar to current understanding of boundary layer theory. It is an irony of hydraulics that Froude's name is associated with a law of similarity which he did not develop, while his great contributions to boundary layer research are today largely unacknowledged.

The earliest known mobile bed river model was built in 1875 by Louis Fargue, according to Ivicsics, while investigating river regulation measures for the Garonne River at Bordeaux. His experiments were rather crude in that the depth and time scales were arbitrarily chosen. It was left to Osborne Reynolds to correlate these parameters properly during his experiments in 1885 on a tidal, mobile bed model of the River Mersey. The work of Reynolds and his successor, Vernon-Harcourt, is largely recognized today as the genesis of mobile bed modeling. Vernon-Harcourt wrote that:

If I succeed in demonstrating with the model that the originally existing conditions can be reproduced typically; and if, moreover, by placing regulating works in the model, the same changes can be reproduced that were brought about by the training works actually built, then I am sure that I can take the third and most important step, namely, of investigating, with every promise of success, the probable effect of the projects that have been proposed.

This hypothesis remains as the most important underlying principle of mobile bed modeling.

With the foundations of open channel modeling established, several major modeling laboratories were set up, many of which continue to operate today. The first river hydraulics laboratories, set up as permanent installations, were established in Germany by Engels in 1898 and Rehbock in 1901, according to Kobus.

The further widespread development of hydraulic laboratories, especially for the design of hydraulic structures and river training schemes, was the predominant phenomenon of early twentieth century hydraulics. In addition to the Froude number, other nondimensional invariant numbers, such as the Reynolds and Weber numbers, had been postulated from physical arguments based on Newtonian physics. The general modeling requirement for dynamic similarity was that these numbers should be the same in model and prototype.

A more formal approach to the determination of governing invariant parameters emerged early in the twentieth century with the development of dimensional analysis. This work was popularized by Buckingham, but as described by Rouse, it was largely based on earlier work by Fourier, Vaschy, Riabouchinsky, and Rayleigh. The basic principle of the so-called Buckingham Pi-theorem is that correct identification of the fluid and flow properties associated with a particular phenomenon leads to the formulation of nondimensional numbers which can then be used as the basis of model-prototype similitude.

The Buckingham Pi-theorem is very easy to apply, involving only the application of some simple mathematical rules. However, its correct application involves a sound understanding of the physics of the problem under study to ensure that all necessary fluid and flow properties are included. Even then, however, as will be shown in the following section, dimensional analysis is of little use to the modeler without specific hydraulic information encapsulated in so-called process functions.

The design of the early mobile bed models developed by Fargue, Reynolds and Vernon-Harcourt was largely intuitive. Provided that the sediment in the model moved, it was assumed that the response of the model bed to spatial and temporal changes in velocity would simulate—at least qualitatively—the response of the prototype to similar changes. In 1936, Shields published his classic work on the inception of sediment transport and this served as the impetus for accelerated research and understanding of many aspects of sediment transport. In the context of this article, Shields' work led to the specific formulation of model design and result interpretation laws for mobile bed models. These are discussed further in the following section.

Over the last fifty years of the twentieth century, physical models have played an increasing role in the design of hydraulic structures, river training works, and coastal works. This has been heightened by the development of sophisticated instrumentation for the simultaneous and highly accurate measurement of model parameters such as velocity, depth, and flow rate.

These advances, however, can have the effect of luring investigators into a false sense of security regarding their own studies. If the model design is based on faulty concepts, then the use of highly sophisticated instrumentation and measurement techniques will only improve the accuracy of the wrong predictions. The issue of sound model design is the focus of the following sections of this article.

3. Model Criteria—Dimensional Analysis and Process Functions

As an example of the use of dimensional analysis, and to illustrate its insufficiency on its own, the simple case of flow in a fixed-bed open channel is first considered. Adopting the mantle of dimensional analysis, the controlling parameters and their dimensional units are first identified as follows:

–	Flow velocity, V,	L/T
–	Channel width, W	L
–	Channel depth, y	L
–	Channel length (distance along), l	L
–	Fluid density, ρ	M/L^3
–	Time, t	T
–	Mass, m	M
–	Fluid viscosity, μ	M/(LT)
–	Fluid surface tension, σ,	M/T^2
–	Surface roughness, ε	L
–	Gravitational acceleration, g	L/T^2

–	Slope (vertical/horizontal), S	L/L
–	Wetted perimeter, P	L
–	Cross-sectional area, A	L^2
–	Hydraulic radius, R (=A/P)	L
–	Discharge, or Flow rate, Q (=AV)	L^3/T
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where L = length unit, M = mass unit, and T = time unit. Dimensional analysis enables the grouping of these parameters in a number of ways. Adopting *V*, *y*, and ρ as the repeating variables, a legitimate set of five nondimensional variables can be developed as follows:

$$f\left(\frac{V^2}{gy},\frac{V^2}{\sigma/y\rho},\frac{Vy\rho}{\mu},\frac{W}{y},\frac{\varepsilon}{y}\right)=0 \quad (1)$$

where ρ, σ, μ, ε are as defined above.Strict similitude would only be possible if all five nondimensional groups in Equation (1) are identical in model and prototype. It can quickly be established, however, that this is not possible, especially if the same fluid is used in model and prototype, unless the scale ratio is unity, in other words a full-scale model. To move forward from this point, use must be made of physical understanding and of a process function derived as follows:

The first term in Equation (1) represents the ratio of inertial forces to gravitational forces. Since the phenomenon under consideration is gravity driven, this parameter must be retained. Requiring equality of the first term at homologous points in the model and the prototype leads to the well-known Froude law of modeling, appropriate to open channel flows:

$$\lambda V=\sqrt{\lambda y} \quad (2)$$

where λ means “the scale of” (model to prototype).

The second term in Equation (1) is a Weber number, representing the ratio of inertial forces to surface tension forces. This ratio increases with model size because the inertial forces act on a volume whereas the surface tension forces act on an area. Thus the surface tension forces become negligible, provided the model is reasonably large, and the second term can be disregarded.

Turning now to the third term in Equation (1), a Reynolds number, Re, is identified, representing the ratio of inertia forces to viscous forces. In the context of an open-channel flow, viscous forces affect the surface resistance, apparently requiring Reynolds number equality between model and prototype for full similarity.

If the same fluid is used in model and prototype, Reynolds number equality at homologous points would require that

$$\lambda V = \frac{1}{\lambda y} \qquad (3)$$

and this condition is clearly incompatible with Equation (2). Indeed, it is readily shown that, if the velocity scale is based on Equation (2),

$$\lambda \mathrm{Re} = \lambda y^{3/2} \qquad (4)$$

This situation is resolved by making use of a process function for flow resistance which links the friction factor, Reynolds number, and relative roughness through the well-known Colebrook-White equation. This function is conveniently plotted as a Moody diagram and is reproduced in Figure 1 (see chapter *Fluid Mechanics in Pipelines*).

The equation for the friction factor, the ordinate of Figure 1, shows that the Froude similitude criterion given by Equation (2) can only be satisfied if the friction factor is the same in model and prototype. Superimposed on Figure 1 is a hypothetical range of prototype Reynolds numbers and a corresponding range of model operation, assuming a model scale of 1:25. For the example given, it is evident that equality of friction factor between model and prototype can only be obtained if the model is relatively smoother than the prototype.

However, it is noted, further, that this equality is only possible for one particular operating condition (characterized by the Reynolds number). For other operating conditions, the model friction factor will be different from that in the prototype, introducing a friction scale effect. The scale effect can be calculated, however, and model results adjusted when scaling up to prototype values. Figure 1 also demonstrates that if the prototype is relatively smooth, it may not be possible to build a model with a low enough friction factor (f) to match that of the prototype. In this situation, the higher model friction factor may be considered as at least being conservative with respect to predicted flow depths, or again, the scale effect may be calculated and the predicted prototype values adjusted.

The discussion above has demonstrated that dimensional analysis is insufficient on its own to provide a basis for the modeling of open channel flow. Indeed, if dimensional analysis was solely relied upon, the conclusion would be drawn that accurate modeling is not possible. It is only by adding knowledge of flow resistance and its corresponding process function to dimensional analysis that an appropriate modeling procedure is identified. Other examples of the necessity for process functions, in addition to dimensional analysis, for physical modeling of weir flows and vortex drop shafts, have been discussed by Ackers.

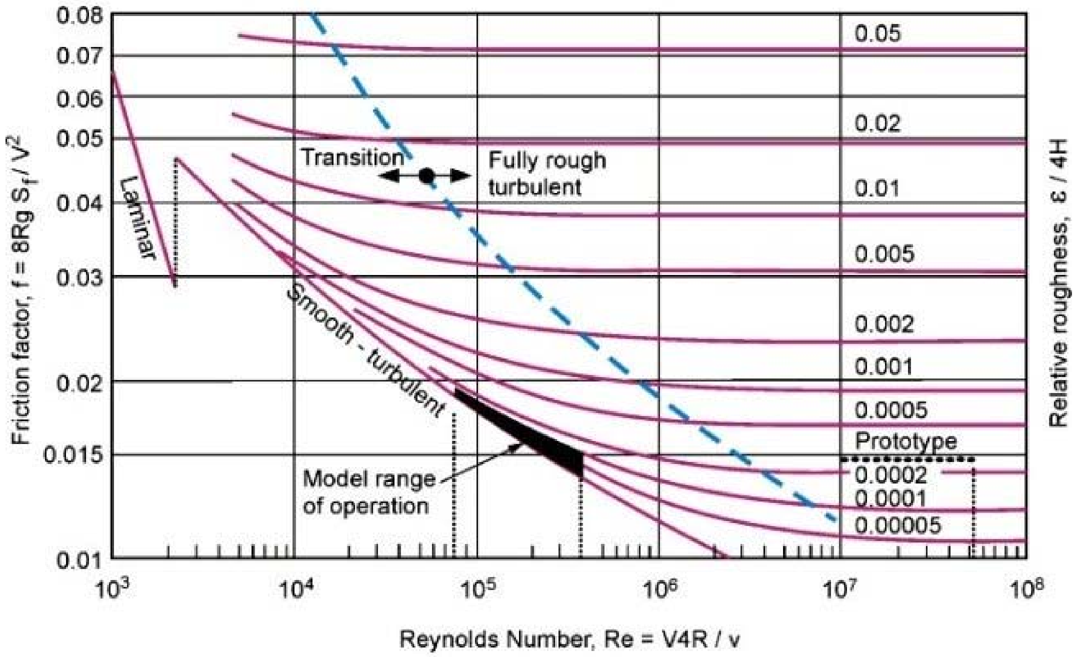

Figure 1. Process function diagram for friction, showing the relationship between friction factor and Reynolds number

The above presentation applies to undistorted models only—i.e., those for which the horizontal and vertical scales are identical. Undistorted models are common in hydraulic structure investigations, but are often impractical for large rivers because of their typically large width to depth ratios. A typical river may have a width of five hundred meters and a depth of perhaps two meters. The corresponding undistorted model of a scale of, say, 1:250 would be two meters wide and eight millimeters deep. Due to surface tension effects and the likelihood of laminar flow, the model flow behavior is likely to be totally different in character from that of the prototype.

This situation is resolved by utilizing a vertical scale which is larger than the horizontal. The Froude relationship is still expressed in the form of Equation (2), where, however, λy represents the vertical scale because it is vertical, rather than horizontal, distances which measure the effect of gravity on velocity.

With reference to Figure 1, the expression for head loss, h_L, is:

$$h_L = f \frac{L}{4R} \frac{V^2}{2g} \quad (5)$$

where R is the hydraulic radius, as defined ahead of Equation (1).

Rearrangement and expression in terms of scale ratios, λ, yields for the scale ratio of the slope, S:

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Biographical Sketch

Robert J. Keller is Associate Professor of Civil Engineering at Monash University, Melbourne, Australia. He is also a Senior Researcher in the Cooperative Research Centre for Catchment Hydrology.Professor Keller received the degrees of Bachelor of Engineering (Honours) in 1967 and Doctor of Philosophy in 1972 from the University of Canterbury in Christchurch, New Zealand. He then devoted two years to research as a Post-doctoral Fellow at the University of Karlsruhe, Germany. After his return to New Zealand in 1974, he worked in a large physical hydraulic model laboratory for five years. He was appointed a lecturer at Monash University in 1979, a Senior Lecturer in 1981, and attained his present position in 1991. Professor Keller's main research interests are in the areas of river engineering, hydraulic structures and physical/numerical modeling. He also maintains an active role in consulting and has undertaken numerous physical modeling assignments. More recently, he has become involved in the restoration and rehabilitation of rivers and other aquatic environments. He is a member of: the American Society of Civil Engineers; the Institution of Engineers, Australia; the Institution of Professional Engineers, New Zealand; and the International Association of Hydraulic Research. He is an Associate Editor of the *Journal of Hydraulic Research*. In 1981, Professor Keller received the Furkert Award of the Institution of Engineers, New Zealand; and in 1993 the Schoemaker Award of the International Association of Hydraulic Research. In 1982, and again in 1987, he carried out advanced research in Germany as a Von Humboldt Scholar. In 1995 he devoted six months to research as a Visiting Scientist at Hydraulic Research Wallingford in Oxfordshire, UK. Professor Keller is the author or coauthor of over seventy publications at conferences and in journals.