NUCLEAR REACTOR KINETICS

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Contents

Summary

The complete neutron cycle from one fission through the moderating process to the next fission is exceedingly rapid taking only about 1 millisecond in a typical reactor. If all neutrons had this same lifetime any slight deviation from a perfectly balanced system (same number of neutrons in each generation) would lead to a rapid collapse or expansion of the

neutron population making control of the reactor impossible. Fortuitously some neutrons are produced shortly after the actual fission and are known as delayed neutrons. This provides a buffer that smoothes the process and allows control of the reactor by conventional neutron measurement and mechanical control devices.

When reactor power is to be changed, such as during startup, load variation or shutdown, neutron absorbing control rods are withdrawn, manipulated or inserted to change the rate of fissioning and hence release of heat. Such changes have to be done very slowly due to the sensitivity of the process and the tremendous range in neutron population from the shutdown condition to the full load condition.

When a reactor is in the shutdown condition, source neutrons are continuously produced by spontaneous fission of fuel and by decay of certain fission products. These source neutrons go on to cause further fissions in a series of diminishing neutron cycles. The reactor is thus never in a true shutdown condition but merely in a state of very low power. When raising power, as the critical condition is approached, the neutron cycles initiated by the source neutrons are extended and take longer to fade away. Thus the neutron population progressively increases with each incremental withdrawal of the control rods when the reactor is in a sub-critical condition. The exact point of criticality is not able to be ascertained exactly. If slightly supercritical the neutron population or power increases exponentially without further withdrawal of the control rods. In practice the reactor continually oscillates between being slightly subcritical and slightly supercritical as the control system corrects slight deviations in power level.

When a reactor is to be shut down, the control rods are inserted to absorb neutrons and create a subcritical condition. This may be done rapidly, as in a reactor trip due to some fault in the system, or slowly to avoid sudden thermal transients. After shutdown, heat is still generated from the decay of fission products. This poses an important safety consideration as this heat must be removed from the reactor for an extended period.

1. Reactor Kinetics

1.1. Introduction

Reactor kinetics is the study of how neutron power changes with time. It is usually associated with short term changes initiated by natural perturbations or imposed transients. Control systems have to be designed to maintain the desired neutron power following both types of short term changes.

In the longer term there are changes due to the build-up of neutron absorbers and the burn-up of neutron producers in the core of the reactor. Since these changes are slow they do not affect the control system as such but the overall reactor configuration has to be adjusted to maintain the desired balanced condition for a steady neutron chain reaction. The control system then maintains equilibrium about this balanced condition.

Short term effects only are considered in this section; however, it should be emphasized that the control systems handling the short term effects must maintain the same effectiveness as long term effects are accommodated.

Long term effects are considered in the next section.

1.2. Neutron Lifetime

An important consideration in reactor kinetics is the extremely high velocity of neutrons (2200 m/s when thermalized at 20°C). With the high probability of neutron scattering during moderation from high energies and of neutron capture to cause fission at thermal energies and the extremely close spacing of fissile nuclei in the fuel, neutrons do not travel very far in the reactor. The overall neutron lifetime during thermalization and diffusion from production due to fission until absorption to produce fission is thus extremely short. The average neutron lifetime in a heavy water moderated CANDU reactor is about 1 millisecond and in a light water moderated reactor such as a PWR or a BWR it is even less. The complete neutron cycle from one generation to the next takes only this amount of time. It is evident therefore that any small deviation from the equilibrium situation, where the number of neutrons in one generation is equal to that of the previous generation, will very rapidly grow in magnitude.

If all neutrons had a lifetime of about 1 millisecond it would be almost impossible to design a control system that would be able to sense changes and effect control before the neutron population grew or shrank out of the control range. Fortuitously some neutrons are produced a short time after the actual fission process. These are known as *delayed neutrons*. The kinetic behavior of a reactor is critically dependent upon the existence of these delayed neutrons as they have the effect of increasing the average lifetime of the neutrons arising from fission. This increase in average lifetime to about 1 second due to less than 1% of the neutrons having much longer effective lifetimes enables control systems to maintain stable operation provided that certain limits are not exceeded.

1.3. Reactor Power

Reactor power P is given by the following formula where φ is the neutron flux, Σ_f the macroscopic fission cross section, V the volume of the reactor and E_R the energy released per fission in joules.

$$P = \varphi \Sigma_\mathrm{f} V E_\mathrm{R} \tag{1}$$

Note that the neutron flux φ and the macroscopic fission cross section Σ_f are given by:

$$\varphi = nv$$

$$\Sigma_\mathrm{f} = N\sigma_\mathrm{f}$$

Here n is neutron density, v the neutron velocity, N the nuclei density (of fissile material) and σ_f the microscopic fission cross section.

For a given reactor configuration it is evident that the reactor power P is proportional to the neutron flux φ and to the neutron density n since the other factors in the equations are constant in the short term. Any variation in neutron density will therefore be reflected as a variation in reactor power. Although the reactor kinetics equations are related to variations in neutron density they are often represented directly as variations in reactor power and the two terms are used synonymously.

In the control room of a nuclear reactor there are instruments which indicate the neutron flux levels in the reactor but ultimately it is the reactor power level which is monitored and controlled. Hence the equations presented below, although derived initially in terms of neutron density, have been converted and rewritten in terms of reactor power.

1.4. Basic Reactor Kinetics

The basic reactor kinetics equation (without delayed neutrons) may be derived by using the definition of the neutron multiplication factor k.

$$k = neutrons\ in\ one\ generation / neutrons\ in\ previous\ generation$$

Consider a slight but steady increase in the number of neutrons in successive generations, that is, k is slightly greater than unity. If the initial neutron density is n then, after one generation, the neutron density will be equal to kn and the change in neutron density Δn will be given by:

$$\Delta n = kn - n$$

If the neutron lifetime is l then this change from one generation to the next will take place in time l, that is, the neutron lifetime l is equal to the time interval Δt from one generation to the next and the equation can be written as:

$$\Delta n / \Delta t = (kn - n) / l$$

Considering k to be very close to unity and Δk its deviation from unity, then substitution into this equation and conversion to differential form, gives the following:

$$dn / dt = \Delta kn / l$$

This equation can be integrated from time zero to time t and converted to exponential form as follows:

$$\int_0^t dn / n = \int_0^t (\Delta k / l) dt$$

$$\ln(n_t / n_0) = (\Delta k / l) t$$

$$n_t = n_0 e^{(\Delta k / l) t}$$

Here n_t is the neutron density at time t while n_0 is the initial neutron density at time zero.

Since reactor power P is proportional to neutron density n this equation may be written as:

$$P_t = P_0 e^{(\Delta k / l)t} \qquad (2)$$

This is the basic reactor kinetics equation assuming that all neutrons have the same lifetime l.

1.5. Reactor Period

Reactor period τ is defined as the time taken for the reactor power to increase by a factor of e. This is a common way of specifying a rate of power increase in a nuclear reactor. After one reactor period the time t will be equal to τ and the reactor power P_t after time t will be equal to the initial reactor power P_0 multiplied by a factor of e.

$$t = \tau$$

$$P_t = P_0 e$$

If these values are substituted into the basic reactor kinetics equation the following is obtained:

$$P_0 e = P_0 e(\Delta k / l)\tau$$

$$\tau = l / \Delta k \qquad (3)$$

Thus the reactor period may be obtained directly from the neutron lifetime l and the change Δk in the value of the neutron multiplication factor k.

1.6. Doubling Time

Reactor *doubling time* is the time taken for the reactor power to rise to twice its initial condition. This is used during reactor commissioning and also during reactor start-up particularly following refueling. It is a convenient practical concept similar to the concept of the half life of radioactive materials.

Fuel *doubling time* is the time taken for a breeder reactor to produce twice as much fissile material as was initially in the reactor. This occurs due to the absorption of excess neutrons in U-238 and its conversion to Pu-239 while the U-235 is consumed to produce power. If more Pu-239 is created than U-235 consumed the reactor produces more fuel than it consumes and is termed a breeder.

It is seen from the above that an important parameter in reactor kinetics is Δk where Δk represents the change in the neutron density. Generally in an operating reactor k is exactly unity at a steady power level and Δk is a small fraction during power transients indicating a departure from steady state conditions. The new value of k arising from this change k' is greater than unity by an amount Δk. Thus the relationship between k' and Δk is as follows:

$$\Delta k = k' - 1$$

The difference Δk is referred to as the *excess multiplication factor* and can be positive or negative. The *reactivity* ρ of a reactor is then defined as follows:

$$\rho = \Delta k / k'$$

Since k' is very nearly unity it follows that the reactivity ρ is almost the same as Δk. It follows therefore that:

$$\rho \approx \Delta k \; \rho \approx \Delta \mathrm{k}$$

In many applications Δk is simply referred to as the reactivity of the reactor and is easily understood. Δk thus represents a departure from the steady state condition in a positive or negative direction and is commonly known as *positive reactivity* or *negative reactivity*. Very conveniently reactivity has been given units of *k* (British and Canadian terminology) although it is really dimensionless. This allows the use of mk for small values of reactivity where 1 mk is equal to 0.001 k. For 1 mk of reactivity Δk would be 0.001 and *k* would be 1.001. Many text books use an alternative terminology where reactivity is expressed in dollars and is related not to the steady state condition but to the margin between the steady state condition and the prompt critical condition which will be defined later.

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Biographical Sketch

Robin Chaplin obtained a B.Sc. and M.Sc. in mechanical engineering from University of Cape Town in 1965 and 1968 respectively. Between these two periods of study he spent two years gaining experience in the operation and maintenance of coal fired power plants in South Africa. He subsequently spent a further year gaining experience on research and prototype nuclear reactors in South Africa and the United Kingdom and obtained M.Sc. in nuclear engineering from Imperial College of London University in 1971. On returning and taking up a position in the head office of Eskom he spent some twelve years initially in project management and then as head of steam turbine specialists. During this period he was involved with the construction of Ruacana Hydro Power Station in Namibia and Koeberg Nuclear Power Station in South Africa being responsible for the underground mechanical equipment and civil structures and for the mechanical balance-of-plant equipment at the respective plants. Continuing his interests in power plant modeling and simulation he obtained a Ph.D. in mechanical engineering from Queen's University in Canada in 1986 and was subsequently appointed as Chair in Power Plant Engineering at the University of New Brunswick. Here he teaches thermodynamics and fluid mechanics and specialized courses in nuclear and power plant engineering in the Department of Chemical Engineering. An important function is involvement in the plant operator and shift supervisor training programs at Point Lepreau Nuclear Generating Station. This includes the development of material and the teaching of courses in both nuclear and non-nuclear aspects of the program.. He has recently been appointed as Chair of the Department of Chemical Engineering.