RESERVOIR SIMULATION

Y. Cheng
Department of Petroleum & Natural Gas Engineering West Virginia University, USA

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Summary

1. Introduction to Reservoir Simulation

A petroleum reservoir is an underground porous medium in which oil or gas or both are trapped structurally or/and stratigraphically. Fluid flow in such a porous medium is very complex phenomena. Generally, analytical solutions to mathematical models are only obtainable after making simplifying assumptions in regard to reservoir geometry, properties and boundary conditions. However, such simplifications are often invalid for most fluid flow problems. In many cases, it is impossible to develop analytical solutions for practical issues due to the complex behaviors of multiphase flow, nonlinearity of the governing equations, and the heterogeneity and irregular shape of a reservoir system. As a result, these models must be solved with numerical methods such as finite difference or finite element. Reservoir simulation provides numerical solutions to hydrodynamic

problems of fluids (oil, gas and water) in petroleum reservoir-well systems on a digital machine. Today, it has become a standard tool in petroleum engineering discipline and been widely used for solving a variety of fluid flow problems involved in recovery of oil and gas from the porous media of reservoirs.

Typical application of reservoir simulation is to predict future performance of the reservoirs so that intelligent decisions can be made to optimize the economic recovery of hydrocarbons from the reservoir. Reservoir simulation can also be used to obtain insights into the dynamic behavoir of a recovery process or mechanism.

In petroleum engineering area, the numerical solution through the reservoir simulators is often the only way to obtain meaningful and reliable solutions for most actual cases due to extreme complexity of reservoir systems. The numerical solution provides results at discrete points in spatial and temporal domains. Development of a reservoir simulator for different types of reservoir-well systems and recovery processes requires substantial background in mathematics and applied science, which starts with establishing the finite difference equations of a mathematical model for fluid flow in a certain type of reservoir-well system, then followed by numerical modeling and computer programming, and generates simulation software for application to the end. A schematic diagram of this process is shown in Figure 1.

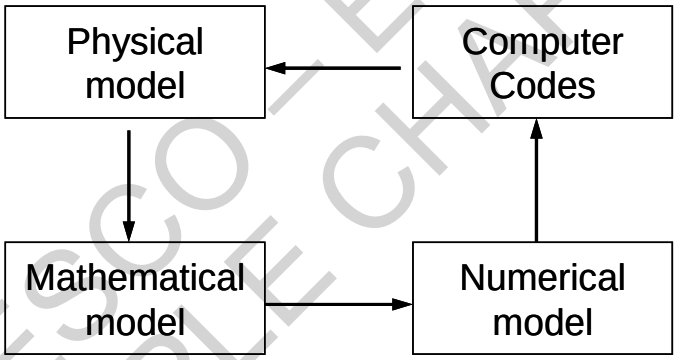


Figure 1. Reservoir simulation process

Reservoir simulation is briefly introduced here. The principles and procedures of the finite difference method are first discussed for the system of a single-phase slightly compressible fluid based on the theoretical basis of Taylor series. For the system of a single-phase compressible fluid, the finite difference equations are built on the individual gridblocks rather than discretization of the diffusivity equation, and are illustrated in Section 2. The heterogeneity of reservoirs, irregular size of gridblocks, and non-linearity of equations caused by pressure-dependent properties are considered in the finite difference equations.

In the section dealing with the system of three-phase flow, the implicit pressure-explicit saturation method is described for obtaining the simulation equations. More detailed information of theory and practice of reservoir simulation was given by Peaceman (1978), Thomas (1977), Aziz and Settari (1979), Mattax and Dalton (1990) and Ertekin, Abou-Kassem and King (2001).

2. Finite Difference Model for Single-phase Slightly Compressible Fluid

Development of reservoir simulators begins from setting up finite difference model for the equations that govern the fluid flow in porous media. These equations are partial differential equations which are constructed following the physical principles, such as continuity equation, Darcy's law and equation of state. For example, Eq. (1) gives the diffusivity equation governing one dimensional single-phase flow of a slightly compressible fluid such as oil

$$\frac{\partial^2 p(x,t)}{\partial x^2} = \frac{\varphi \mu c_t}{k} \frac{\partial p(x,t)}{\partial t} \tag{1}$$

where p represents pressure, which is the function of location x and time t. For other parameters in Eq. (1), ϕ stands for porosity, k for permeability, μ for fluid viscosity and c_t for total compressibility of rock and fluid. Eq. (1) describes the flow of a slightly compressible fluid in a homogeneous and isotropic reservoir. The fluid has constant viscosity and compressibility.

The finite-difference approach is the most commonly used numerical method in reservoir simulation and therefore will be introduced here.

2.1. Discretization

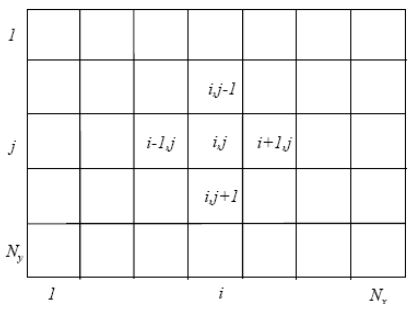
Unlike analytic methods which give continuous solution in time and space (if an analytic solution can be found), numerical approaches find solutions at discrete points in time and space. The spatial domain is divided into a number of grids (also called cells or blocks) and the time domain discretized to a number of time steps. Continuous partial differential equation is then transformed to an equivalent discrete form of the equation by finite-difference.

Spatial discretization

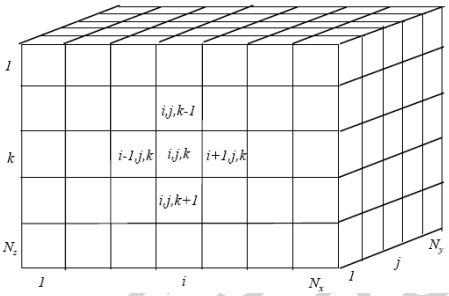
The grids in the numerical model are usually rectangular in form. Radial grids are sometimes used in single-well modeling or local hybrid gridding system. Discretization in spatial domain for one-dimensional, two-dimensional and three-dimensional reservoir problems is shown in Figure 2 (a)-(c) for illustrations.

In these illustrations, the grid system is defined with N_x gridblocks for a one-dimensional model, with N_x by N_y gridblocks for a two-dimensional model, and N_x by N_y by N_z for a three-dimensional model. The index is referred to the center and the unknowns such as pressure are calculated at the center of a gridblock. This type of gridding systems is called the block centered grid. The grid systems presented in Figure 2 have a uniform gridblock for each of them. In practice, variable sizes of gridblocks are often used, which will be demonstrated in Section 3.

(a) One dimensional grid system



(b) Two dimensional grid system



(c) Three dimensional grid system

Figure 2. Discritization in spatial domain

Time discretization

Discretization in time domain is illustrated in Figure 3.

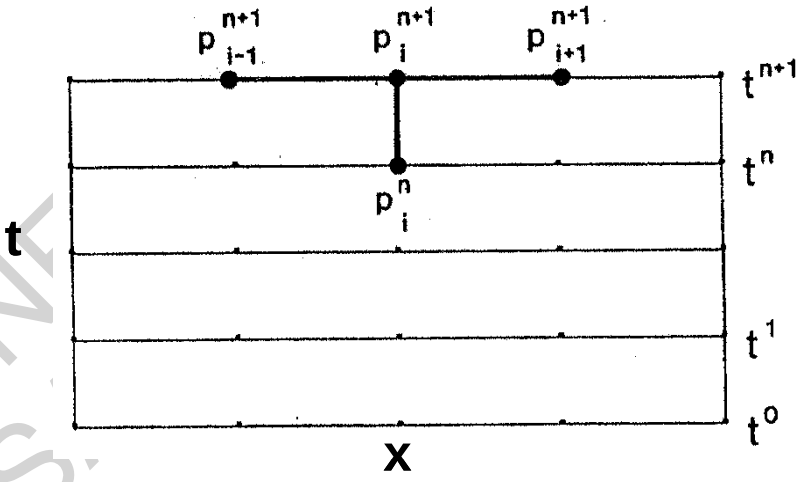


Figure 3. Discritization in time domain

2.2. One-Dimensional Finite Difference

When the partial derivatives are replaced by finite difference approximation based on Taylor's series expansion, a partial differential equation is discretized into a finite-difference form. With the finite-difference equation, a numerical solution can be obtained for the problems described by the governing equations with the specified conditions including initial and inner/outer boundary conditions.

For a one-dimensional problem, the value of pressure p at $x+\Delta x$ can be approximated with Taylor series expansion if the values of p and its all derivatives at x are known.

$$p(x+\Delta x)=p(x)+\Delta x\frac{\partial p(x)}{\partial x}+\frac{\Delta x^2}{2!}\frac{\partial^2 p(x)}{\partial x^2}+\frac{\Delta x^3}{3!}\frac{\partial^3 p(x)}{\partial x^3}+\frac{\Delta x^4}{4!}\frac{\partial^4 p(x)}{\partial x^4}+\cdots$$
$$+\frac{\Delta x^n}{n!}\frac{\partial^n p(x)}{\partial x^n}+\cdots \qquad (2)$$

Similarly, the value of p at $x-\Delta x$ can be approximated as

$$p(x-\Delta x)=p(x)-\Delta x\frac{\partial p(x)}{\partial x}+\frac{\Delta x^2}{2!}\frac{\partial^2 p(x)}{\partial x^2}-\frac{\Delta x^3}{3!}\frac{\partial^3 p(x)}{\partial x^3}+\frac{\Delta x^4}{4!}\frac{\partial^4 p(x)}{\partial x^4}-\cdots$$
$$+(-1)^n\frac{\Delta x^n}{n!}\frac{\partial^n p(x)}{\partial x^n}+\cdots \qquad (3)$$

where Δx is the distance of the centers of two adjacent gridblocks and for a uniform gridding system Δx is a constant equal to the gridblock length.

Combining Eqs. (2) and (3), the second order partial derivative in the left side of Eq. (1) may be approximated as below

$$\frac{\partial^2 p(x)}{\partial x^2}=\frac{p(x-\Delta x)-2p(x)+p(x+\Delta x)}{\Delta x^2}-\frac{2\Delta x^2}{4!}\frac{\partial^4 p(x)}{\partial x^4}- \qquad (4)$$

For convenience, use i to label an arbitrary gridblock between 1 and N_x (the last gridblock) in Figure 2, i.e., $i=1, 2, \ldots, N_x$. The location of the center of gridblock i is represented by x_i. Similarly, the neighboring gridblocks $i+1$ is centered at x_{i+1} and $i-1$ at x_{i-1}. To simplify the notation, $p(x_i)$ is denoted as p_i, similarly, $p(x_{i+1})$ by p_{i+1} and $p(x_{i-1})$ by p_{i-1}. Then Eq. (4) may be written as

$$\frac{\partial^2 p(x)}{\partial x^2}\approx\frac{p_{i-1}-2p_i+p_{i+1}}{\Delta x^2}+0(\Delta x^2) \qquad (5)$$

The first term on the right side in Eq. (5) is a central difference. Eqs. (2 & 3) are theoretically exact for an infinite number of terms in the series. When the series is truncated, the error will be introduced, which is called truncation error. The approximation of the second order partial derivative by the central difference has a truncation error of the order of $O(\Delta x^2)$.

For the first-order partial derivative with respect to time in the right side of Eq. (1), the forward difference is used. If the values of p_i and its all derivatives are known at t_n, the value of p_i at t_{n+1} can be approximated with Taylor series expansion as

$$p_i^{n+1} = p_i^n + \Delta t \frac{\partial p_i^n}{\partial t} + \frac{\Delta t^2}{2!}\frac{\partial^2 p_i^n}{\partial t^2} + \frac{\Delta t^3}{3!}\frac{\partial^3 p_i^n}{\partial t^3} + \frac{\Delta t^4}{4!}\frac{\partial^4 p_i^n}{\partial t^4} + \cdots + \frac{\Delta t^n}{n!}\frac{\partial^n p_i^n}{\partial t^n} + \cdots \quad (6)$$

where the superscripts n and $n+1$ on p_i correspond to the beginning and end of the time interval, Δt.

Rearranging Eq. (6) yields the forward difference as

$$\frac{\partial p_i}{\partial t} \approx \frac{p_i^{n+1} - p_i^n}{\Delta t} + 0(\Delta t) \quad (7)$$

This approximation of the first order partial derivative by the forward difference has a truncation error of the order of $O(\Delta t)$.

Substituting Eqs. (5 and 7) into Eq. (1) and dropping the error terms, the finite difference approximation to the diffusivity equation is

$$\frac{p_{i-1} - 2p_i + p_{i+1}}{\Delta x^2} = \frac{\varphi \mu c_t}{k}\frac{p_i^{n+1} - p_i^n}{\Delta t} \quad (8)$$

Eq. (8) is a linear equation. It is the finite difference equation to the diffusivity equation (Eq.(1)), and the error is of the order of $O(\Delta x^2, \Delta t)$.

Initial condition

The initial condition is specified to begin the timestep sequence. For the initial condition, n = 0, an initial pressure before production is usually assigned for all the gridblocks.

$$p_i^0 = p_{\text{initial}}, \quad i = 1, \ldots N_x \quad (9)$$

Boundary conditions

The boundary condition specifies the equations at the first and last gridblocks. In general, there are two types of boundary conditions: fixed pressures and fixed first partial derivatives.

For fixed pressures (Dirichlet condition),

$$p_1^{n+1} = p_{C1}; p_{N_x}^{n+1} = p_{C2}, \quad (10)$$

For fixed first partial derivatives (Neumann condition),

$$\frac{p_2 - p_1}{\Delta x} = p_{C3}; \frac{p_{N_x} - p_{N_x-1}}{\Delta x} = p_{C4} \quad (11)$$

The fixed values p_{C1} through p_{C4} may change with time, but remain constant during a timestep.

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Biographical Sketch

Dr. Yueming Cheng is an Assistant Professor in the Department of Petroleum and Natural Gas Engineering at West Virginia University. She holds Ph.D. from Texas A&M University in College Station, U.S.A., M.S. from Research Institute of Petroleum Exploration and Development (RIPED) in Beijing, China and B.S. from Southwestern Petroleum University in Sichuan, China, all in Petroleum Engineering. She has more than 17 year experiences in petroleum industry and academics. Her expertise includes mathematical modeling of fluid flow and heat transfer in well-reservoir system, reservoir simulation, pressure transient analysis, reservoir characterization, hydraulic fracturing, unconventional gas recovery and uncertainty quantification. She is a SPE technical editor for the Journal of SPE Reservoir Engineering and Evaluation.