MISPERCEPTIONS AND HYPERGAME MODELS OF CONFLICT

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Contents

Summary

Environmental decision making is often characterized by controversies and disputes. Hypergame analysis is a useful tool for modeling and analyzing environmental conflicts with misperceptions. This article presents the basic concepts and structures of hypergame models; the stability analysis procedure that relaxes the assumption of common knowledge in game theory; and uses perceptual games to represent decision makers' beliefs about a conflict. Hypergame analysis is also extended to incorporate the idea of cooperative games involving bargaining and negotiation. Moreover, the effect of misperception is systematically incorporated into the analysis so that the behavior of decision makers in a complicated situation can be fully explored and understood. To demonstrate how hypergames can be used in a practical application, the hypergame approach is used to model and analyze a water aquifer dispute that took place in San Antonio, Texas, USA, in the 1980s between the city council and developers.

1. Introduction

Dispute and controversy often characterize decision making in environmental management and impact assessment. Many development proposals subject to impact assessments, or permit approval, are the focus of disputes involving governments, developers, private citizen groups, and environmental organizations. This is inevitable because people have diverse understandings of the facts, different preferences over the states, separate interests over unevenly distributed benefits and costs, and contrasting perspectives of how ecosystems are likely to respond to various human activities and what constitutes good policy in managing the environment. Modern society's development has greatly increased the interdependence of ecosystems and socioeconomic systems, which is further complicated by uncertainty and our limited knowledge. The increasing demands for natural resources, complexity, and uncertainty have greatly increased environmental disputes in recent years.

In an environmental conflict, two or more participants with separate interests are involved in a dispute over issues, such as natural resources allocation, industrial expansion, site selection for landfill, solid or liquid waste disposal, zoning, or energy development projects. Such a situation could be modeled as a game and studied by game theory. In a game model, each decision maker (DM), also commonly referred to as a player, participant, stakeholder or actor, must decide upon the strategy to follow from a set of possible strategies formed from choosing various courses of actions called options. When each DM involved in a conflict chooses a strategy, a state or possible scenario in a game is created. A DM's strategy choice is affected by the DM's preferences assigned over the states, and directed by certain decision rules referred to as solution concepts. Based on the results of a conflict analysis, each DM can select the strategy that would be most effective in reaching her goals.

Although the game-theoretic model is one of the most powerful analytical tools in environmental management, the fundamental assumption underlying the analysis is that a solution (called "equilibrium") is obtained from mutual knowledge of rationality and common knowledge of belief. That is, all of the DMs are assumed to understand the conflict fully, and thereby have common knowledge about all the components of the game—all of the DMs' strategies, preferences, and decision rules used, and so on—that are usually not completely known in reality. In many real-world conflicts, DMs do not always have all the information to access the parties' true intentions, secret options, strategies, or preferences. Consequently, they have to perceive the conflict from their own points of view, and may err in their perceptions. Sometimes misperceptions may be deliberately introduced because it is advantageous for DMs to deceive their opponents. In more complicated situations, sophisticated participants make their decisions based on consideration of what others think about them. Therefore, multilevel perceptions may be involved when modeling a conflict as a game.

Since the early development of game theory, many researchers have pointed out the lack of perceptual considerations in game theory. In 1957, Luce and Raiffa discussed the possibility of extending the game theory framework such that the assumption of complete information was relaxed. This generalization was referred to as "a game with misperceptions." In order to deal with perceptual issues systematically, there has been a great deal of research regarding misperception. For example, in 1967, Harsanyi analyzed in detail the properties of games with incomplete information played by “Bayesian” players. Some authors initiated the study of sequential games with incomplete information, while in 1977, Brams and Zagare described deception in strictly ordinal 2 x 2 games.

One area of conflict research that has seen significant advances is hypergame analysis. A hypergame refers to a mathematical structure that models the ways participants perceive a conflict, whereas the hypergame analysis is the application of a stability analysis algorithm or a solution concept to a hypergame model in order to predict possible resolutions of the hypergame. The idea of the hypergame was first proposed in 1977 by Bennett, who also furnished definitions for various kinds of hypergames. Utilizing Bennett’s definitions, hypergames have been shown to be useful in modeling sports, military, and business disputes. In 1984, Takahashi, Hipel, and Fraser developed an operational procedure for conveniently analyzing a hypergame. In particular, they explained how sequential stability, a solution concept designed by Fraser and Hipel in 1984, can be employed in finding equilibria envisioned by each DM as well as the overall equilibria to a hypergame. Using this approach, hypergame analysis has been applied to business, water resources, and military conflicts.

Wang, Hipel, and Fraser presented a comprehensive approach to hypergame modeling and analysis in 1988 and 1989. Based on rigorous definitions of perception, misperception, and order of expectation, hypergames may be constructed at different levels and analyzed by a general procedure of stability analysis independent of the particular solution concept used. They further introduced various solution concepts that were originally designed for simple games (games without misperceptions), and discussed solution properties of various solution concepts. Hypergame analysis has thus become not only mathematically rigorous, but also flexible and practical. The authors also discussed in detail hypergame modeling and analysis for conflicts involving only two DMs. The approach has been applied to international conflicts, environmental disputes, and bargaining and negotiation in international economic cooperation.

Efforts have been made to apply hypergame analysis in bargaining and negotiation, where various types of emotions may play a role in decision making. In 2007, Inohara, Hipel and Walker showed how attitudes can be combined with misperceptions (hypergames) when carrying out a strategic investigation of a given conflict. The strategies of adaptive learning based on genetic algorithms may also be employed among a group of DMs to reach consensus or an agreement.

In order to manage environmental disputes more effectively, it is important to provide DMs such as developers, planners, environmental managers, lawyers, consultants, and government officials with more comprehensive decision analysis tools. With the help of the procedures presented in this article, DMs can model their strategic problems in a systematic way, better understand the conflict situations, and find the most reliable compromise resolution. The purpose of this article, therefore, is to present the methodology of hypergame analysis for resolving environmental conflicts with misperceptions. When an environmental conflict involves misperceptions, one or more of the DMs will see other participants’ intentions, strategies, or preferences in a different manner from how DMs do in reality. In this article, the water aquifer dispute in San Antonio, Texas, USA, between the city council and the developers is modeled and analyzed by hypergame analysis. In particular, the hypergame analysis is extended to incorporate the idea of cooperative games involving bargaining and negotiation. The effect of misperception is systematically incorporated into the analysis so that the behavior of the DMs in a complicated situation can be fully explored and understood. In the next section, a brief discussion of hypergame models and stability analysis is presented, while background information about the water aquifer dispute is given in Section 3. Sections 4 and 5 are devoted to hypergame modeling and analysis of the case, respectively, and conclusions are presented in the last section.

2. Hypergame Models and Stability Analysis

2.1. Basic Structure of Game Models and Stability Analysis

As defined in the game theory and conflict analysis literature, a conflict can be conveniently modeled as a game in various formats, such as the extensive (see *Compliance Models for Enforcement of Environmental Laws and Regulations*), normal, characteristic function (see *Cost Allocation*), option (see Tables 3, 4, and 6; also see *The Graph Model for Conflict Resolution*), and graphic (see *The Graph Model for Conflict Resolution*) forms (see *Formal Models for Conflict Resolution and Case Studies*). No matter what form is used to describe a conflict, a game model is usually constructed in terms of DMs, options, strategies, states, and payoff functions:

- DMs: the participants in a dispute are called the DMs. The set of n DMs is denoted by $N = \{1, 2, \ldots, i, \ldots, n\}$.
- Options: the possible courses of actions available to each of the DMs are referred to as options, and defined as $o_i \in O_i, \forall i \in N$, where o_i is an option of i, while O_i represents all the options of i.
- Strategies: any set of options that could be taken by DM i is called a strategy, and denoted by s_i, where $s_i \in S_i, \forall i \in N$.
- States: a state is formed when each of the DMs chooses a strategy that represents a possible scenario in a conflict. A state is an n-tuple vector of strategies, denoted by $u = [s_1, s_2, ..., s_i, ... s_n]$. The set of states is $U = S_1 \times S_2 \times ... \times S_i \times ... \times S_n$, $u \in U$, where $\times$ stands for the Cartesian product.
- Payoff functions: a payoff function $P_i, \forall i \in N$ reflects DM i's preference over the state space $U = S_1 \times S_2 \times ... \times S_i \times ... \times S_n$.

Hence, a game can be defined as $G = \{S_1, S_2, ..., S_n, P_1, P_2, ..., P_n\}$, which incorporates all the elements mentioned above. When the states are ranked from the most to the least preferred according to a DM i's payoff function P_i, where ties are allowed, a preference vector (PV) is formed for the DM and denoted by V_i. Hence, PV_i contains DM i's ordinal preferences over the states. Considerable information about the game is embodied in the set of PVs; therefore, a game model can also be represented as $G = \{V_1, V_2, ..., V_i, ..., V_n\}$.

In a simple game (or a game without misperception), the game structure is common knowledge to all the DMs, and each DM is represented by one *PV* only. The stability analysis is then performed using the same set of *PV*s according to a certain solution concept to determine the stability of each state for each DM. The states that possess group stability constitute the possible resolutions or equilibria to a conflict. Therefore, the analysis is based on the concept of an equilibrium, which is calculated assuming the DMs are optimizing their payoffs against one another. Moreover, everyone is aware of this, and the DMs' belief is common knowledge.

2.2. Hypergame Models

In a hypergame, the assumption of common knowledge is relaxed, and the DMs may look at the situation in different ways. Specifically, they may have incorrect interpretations about the conflict. Six distinct kinds of misperceptions could be modeled and analyzed by hypergame analysis. These are misperceptions about:

- DMs,
- options,
- strategies,
- preferences,
- decision rules used, and
- higher order expectations,

as well as any combination of the above misperceptions by one or more DMs.

Nevertheless, each DM constructs her perceptual game according to what she imagines (believes). This imagination could be defined as a DM's expectation. In an *n*-person game, for example, DM *i*'s decision may depend on what she thinks of *j*'s viewpoint about the game. The expectation about *j*'s viewpoint is called DM *i*'s first-order expectation. If at least one of the DMs knows that they are playing different games, she will also consider what others' games are. Thus, the DM's decision is based on second-order expectation. When DM *i* perceives DM *j*'s perception about DM *k*'s idea of how DM *q* views the game, then it is a third-order expectation. In general, the idea of expectation could be extended to any order, although the first, second, and third orders are usually sufficient for modeling most real-world conflicts.

When misperception occurs, an individual DM may be represented by more than one *PV* in a hypergame. Suppose V_i is the true *PV* for DM *i*, and V_{ij} is the expectation of *j* for *i*; this indicates what DM *j* thinks *i*'s *PV* is. Misperception occurs if $V_{ij} \neq V_i$, that is, DM *j* incorrectly interprets *i*'s *PV*. Expectations differ in orders. For example, V_{kji} is a second-order expectation, which describes what DM *i* believes with regard to how DM *j* interprets *k*'s intentions, where $i \neq j \neq k$, and $i, j, k \in N$. Such an expectation can go to any required order or level to depict a real-world situation.

Since perception and misperception may occur at any order of expectation, hypergames are modeled at different levels. The level of a hypergame depends on the highest order of expectation involved. If all the DMs are playing the same game (no misperceptions involved), then the hypergame is level zero (or a simple game), where each DM is described by only one *PV*, which is the true one.

$$H^0 = G = \{V_1, V_2, ..., V_i, ..., V_n\} \tag{1}$$

The game of even one DM who makes a mistake in interpreting others' *PV*s is different from the games of the others. The situation is modeled as a first-level hypergame, where the DMs are playing different games, no one realizes this, and at least one perceived *PV* is different from the true one.

$$\begin{aligned} H^1 &= \{H_1^0, H_2^0, ..., H_i^0, ..., H_n^0\} \\ &\forall i \in N, \quad \exists i, j \in N : H_i^0 \neq H_j^0; \\ &= \{G_1, G_2, ..., G_i, ..., G_n\} \\ &= \left\{ \begin{bmatrix} V_1 \\ V_{21} \\ ... \\ V_{n1} \end{bmatrix} \begin{bmatrix} V_{12} \\ V_2 \\ ... \\ V_{n2} \end{bmatrix} \begin{matrix} ... \\ ... \\ ... \\ ... \end{matrix} \begin{bmatrix} V_{1n} \\ V_{2n} \\ ... \\ V_n \end{bmatrix} \right\} \\ &\qquad \exists i, j \in N : V_{ij} \neq V_i \end{aligned} \tag{2}$$

In a second-level hypergame, at least one of the DMs is aware that they are playing different games and would therefore perceive what the other DMs' games are. This can be interpreted as the DMs playing different first-level hypergames, resulting in a second-level hypergame.

$$\begin{aligned} H^2 &= \{H_1^1, H_2^1, ..., H_i^1, ..., H_n^1\} \\ &\forall i \in N, \quad \exists i, j \in N : H_i^1 \neq H_j^1; \\ &= \left\{ \begin{bmatrix} H_1^0 \\ H_{21}^0 \\ ... \\ H_{n1}^0 \end{bmatrix} \begin{bmatrix} H_{12}^0 \\ H_2^0 \\ ... \\ H_{n2}^0 \end{bmatrix} \begin{matrix} ... \\ ... \\ ... \\ ... \end{matrix} \begin{bmatrix} H_{1n}^0 \\ H_{2n}^0 \\ ... \\ H_n^0 \end{bmatrix} \right\}, \end{aligned} \tag{3}$$

where

$$H_{ji}^0 = [V_{1ji}, V_{2ji}, ..., V_{kji}, ..., V_{nji}], \qquad \forall k \in N_{ji}, \forall j \in N_i, \forall i \in N$$

A third-level hypergame has to be employed if at least one of the DMs makes his decision based on how others are thinking about him. Thus, he tries to perceive the other DMs' first-level hypergames, which form his second-level perceptual game, thereby forming a third-level hypergame. The more sophisticated the DMs are, the higher is the level of a perceptual game. The level of the overall hypergame is determined by the most sophisticated DM.

$$H^3 = \{H_1^2, H_2^2, .., H_i^2, ..., H_n^2\}$$
$$\forall i \in N, \quad \exists i, j \in N : H_i^2 \neq H_j^2; \tag{4}$$
$$= \left\{ \begin{bmatrix} H_1^1 \\ H_{21}^1 \\ ... \\ H_{n1}^1 \end{bmatrix} \begin{bmatrix} H_{12}^1 \\ H_2^1 \\ ... \\ H_{n2}^1 \end{bmatrix} \begin{matrix} ... \\ ... \\ ... \\ ... \end{matrix} \begin{bmatrix} H_{1n}^1 \\ H_{2n}^1 \\ ... \\ H_n^1 \end{bmatrix} \right\},$$

where

$$H_{ji}^1 = \{H_{1ji}^0, H_{2ji}^0, ..., H_{kji}^0, ..., H_{nji}^0]$$
$$= \left\{ \begin{bmatrix} V_{1ji} \\ V_{21ji} \\ ... \\ V_{n1ji} \end{bmatrix} \begin{bmatrix} V_{12ji} \\ V_{2ji} \\ ... \\ V_{n2ji} \end{bmatrix} \begin{matrix} ... \\ ... \\ ... \\ ... \end{matrix} \begin{bmatrix} V_{1nji} \\ V_{2nji} \\ ... \\ V_{nji} \end{bmatrix} \right\} \qquad \forall k \in N_{ji}, \forall j \in N_i, \forall i \in N$$

An Lth-level hypergame consists of n individual games, where at least one of the individual games is different from the others, and the highest order of expectation involved in the individual games is L. A formal definition of Lth-level hypergame is given below.

$$H^L = \{H_1^{L-1}, H_2^{L-1}, ..., H_i^{L-1}, ..., H_n^{L-1}\}$$
$$= \{H_i^{L-1}\}$$
$$\forall i \in N, L = 1, 2, 3, ... \tag{5}$$

$$\exists i, j \in N : H_i^{L-1} \neq H_j^{L-1}$$

2.3. Hypergame Stability Analysis and Solution Concepts

Hypergame stability analysis has some unique characteristics due to misperceptions. Each DM is pursuing the best state in her perceptual game. Instead of choosing the state, however, a DM can only select the strategy related to that state. The realization of the state depends on what the other DMs do. This is not a problem in a simple game, where DMs envision the same game and the identical set of equilibria. In a hypergame, DMs are faced with different perceptual games and perceive diverse solutions. Nevertheless, DMs make decisions, or take strategies, according to their perceived equilibria, even if some DMs possess misperceptions. An overall equilibrium is formed when each DM's stable strategy selection is invoked. In a higher-level hypergame ($L > 1$), a DM's decision, or strategy selection, is influenced by her perception of others' perceptual equilibria.

Let E be the set of overall equilibria in a hypergame and E_i be the set of perceptual equilibria of DM i. The strategy $s_i^* \in S_i^*$ is a stable strategy selection for DM i, which is related to a perceived equilibrium of i. Hypergame stability analysis for an Lth-level hypergame is carried out in two stages.

- The individual stability analysis is performed for each of the DMs in H_i^{L-1} by analyzing the individual games separately to obtain the sets of perceptual equilibria E_i and stable strategies S_i^*.
- The overall stability analysis is performed for the whole hypergame to produce the overall solution set E according to all perceptual solutions E_i, $\forall i \in N$.

Obviously, what constitutes perceptual equilibria is strongly affected by what the DMs believe. However, the overall equilibria may differ from what is expected, and a DM may be given new information about his misperception. Therefore, a hypergame may collapse when some of the DMs obtain more information through the process of interaction. Moreover,

- If $\bigcap_{i=1}^{n} E_i \neq \emptyset$, then the overall equilibrium $e \in \bigcap_{i=1}^{n} E_i$ is a *hypergame-preserving* equilibrium.
- If $\bigcap_{i=1}^{n} E_i = \emptyset$, then the hypergame has a *hypergame-destroying* equilibrium formed by the DMs' stable strategy selections s_i^* related to $e_i \in E_i$ in H_i^{L-1}.

Further,

- e is a *persistent* equilibrium if it is stable for all the DMs in the new game after the original hypergame collapses.
- e is a *snapshot* equilibrium if the conflict is a snapshot decision-making problem.
- Otherwise, e is a *transitory* equilibrium and the conflict will either involve a dynamic decision process or transfer to another phase of analysis.

A hypergame will always have at least one overall solution if each individual game has one. Whether or not each individual game has a solution depends on the problem studied and the solution concept used in the analysis. Various solution concepts, such as Nash stability, general metarationality, symmetric metarationality, sequential stability and limited-move stability have been introduced into hypergame analysis.

These solution concepts are defined by the consideration of whether or not a DM, at a particular state, would like to move away from it unilaterally, given all the other DMs' fixed strategy choices. The choice of such a unilateral movement is also affected by contemplating possible consequences of other DMs' sequential counter-moves. The water aquifer conflict reported in this article uses the sequential stability defined by Fraser and Hipel, although other solution concepts are also employed in the analysis of the conflict.

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Biographical Sketches

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