# INPUT-OUTPUT MODELS

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## Contents



## Summary

Description and analysis of input-output model, static input-output model, and dynamic input-output models in form of equalities and optimization form is given. The development of input-output method [the concept of dynamic equilibrium, environmental (or ecological) factor reckoning] is examined.

## 1. History

The complex analysis of production and distribution of goods and services at input-output level of national economy was first implemented in the Soviet Union in the national economy balance of the USSR in 1923—1924 fiscal year. Its scheme represented the prototype of current input-output model. The article of W.W. Leontief ("The National Economy Balance of the USSR"), where he analysed the results of the balance research, was published in the journal "Planowoye hoziaystvo" (or "The Planned Economy", 12/1925). W.W. Leontief propounded the scheme of an input-output model that was close to the scheme of the national economy balance of the USSR (the first publication was in 1936). The suggested and implemented by W.W. Leontief idea of input-output coefficients (direct yield costs) of a single industry production for other industries production is the basement of contemporary input-output model. Analogous coefficients were used earlier by L. Walras and V.K. Dmitriev. The results of W.W.Leontief's and the scientists' researches

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were published in a series of works. The most complete representation of the results was given in a famous monograph "Studies in the Structure of the American Economy by Leontief et al" (Oxford University Press, 1952). W.W. Leontief was awarded in 1972 the Nobel Prize of The Bank of Sweden in economic sciences "for the development of the input-output method and for its application to important economic problems." With the appearance of first computers in the late 1940s the input-output method started to be widely practised in many countries as the method of analysis, forecasting and planning national and regional economic structures. Input-output models developed and improved theoretically and practically, i.e. the transition from static to dynamic version with the capital investments lag of one or more years, reckoning environmental factor, the usage in analysis and estimation of the world economy further development, the use as a tool of forecasting and forming long-term and medium-term indicative planning of national economy (for example, in Japan and France).

The integration of the input-output model in the system of national accounts after World War II was based in 1968 ("System of National Accounts," UN, 1968). Russian input-output lists of the year 1995 ("Input-Output Lists of the Year 1995," "Moskva" 2000) were elaborated according to the methodological principles and standards of the latest version of the system of national accounts ("System of National Accounts," UN, 1993).

## 2. Input-Output model

| | Pure industries intermediate demand | | | Final demand | | | | |
|---|---|---|---|---|---|---|---|---|
| | Yield | Manufact. | Services | Total | Final consumption | Gross accumulation | Net exports | Output |
| Yield | $x_{11}$ | $x_{12}$ | $x_{13}$ | $y_1$ | $y_{11}$ | $y_{12}$ | $y_{13}$ | $x_1$ |
| Manufact. | $x_{21}$ | $x_{22}$ | $x_{23}$ | $y_2$ | $y_{21}$ | $y_{22}$ | $y_{23}$ | $x_2$ |
| Services | $x_{31}$ | $x_{32}$ | $x_{33}$ | $y_3$ | $y_{31}$ | $y_{32}$ | $y_{33}$ | $x_3$ |
| Gross value added | $z_1$ | $z_2$ | $z_3$ | | | | | |
| Output | $x_1$ | $x_2$ | $x_3$ | | | | | |

Table 1. Input-output model of production and distribution of goods and services of a country (home currency).

Input-output model of production and distribution of goods and services is a table based on methodological principles and standards of the latest version of the system of national accounts (1993). For simplification, Table 1 consists only of three input-output matrix quadrants (sections). It contains data on three pure industries: yield (for example, mining industry), manufacturing (the production of ready-made goods, for example, mechanical engineering) and services (for example, housing facilities and public utilities, nonproductive consumer service establishment). Each of the pure industries produces only one good in contrast to administrative industry that is able

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to produce more than one.

The first quadrant in Table 1 is a square matrix with elements $x_{11},\ldots,x_{33}$. It describes *input-output ties* within the context of singled out production and services. *Production* costs are placed in the columns of the first quadrant, the *use* of production and services for further production from the point of view of the *pure industries* (here, three industries) in a period of time is placed in the rows. It is accepted that time in the model is discrete; the unit of time is a period normally equal to a year. For example, figure $x_{12}$ shows how many units (in money form of values) of the first product are needed for producing the second product in the quantity of $x_2$ units in money form of values. The row of elements $x_{11}$, $x_{12}$, $x_{13}$ shows the amount of production units (in money form of values) of the first industry that it *supplies* to itself ($x_{11}$), to the second industry ($x_{12}$) and to the third industry ($x_{13}$). The column of elements $x_{12}$, $x_{22}$, $x_{32}$ shows how many units (in money form of values) of the first, second and third products (in compliance with $x_{12}$, $x_{22}$, $x_{32}$) are needed for the gross production of the second product in the quantity of $x_2$ in money form of values. Double counting is reckoned in the first quadrant.

The second quadrant in Table 1 is vector $y$ with coordinates $y_1$, $y_2$, $y_3$. It describes the *final demand* in money form of value within the context of singled out production and services. The second quadrant in the Table 1 is also a square matrix with the elements $y_{11},\ldots,y_{33}$, which show the final demand for its functional elements (the costs of final consumption, gross accumulation, net exports) within the context of singled out production and services. Generally speaking the matrix of the second quadrant is not square. Vector $y$ of the final demand shows the annual result of the economic system functioning. The first quadrant elements describe the intermediate demand in the industries that ensures their development. Double counting is not reckoned in the second quadrant.

| | Yield | Manufacturing | Services |
|---|---|---|---|
| Salaried employees payments | $z_{11}$ | $z_{12}$ | $z_{13}$ |
| Gross profit | $z_{21}$ | $z_{22}$ | $z_{23}$ |
| Gross mixed income | $z_{31}$ | $z_{32}$ | $z_{33}$ |
| Social insurance assignments | $z_{41}$ | $z_{42}$ | $z_{43}$ |
| Production taxes and grants | $z_{51}$ | $z_{52}$ | $z_{53}$ |
| Production taxes and grants | $z_{61}$ | $z_{62}$ | $z_{63}$ |

Table 2. The elements of the gross added value within the context of the pure industries

The third quadrant in Table 1 is vector $z$ with coordinates $z_1$, $z_2$, $z_3$. It describes the structure of *gross value added* (of *gross domestic product*) within the context

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of the pure industries (here, three industries). The third quadrant can be represented as matrix with values $z_{11},\ldots,z_{63}$ (Table 2) that show the elements of the gross value added within the context of the pure industries.

Gross profit includes the consumption of fixed capital, net profits, and net composite income. In Table 2 element $z_{13}$ equals to the labor payments of salaried employees in the third industry (services). Element $z_{21}$ is the first industry gross profit. Element $z_{42}$ shows the amount of social insurance assignments in the second industry, etc.

The fourth quadrant is not elaborated in the national accounting concept.

Attention should be paid to the fact that the right-most column "Output" and the bottom row "Output" (Table 1) do not refer respectively to the second and third quadrants, i.e. they are not part of the input-output model.

All values in input-output model ( $x_{11}\ldots x_{33}$ , $y_1$ , $y_2$ , $y_3$ , $z_1$ , $z_2$ , $z_3$ ) are given in money form, therefore the model has the monetary form. The physical input-output model contains some data in physical form. It is correct to sum by rows as well as by columns in the first version, whereas summing by columns is impossible in the second version. Today monetary input-output models are more widespread. It is possible to put in table the elements of the monetary input-output model in *demand prices* and *basis prices*. Demand prices of production and resources purchased by a industry include transportation and marketing margins and taxes less grants. The count of basis prices of production and resources requires detraction of transportation and marketing margins and net taxes.

The input-output model for n pure industries is constructed the same way. The first quadrant is an *n*-dimensional square matrix, the second squadrant is an *n*-dimensional column vector or $m \times n$ matrix, where *m* is the number of additive components of final demand vector, i.e. the number of its functional elements. The third quadrant is *n*-dimensional row vector of gross value added or $k \times n$ matrix, where *k* is the number of additive elements of gross value added vector.

The figures in Table 1 are linked in the following way:

$$\begin{aligned} y_1 &= y_{11} + y_{12} + y_{13}, \\ y_2 &= y_{21} + y_{22} + y_{23}, \\ y_3 &= y_{31} + y_{32} + y_{33}. \end{aligned} \qquad (1)$$

The figures in Tables 1 and 2 are linked in the following way:

$$\begin{aligned} z_1 &= z_{11} + z_{21} + z_{31} + z_{41} + z_{51} + z_{61}, \\ z_2 &= z_{12} + z_{22} + z_{32} + z_{42} + z_{52} + z_{62}, \\ z_3 &= z_{13} + z_{23} + z_{33} + z_{43} + z_{53} + z_{63}. \end{aligned} \qquad (2)$$

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First three rows in Table 1 are represented in the following way:

$$x_1 = x_{11} + x_{12} + x_{13} + y_1,$$
$$x_2 = x_{21} + x_{22} + x_{23} + y_2, \quad (3)$$
$$x_3 = x_{31} + x_{32} + x_{33} + y_3.$$

First three columns in Table 1 are represented in the following way:

$$x_1 = x_{11} + x_{21} + x_{31} + z_1,$$
$$x_2 = x_{12} + x_{22} + x_{32} + z_2, \quad (4)$$
$$x_3 = x_{13} + x_{23} + x_{33} + z_3.$$

In formulae (1) the final demand for the production of every industry is represented as the sum of its functional elements. In formulae (2) gross added value of every industry is represented as the sum of its elements. Formulae (3) show output *distribution* of every industry (for example, $x_1$ for the fisrt one) for intermediate demand (for example, the sum of $x_{11}$ , $x_{12}$ , $x_{13}$ for the fisrt industry) and final demand (for example, $y_1$ for the first industry). Formulae (4) show every industry *production*. For instance, output $x_1$ of the first industry (in money form of value) equals to the sum of its intermediate consumption ( $x_{11} + x_{21} + x_{31}$ ) and gross added value.

Summing up equations (3):

$$x_1 + x_2 + x_3 = x_{11} + \ldots + x_{33} + y_1 + y_2 + y_3. \quad (5)$$

Summing up equations (4):

$$x_1 + x_2 + x_3 = x_{11} + \ldots + x_{33} + z_1 + z_2 + z_3. \quad (6)$$

Equations (5) and (6) lead to

$$y_1 + y_2 + y_3 = z_1 + z_2 + z_3, \quad (7)$$

i.e. total final demand being equal to the total gross value added, or to the GDP of a country. Equation (7) is called input-output model balance term. All expressions (1)—(7) mentioned above can be generalized in case of *n* pure industries.

Various conditions of pure industries aggregation are used in real input-output models. For example, Russian input-output lists of the year 1995 were created in brief scheme representing 22 production and service groups (i.e. 22 pure industries) and in detailed way representing 110 production and service groups (i.e. 110 pure industries). Today the studies extend to 600 pure industries.

Input-output model (table) is a unique reflection of economy life of a country or a single

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region in multiproduct and multisector view. Input-output model (table) is a computational model for the elements real (and partly, expert) data forming, which introduction requires large funds. The detailed input-output table (more than 100 pure industries) is formed more often than not one time per five years even in developed countries.

An input-output model (table) gives figures to direct analysis of national economy input-output flows, and can be regarded as a database for more complex and sophisticated national economy models used for solving analysis problems, forecasting and in some way indicative planning.

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**Biographical Sketch**

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