SERVO CONTROL DESIGN

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Keywords: servo control, servomechanism, feedback, feedforward control, regulation, disturbance rejection, compensation, input shaping, lag compensator, lead compensation, phase margin, steady-state error, integral control, industrial regulator, stabilizing compensator, PID control, state feedback, frequency response, parameter optimization.

Contents

1. Introduction
2. Classical Servo Control Design
2.1. Integrator Based Control
2.1.1. Design Example: Industrial Regulator
2.2. Phase Lag Control
2.2.1. Design Example: Phase Lag Compensation
2.3. Phase Lead Control
2.3.1. Design Example: Phase Lead Compensation
3. Modern Servo Control Design
3.1. Feedforward Control: Input Shaping
3.1.1. Mathematical Analysis of the Input Shaping Scheme
3.1.2. Design Example: Input Shaping for Unit Step Command
3.2. Feedback Control
3.2.1. Controller Parameterization
3.2.2. Time Domain Parameter Optimization
3.2.3. Frequency Domain Parameter Optimization
4. Conclusions
Acknowledgements
Glossary
Bibliography
Biographical Sketch

Summary

Elements and design of servo control systems are discussed in this paper. The primary purpose of a servo system is to regulate the output of a dynamical system by means of feedback control. For pedagogical and historical reasons, the discussions of servo control are divided into two parts: classical and modern servo systems aiming at audience with an undergraduate and graduate level of engineering background respectively. Classical servo control deals with single-input, single-output systems in either frequency or time domain. It remains popular with many industrial applications due to its simplicity in design and implementation. The methods presented here include: integrator based control, lag, and lead compensation. A running design example is provided to illustrate the key features and comparative properties. Modern servo control design deals with multiple-input, multiple-output systems. Input shaping (or command shaping) is first introduced as a feedforward

control strategy. It has found widespread use in robotics and various engineering systems when the plant exhibits oscillatory transient response. The input shaper convolves with the command signal in such a way that the command-induced oscillations are cancelled. Feedback control is then discussed from the perspective of internal model principle, i.e. the models of the exogenous signals (command as well as disturbance signals) are obtained and translated into a servocompensator. Stabilization of the closed loop system can then be carried out using various multivariable control synthesis techniques such as estimator-based (H_2, H_∞) and parameter optimization (time, frequency domains). The latter approach is discussed in this paper whereas details of estimator-based designs may be found in Sections *Control of Linear Multivariable Systems* and *Robust Control*. A number of design examples are also provided to illustrate the properties of the parameter optimization methods.

1. Introduction

The term servo control is used synonymously with servomechanism, which is concerned with using automatic control system to regulate the output(s) of a dynamic system. Originally focused on the improvement of WW2 firing mechanisms, servo control has evolved into a broad based scientific and technological subject with applications in aerospace, biomedical systems, chemical processes, manufacturing systems, mechatronics, power plants/networks, traffic/transportation systems, etc., to name a few. The essential elements of a servo system consist of the plant, the control, and signals. The plant represents the dynamical system to be regulated, is either expressed as transfer function(s) or state space equations. The control system is primarily of the feedback type but occasionally feedforward control is also applied to speed up system response. The following block diagram illustrates a typical servo control system:

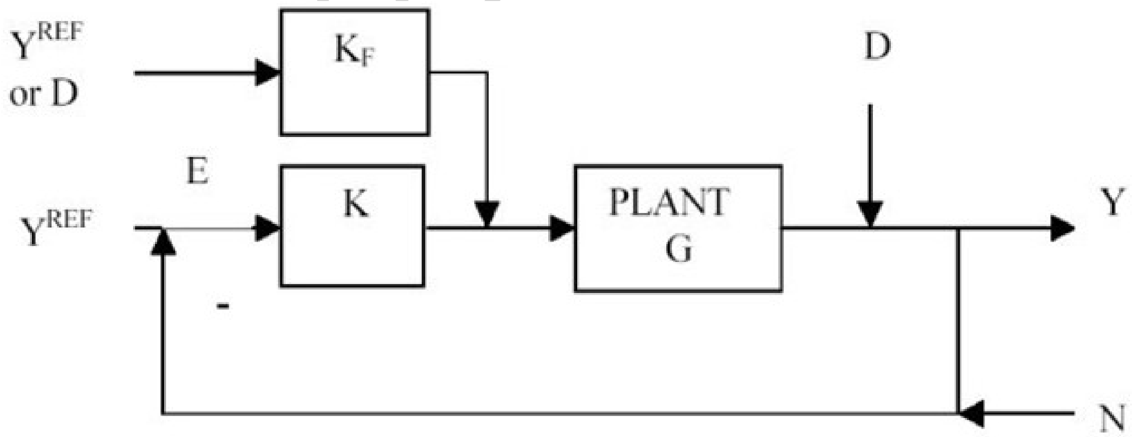

Figure 1: Typical Servo Control Configuration

The closed loop system is described by the configuration shown in Figure 1. The plant $G(s)$ is assumed to be linear, time-invariant. The servo controller $K(s)$ acts on the error signal $E = Y^{\mathrm{REF}} - Y - N$ to generate a control signal to the plant so that the output Y tracks the reference command signal Y^{REF}. Two exogenous variables D and N are usually included to account for the effects of disturbance and measurement (sensor) noise. The block K_{F} represents a feedforward control that serves to speed up the tracking response. It may include command Y^{REF} or disturbance D feedforward algorithms. For the latter, the disturbance must be known on an a priori basis. From the above block diagram, it is straightforward to show that:

$$\begin{aligned} \mathrm{Y} &= [(\mathrm{I}+\mathrm{GK})^{-1}]D + [(I+GK)^{-1}GK]N + [(I+GK)^{-1}GK]Y^{\mathrm{REF}} \\ &= SD + TN + TY^{\mathrm{REF}} \end{aligned} \quad (1)$$

To simplify notations, the independent variable s is omitted, i.e. Y is to be interpreted as $Y(s)$, etc. The quantities S and T are known as the sensitivity function and closed loop transfer function, respectively. Obviously, to suppress the effects of D while achieving tracking ($Y \to Y^{\mathrm{REF}}$), it is desirable to minimize $\|S\|$ while maintaining $\|T\|$ to be closed to unity where $\|\bullet\|$ is some suitable norm. However, brute force wideband high gain design on K almost always results in unacceptable performance such as:

- Instability or low stability margins.
- Measurement noise propagation into the loop.
- Robustness problems (especially with respect to unmodeled high frequency dynamics).
- Saturation and other nonlinear effects.

A closer examination of the spectral characteristics of Y^{REF}, D, N shown in Figure 2 reveals that the signals generally occupy different frequency ranges. The reference command $y^{\mathrm{ref}}(t)$ (or $Y^{\mathrm{REF}}(s)$) and the input disturbance $d(t)$ (or $D(s)$) are assumed to be lowpass with effective cutoff frequencies of Ω_{ref} and Ω_D respectively. The sensor noise $n(t)$ (or $N(s)$), on the other hand, is assumed to be highpass with an effective cut in frequency Ω_N. It is further assumed that $\Omega_N \gg \Omega_{\mathrm{REF}}$ and $\Omega_N \gg \Omega_D$. This spectral disjoint can be exploited in servo control designs.

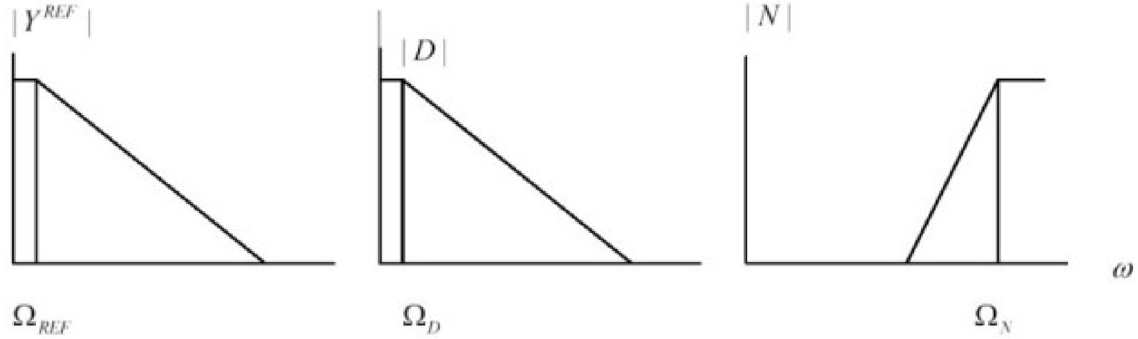

Figure 2: Typical Spectral Characteristics of Y^{REF}, *D*, and *N*.

Essentially, the sensitivity function S is to be attenuated at low frequencies while T is to be attenuated at high frequencies without violating the following condition:

$$S + T = I \tag{2}$$

This is possible if the design of K is frequency dependent. In particular, applying K to shape the frequency response T is the primary objective of servo control design. Over the decades, servo design can be roughly classified into (I) classical design and (II) modern design. In classical design, the input/output variables are scalar functions while the plant G is assumed linear, time-invariant. Such methodology remains popular in small industrial systems as well as multivariate systems admitting a decentralized control structure (i.e. the plant outputs can be regulated by a series of SISO loop controllers). For the more general case where the plant may be Multiple-Input, Multiple-Output (MIMO), nonlinear, etc., modern designs are generally more effective but also more complex. A discussion of the classical design methods is now given. This is followed by modern servo control designs.

2. Classical Servo Control Design

For SISO systems, the design criteria may include:

1. Closed loop stability.
2. Sensitivity reduction ($S_G^{\mathrm{T}} = S$ or other sensitivity functions).
3. Steady-state accuracy.
4. Disturbance rejection.
5. Transient response.

For closed loop stability, it is sufficient to examine if all of the closed loop poles of T are located in the left half s-plane (see *Stability Concepts* for a detailed discussion). Assuming that closed loop system is stable, analyzing the error signal can combine the stead-state accuracy and disturbance rejection criteria:

$$\frac{Y}{Y^{\mathrm{REF}}} = \frac{GK}{1+GK} \tag{3}$$

$$\frac{E}{Y^{\mathrm{REF}}} = \frac{1}{1+GK} \tag{4}$$

Or equivalently in time domain with $e(t) = L^{-1}E(s)$, the steady-state error e_{ss} is calculated as:

$$\begin{aligned} e_{\mathrm{ss}} &= \lim_{t \to \infty} e(t) \\ &= \lim_{s \to 0} sE \\ &= \lim_{s \to 0} \frac{sY^{\mathrm{REF}}}{1+GK} \end{aligned} \tag{5}$$

For classical servo control, the class of command and disturbance signals Y^{REF} and D are usually polynomial types, i.e. step, ramp, parabola (acceleration), etc. corresponding to $\frac{1}{s^L}, L = 1, 2, \ldots$ in the frequency domain. The steady-state errors for these signals are listed below:

- Under unit step command

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1+GK} \frac{1}{s} = \frac{1}{1+G(0)K(0))} \tag{6}$$

- Under ramp command:

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1+GK} \frac{1}{s^2} = \lim_{s \to 0} \frac{1}{sG(s)K(s)} \tag{7}$$

- Under parabola command:

$$e_{ss} = \lim_{s \to 0} s \frac{1}{1+GK} \frac{1}{s^3} = \lim_{s \to 0} \frac{1}{s^2 G(s)K(s)} \tag{8}$$

It is customary to define the following error constants:

position error constant	$K_p = \lim_{s \to 0} G(s)K(s)$
velocity error constant	$K_v = \lim_{s \to 0} sG(s)K(s)$
acceleration error constant	$K_a = \lim_{s \to 0} s^2 G(s)K(s)$

Table 1: Error Constants

In term of the error constants, the steady-state errors become:

- Under unit step command

$$e_{ss} = \frac{1}{1+K_p} \tag{9}$$

- Under ramp command:

$$e_{ss} = \frac{1}{K_v} (10) \tag{10}$$

- Under parabola command:

$$e_{ss} = \frac{1}{K_a} \tag{11}$$

Therefore, to produce zero steady-state error under a step command/disturbance, it is necessary that $K_p \to \infty$. Similarly, an infinite K_v and K_a will produce zero steady-state error under a ramp and a parabola command/disturbance, respectively. Essentially, reducing steady-state error depends on matching the number of poles at the origin against the number of exogenous poles (in Y^{REF} or D) at the origin. This consideration is summarized in the table below:

Number of GK poles at the origin	System Type	K_p	K_v	K_a
0	0	finite	0	0
1	1	∞	finite	0
2	2	∞	∞	finite
3	3	∞	∞	∞

Table 2: System Type Number and Error Constants

In other words, system type number equals to the number of integrators ($\frac{1}{s}$) in GK. Now since an integrator has the unique characteristics of infinite DC gain while rolling off rapidly as frequency increases, the use of integrators to shape low frequency characteristics of T remains the centerpiece of servo control, classical or modern. Furthermore, systems with type number 0 are known as regulators whereas systems with type number 1 or 2 are known as position or velocity servos respectively. It should be noted that systems with a high type number are generally difficult to stabilize using single parameter compensation methods.

As an example, consider a unity gain feedback system with

$$G = \frac{10}{(s+1)(s+3)},\ K = 1 \qquad (12)$$

as shown in Figure 3 below:

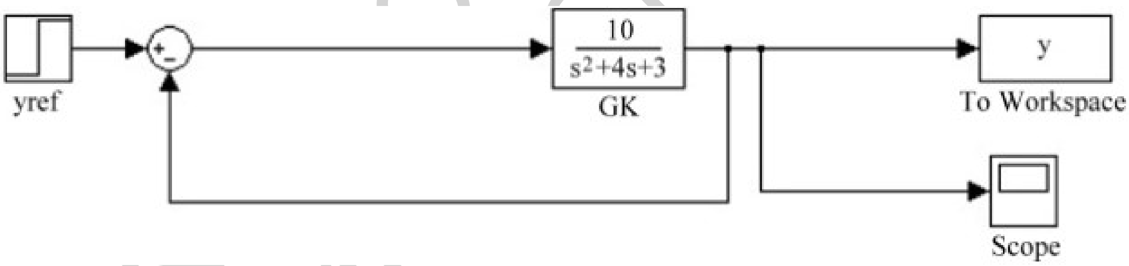

Figure 3: Simulation Block Diagram for (12)

The closed loop poles are determined from the denominator of $T = \frac{GK}{1+GK}$ as $-2 \pm j3$ and hence the closed loop system is stable. The steady-state error under a unit step command can be calculated in a number of ways:

1. From the output $y(t)$:
   $\lim_{t \to 0} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} s\frac{GK}{1+GK}\frac{1}{s} = \frac{10}{13}$. Thus $e_{ss} = \frac{3}{13}$.

2. From the error $e(t)$: $\lim_{t \to 0} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s\frac{1}{1+GK}\frac{1}{s} = \frac{3}{13}$.

3. From the error constants: $K_p = \lim_{s \to 0} G(s) = \frac{10}{3}$ and hence $e_{ss} = \frac{1}{1+K_p} = \frac{3}{13}$.

Unit step response of the closed loop system is shown in Figure 4. It is evident that the simulated steady-state error agrees with the calculated value of 3/13.

If zero steady-state error is desired for step command/disturbance, at least one integrator must be appended to K resulting in $GK = \frac{10}{s(s+1)(s+3)}$. The closed loop poles are calculated as: $-3.8897, -0.0552 \pm j1.6025$, stable. In this case $K_p = \infty$ and $e_{ss} = 0$. The corresponding step response is shown in Figure 5.

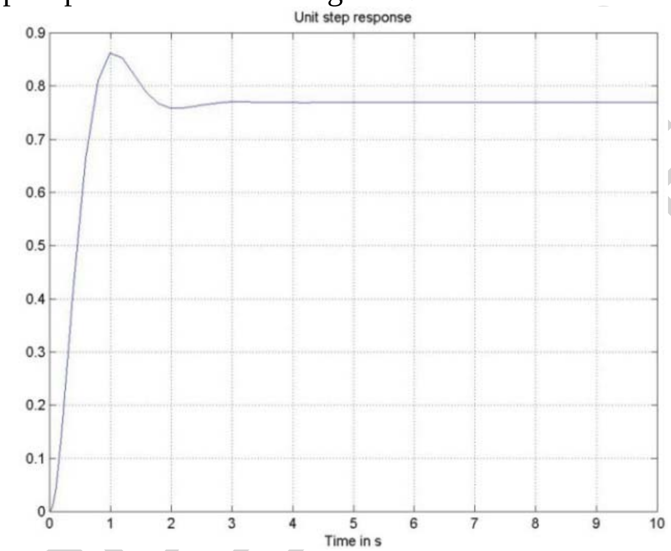

Figure 4: Unit Step response of (12)

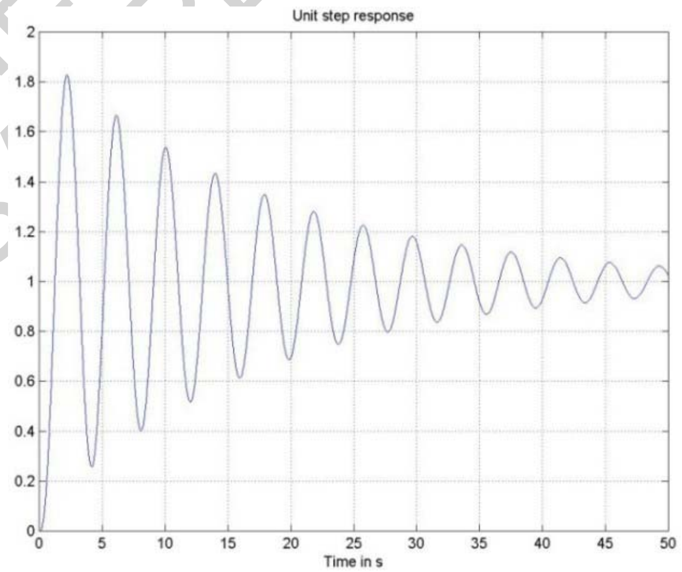

Figure 5: Unit Step response for (12) augmented with integrator

Comparison between Figures 4 and 5 indicates that adding an integrator improves steady-state error but also worsens transient response. Indeed, introduction of high magnitude, low frequency gain (such as an integrator) must be accompanied by stabilizing dynamics, directly or indirectly. In classical design there are primarily two ways to add in high gain at low frequency to attenuate steady-state error and minimizing sensitivity (note if, $|S|\to 0$ then $|T|\to 1$so the design criteria are fairly congruent). These two approaches will be discussed in Sections 2.1-2.3.

In terms of robustness and transient response, gain and phase margins are most commonly used. Refer to *Elements of Control Systems* for definitions of these quantities. Generally speaking, it is desirable to have 6dB gain margin and about 30 degrees phase margin for a servo loop. For a transfer function dominated by second order dynamics, the phase margin is approximately related to the damping factor (ζ) as follows:

$$\zeta \approx \frac{PM}{100\text{ degrees}} \text{ for } PM < 65 \text{ degrees} \tag{13}$$

where PM is phase margin in degrees. Now since ζ is tied to a number of time/frequency domain characteristics such as overshoot and peak frequency response, designing with phase margin as a target also serves to shape the transient response of the closed loop system. For example, it can be readily derived that, for system dominated by second order dynamics:

$$M_{\mathrm{p}}(\%) + PM \approx 75 \tag{14}$$

where M_{p} is the percentage overshoot in time domain. Among the existing classical servo controllers, the integrator based control and the lag/lead families have been most popular. A discussion of these controllers is now given:

2.1. Integrator Based Control

Integral-based control, also known as industrial regulator, is a time domain technique employing state space formulation in which the plant is described by:

$$\begin{aligned} \dot{x} &= Ax + Bu + Ew \\ y &= Cx + Fw \\ e &= y^{\mathrm{ref}} - y \end{aligned} \tag{15}$$

where x represents the plant states, y^{ref} and w are constant reference and constant, unknown disturbance. $A \in \Re^{n\times n}, B \in \Re^{n\times 1}, C \in \Re^{1\times n}$ is the state space equivalent of $G(s)$ (most commonly the minimal realization but uncontrollable/unobservable modes may also be present). The goal is to synthesize an integrator based control so that $\lim_{t\to\infty} e(t) = 0$ for polynomial type of reference signal y^{ref} and/or disturbances w. The controller structure, expressed in companion form, is essentially a chain of L integrators where L is required system type:

$$\dot{\eta} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \eta + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} e \tag{16}$$

For this section, the most common case $L = 1$ is considered. Higher type augmentation strategy will be discussed in the Modern Servo Control Design Section.
With $L = 1$, the controller simplifies to:

$$\dot{\eta} = e \tag{17}$$

Combining the plant and controller state variables and set $\overline{x} = \begin{bmatrix} x \\ \eta \end{bmatrix}$, the overall system becomes:

$$\dot{\overline{x}} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \overline{x} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ y^{\mathrm{ref}} \end{bmatrix} + \begin{bmatrix} E \\ -F \end{bmatrix} w \tag{18}$$

with

$$\begin{aligned} u &= -K_{\mathrm{I}} \overline{x} \\ &= -K^2 \overline{x} - K^1 \eta \end{aligned} \tag{19}$$

as the feedback law. This is essentially a state feedback problem. Explicit pole placement or LQR techniques can be readily applied to determine the feedback gains K^2 and K^1. It should be noted that for a second order system, the industrial regulator above is equivalent to a PID control (with plant state feedback accounting for the "PD" aspect).

This may explain the popularity of PID controllers and their proven success for general applications where the plant is dominated by second order dynamics at baseband. For in-depth discussions of PID controller, the reader is referred to section *Classic Design Methods for Continuous LTI- Systems*.

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Biographical Sketch

Timothy Chang is an Associate Professor at the Department of Electrical & Computer Engineering and Coordinator of the Intelligent Systems Area, NJIT. He received his B.Eng (honours) degree from McGill University, M.A.Sc. and Ph.D. degrees from University of Toronto. Prior to joining NJIT in 1991, he was a Senior Research Specialist and Program Manager at Kearfott Guidance and Navigation Corp., NJ, in charge of the Doppler mirror ring laser gyroscope program. Dr. Chang is the Chairman of the North Jersey IEEE Control Systems Chapter and Advisor to the IEEE student Branch at NJIT. He holds 4 US patents with 3 patents pending. His areas of interest include: ultra-high precision systems, robotics, embedded real time systems, decentralized control systems, and DNA microarray systems. His research has been supported the National Science Foundation, National Institute of Justice, National Institute of Standards and Technology, Department of Transportation, US Army ARDEC, New Jersey Commission on Science and Technology, and New Jersey Commission on Higher Education. He received the title of Master Teacher, NJIT in 2003.