UNCERTAINTY MODELS FOR ROBUSTNESS ANALYSIS

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Summary

In any engineering context, it is common practice to represent physical systems by mathematical models. Clearly, this representation is not exact. The discrepancy between the physical system and the mathematical model is due to two main sources: i) the lack of information on the structure or the behavior of the physical system; ii) the need for a simple model in order to apply available analysis and design techniques.

Such a discrepancy, although unknown, must be modeled in a more or less accurate way in order to account for it in the design of the control law. In this chapter an overview of the most used approaches to uncertainty modeling is provided. Several unstructured and structured uncertainty models for both input-output and state-space settings are presented. The properties of these models in control design problems involving robustness issues are also discussed.

1. Introduction

When modeling physical systems for control purposes, it is necessary to provide model descriptions that capture the main features of the system behavior and are mathematically tractable at the same time. An extremely accurate model of a physical process may turn out to be unsuitable for application of the available analysis and design techniques. By contrast, an over-simplified model, which misses significant information on the system structure, may lead to unacceptable design performance.

A careful balance between capturing the true behavior of the physical system and generating mathematically tractable models requires a great effort from the control designer. A standard way is to assume a simplified model as the *nominal* system. The discrepancy between the system and the adopted nominal model is usually represented as a perturbation on the nominal model. The resulting model, which is therefore composed of the nominal one and the perturbation, is usually referred to as the *uncertain model* or *model set*. For example, an infinite-dimensional system is usually represented by a finite-dimensional approximation as the nominal model, and a perturbation accounting for the neglected dynamics.

In order to obtain a satisfactory control design, it is mandatory that the control system performs well, not only on the adopted nominal model, but also on the actual physical process. This leads directly to the requirement of control design *robustness*, which demands that satisfactory performance is achieved for the uncertain model, i.e., the nominal model and the class of possible perturbations.

Two approaches are commonly used to describe the uncertainty involved in the physical system description: i) unstructured; ii) structured. Roughly speaking, the *unstructured uncertainty* representation is used to describe unmodeled or difficult to model dynamics and it is usually given as a bound on some measure of the error signal between system and nominal model outputs for a chosen class of input signals. The *structured uncertainty* is represented by an element (e.g. a finite dimensional vector or an operator) in some pre-specified uncertainty set of a suitable space. For example, in *highly structured (or parametric) uncertainty* description, the uncertain elements may be the coefficients of a transfer function, or the components of system matrices in a state-space realization.

After introducing the employed definitions and notation in Section 2, the main features of the two approaches to describe uncertainty are outlined in Section 3, by focusing on an input-output setting. Section 4 summarizes the most popular unstructured uncertainty models employed for modeling feedback control systems, also illustrating the related sources of system uncertainty. More structured uncertainty models are introduced in Section 5, where a standard model for uncertain control systems is presented. Section 6 is devoted to highly structured, parametric uncertainty models. Finally, state-space models are discussed in Section 7.

2. Notation and Definitions

In this section, the basic material required for the representation of uncertainty models is introduced.

Vector and matrix norms are defined in the usual way. In particular, the 2-norm of a vector $v \in R^n$ is equal to $\sqrt{\sum_{i=1}^{n} v_i^2}$, while the 2-norm of matrix A is given by the maximum singular value of A, denoted by $\bar{\sigma}[A]$.

Linear time-invariant dynamic systems are addressed via the associated real rational transfer function matrices. Figure 2 shows a generic multi-input multi-output (MIMO) system, with input signal $u(t) \in R^m$ and output signal $y(t) \in R^p$.

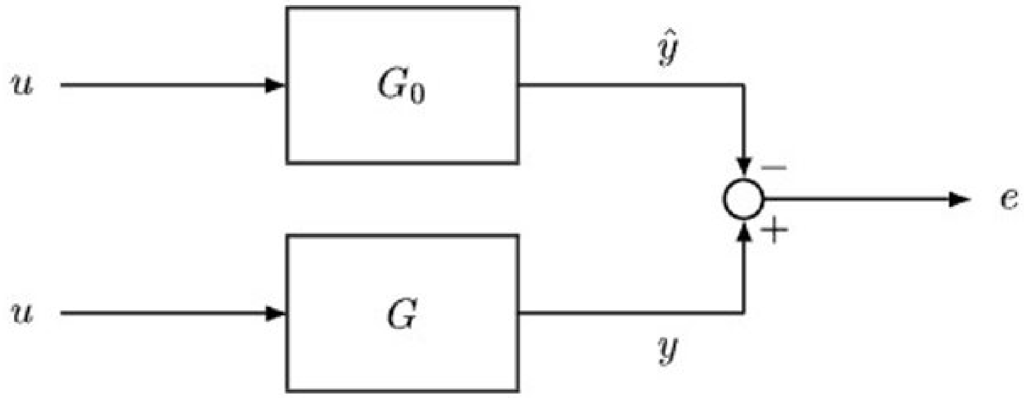

Figure 1: System G with input u and output y.

The corresponding $p \times m$ transfer function matrix is denoted by $G(s) = N(s)D^{-1}(s)$, where N and D are polynomial matrices in the complex variable s, and $G(s)$ is the Laplace transform of the system impulse response $g(t) \in R^{p \times m}$ satisfying $y(t) = \int_0^\infty g(t-\tau)u(\tau)d\tau$. When $m = p = 1$, the system is said single-input single-output (SISO) and its transfer function $G(s) = \dfrac{N(s)}{D(s)}$ is a rational function of s.

In order to evaluate the performance of a control system, it is customary to quantify the size of the involved signals. This is usually done by means of suitable signal norms. For a signal $u(t) \in R^m$, norms that frequently arise in control systems are:

- the L_2 norm (or *energy* norm)

$$\|u\|_2 = \sqrt{\int_0^\infty \left(\sum_{i=1}^{m} u_i^2(t) \right) dt};$$

- the L_∞ norm

$$\|u\|_\infty = \sup_t \max_{i=1,\dots,m} |u_i(t)|.$$

For a scalar signal, the L_2 norm represents the amount of energy associated with the signal, while the L_∞ norm is the maximum magnitude attained by the signal.

A standard way to assess the performance of a control system is to look at the size of the output signal, once the size of one or more input signals (commands and/or disturbances) is fixed. If the system G is considered as an operator from the input space to the output space, the achievable performance of the system can be measured according to the induced norm of G, defined as

$$\|G\| = \sup_{u\neq 0} \frac{\|Gu\|}{\|u\|}. \tag{1}$$

Depending on the norms used in (1) for signals u and $y = Gu$, different system induced norms are obtained.

Let $G(s)$ be a matrix transfer function with poles in the open left half plane, and $g_{ij}(t)$ be the entry (i, j) of the corresponding impulse response matrix $g(t)$. The most popular system norms when dealing with uncertainty models for robust control are:

- the H_∞ norm

$$\|G\|_\infty = \sup_{\omega\in R} \overline{\sigma}\left[G(j\omega)\right];$$

- the H_2 norm

$$\|G\|_2 = \sqrt{\frac{1}{2\pi}\int_0^\infty \text{trace}\left[G^*(j\omega)G(j\omega)\right]d\omega};$$

- the L_1 norm

$$\|G\|_1 = \max_{i=1,\dots,p} \int_0^\infty \sum_{j=1}^{m} |g_{ij}(t)|\,dt.$$

The interpretation of the above system norms in terms of input-output gain (1) are reported in Table 1 for SISO systems.

An uncertain polynomial family of order n is defined as

$$\left\{\delta(s;p) = a_n(p)s^n + a_{n-1}(p)s^{n-1} + \ldots + a_1(p)s + a_0(p), \quad p\in B\right\}, \tag{2}$$

where $p = (p_1 \ldots, p_q)'$ is the parameter vector, $B \subset R^q$ is the parameter set, and $a_i : B \longrightarrow R$, $i = 1, \ldots, n$ are given functions. It is usually assumed that B is arcwise connected and $a_i(\cdot)$ are continuous functions. Also, $a_n(p) \neq 0$, $\forall p \in B$, to guarantee an invariant degree polynomial family.

input norm	output norm	system induced norm
$\mathcal{L}_2$	$\mathcal{L}_2$	$\|\|G\|\|_\infty$
$\mathcal{L}_\infty$	$\mathcal{L}_\infty$	$\|\|G\|\|_1$
$\mathcal{L}_2$	$\mathcal{L}_\infty$	$\|\|G\|\|_2$
$\mathcal{L}_\infty$	$\mathcal{L}_2$	unbounded

Table 1: System induced norms for SISO systems.

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Biographical Sketches

Andrea Garulli was born in Bologna, Italy, in 1968. He received the Laurea in Electronic Engineering from the Universit`a di Firenze in 1993, and the Ph.D. in System Engineering from the Universit`a di Bologna in 1997. In 1996 he joined the Dipartimento di Ingegneria dell'Informazione of the Universit`a di Siena, where he is currently Associate Professor. He serves as Associate Editor for the *IEEE Transactions on Automatic Control* and he is member of the Conference Editorial Board of the IEEE Control Systems Society. He is author of more than 60 technical publications and co-editor of the book "Robustness in Identification and Control", Springer, 1999. His present research interests include robust identification and estimation, robust control, LMI-based optimization, mobile robotics and autonomous navigation.

Alberto Tesi received the Laurea in Electronic Engineering from the Universit`a di Firenze, Italy, in 1984. In 1989 he obtained the Ph.D. degree from the Universit`a di Bologna, Italy. In 1990 he joined the Dipartimento di Sistemi e Informatica of the Universit`a di Firenze, where he is currently a Professor of Automatic Control. He has served as Associate Editor for the *IEEE Transactions on Circuits and Systems* from 1994 to 1995 and the *IEEE Transactions on Automatic Control* from 1995 to 1998. Presently he serves as Associate Editor for *Systems and Control Letters*. His research interests are mainly in robust control of linear systems, analysis of nonlinear dynamics of complex systems, and optimization.

Antonio Vicino was born in 1954. He received the Laurea in Electrical Engineering from the Politecnico di Torino, Torino, Italy, in 1978. From 1979 to 1982 he held several Fellowships at the Dipartimento di Automatica e Informatica of the Politecnico di Torino. He was assistant professor of Automatic Control from 1983 to 1987 at the same Department. From 1987 to 1990 he was Associate Professor of Control Systems at the Universit`a di Firenze. In 1990 he joined the Dipartimento di Ingegneria Elettrica, Universit`a di L'Aquila, as Professor of Control Systems. Since 1993 he is with the Universit`a di Siena, where he founded the Dipartimento di Ingegneria dell'Informazione and covered the position of Head of the Department from 1996 to 1999. From 1999 he is Dean of the Engineering Faculty. In 2000 he founded the Center for Complex Systems Studies (CSC) of the University of Siena, where he presently covers the position of Director. He has served as Associate Editor for the *IEEE Transactions on Automatic Control* from 1992 to 1996. Presently he serves as Associate Editor for *Automatica* and Associate Editor at Large for the *IEEE Transactions on Automatic Control*. He is Fellow of the IEEE. He is author of 170 technical publications, co-editor of 2 books on `*Robustness in Identification and Control'*, Guest Editor of the Special Issue `*Robustness in Identification and Control*' of the *Int. Journal of Robust and Nonlinear Control*. He has worked on stability analysis of nonlinear systems and time series analysis and prediction. Presently, his main research interests include robust control of uncertain systems, robust identification and filtering, mobile robotics and applied system modeling.