ON THE ECONOMICS OF NON-RENEWABLE RESOURCES

Neha Khanna
Department of Economics and Environmental Studies Program, Binghamton University, Binghamton, NY, USA

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Contents

Summary

This paper presents an overview of the key economic results associated with the use of non-renewable resources. The basic Hotelling model of resource depletion is discussed, followed by several extensions. The fundamental result is that scarcity rent rises at the discount rate, and that, at equilibrium, marginal benefits from extraction must equal the marginal economic cost. If marginal extraction cost is determined by the remaining stock of the resource, then the result is that the scarcity rent rises at the discount rate less the percentage increase in marginal cost caused by the marginal reduction in remaining reserves. The versatility of the Hotelling model is clearly brought out in the various qualitatively distinct outcomes possible for the equilibrium production and price trajectories. Production may be monotonically increasing, decreasing, or inverted u-shaped.

The equilibrium price trajectory is determined by the interaction between the marginal extraction cost and the scarcity rent. Typically, it is increasing throughout the production horizon. However, if the marginal cost is declining rapidly, it may exceed the scarcity rent in the early part of the production horizon and price will decline. Eventually, however, it must rise as scarcity rent rises sharply. The results obtained under the Hotelling model are robust to the assumption of market structure – both monopoly and perfect competition yield qualitatively similar outcomes, though, of course, the price and quantity paths are different. The paper concludes with a case study of the global oil market, using a particular extension of the Hotelling model that is applicable in this case. Several qualitatively different scenarios are presented, each of

which is consistent with the crude oil production and price trajectories observed since the early 1970s.

1. Introduction: Renewable Versus Non-renewable Resources

The New Webster's Dictionary defines "renewable" as "replaceable naturally or by human activity". Examples of renewable resources include trees and other plants, animal populations, groundwater, etc. In contrast, non-renewable resources are those that are not replaceable, or replaced so slowly by natural or artificial processes that for all practical purposes, once used they would not be available again within any reasonable time frame. Obvious examples are oil and mineral deposits.

Economists add another dimension to this distinction between renewable and non-renewable resources. Since economics is concerned with the allocation of scarce resources, for an economist non-renewable resources not only have a fixed stock, they are also in limited supply relative to the demand for them. Thus, old growth trees with life spans of as much as 1000 years, while renewable by the common definition, may be classified as non-renewable by economists due to their relatively slow growth to maturity and few remaining stands. They may also be ecologically unique and not reproducible. Similarly, while coal would be considered non-renewable by some, most resource economists would consider it renewable due to the vast remaining stock. It is estimated that at current rates of consumption of about one billion tons per year, there is enough coal to last approximately 3000 years. From an economic perspective, there is no immediate coal scarcity simply due to its fixed stock. It is as if it were renewable. There is no scarcity rent associated with its extraction.

Traditionally, the major economic issues in the study of non-renewable resources have involved predicting the future production and price trajectories, as well as the date of possible resource exhaustion. In addition, there has been an effort to understand the impact of alternative market structures, such as pure competition, monopoly, and a cartel-dominated oligopoly, on the predicted trajectories. More recently, the environmental costs associated with the extraction and consumption of non-renewable resources have also come into focus.

2. The Hotelling Model of Resource Depletion

The central question in non-renewable resource economics is: given consumer demand and the initial stock of the resource, how much should be harvested in each period, so as to maximize profits? A simple example brings out the underlying intuition. For now, assume away any extraction costs and focus on the price per unit, p, of the resource in the market. Assume also that the real (inflation adjusted), risk-free interest rate on investments in the economy is r per cent per year. Then, the owner of the resource can either extract the resource now or hold on to it to extract in the future. Any amount of the resource extracted today will not be available in the future, and any resource left untouched today may fetch a higher price in the market in the future. These are the two fundamental factors influencing the resource owner's extraction decision. If the owner extracts the resource today she can invest the proceeds and earn r per cent year. However, if she expects the price of the resource to rise faster than r per cent per year, then it would make sense to hold on to the resource, forgo the interest earned on the proceeds but earn a higher total income by selling the resource at a higher price per unit. The opposite argument would hold if the resource price was expected to rise slower than r per cent per year.

In a competitive market where there are a large number of sellers, and each seller can sell any quantity at the going market price, each resource owner would be faced with the same options and would follow the same logic. The theoretical result is that in this market the quantity extracted will be such that resource price will rise at exactly r per cent per year. If it were to rise slower, resource owners would begin to sell off current stocks and the current market price would fall. If the resource price were to increase at a rate faster than r per cent per year, all owners of the resource would hold on to their stock, decreasing the current supply in the market, thereby inducing the current market price to rise. The equilibrium price trajectory for a non-renewable resource would, therefore, be rising exponentially as shown in Figure 1, where P_0 is the initial price and T indicates the time period of resource exhaustion.

An implication of the continuously rising price is that the quantity extracted would be continuously falling until such time as the resource is exhausted. As the price rises the demand for the resource is slowly choked off. Eventually the price would be so high that demand would be eliminated altogether. In the basic model, this is precisely when the resource stock would also be completely exhausted. To understand why, suppose that when the price is sufficiently high to choke off entirely all the demand, resource owners are left with some positive quantity of the resource. This remaining stock would be completely worthless to the owner since no one would want to buy it. Realizing this, the resource owners would begin to sell off the stock at lower prices before the demand is choked off by the high prices. However, this would mean that there would be an excess supply of the resource in the market which would lower current prices. The production trajectory would be extended in time and again the price would continue to rise at r percent per year until all the stock is completely depleted. The equilibrium production (or extraction) trajectory for a non-renewable resource is also shown in Figure 1.

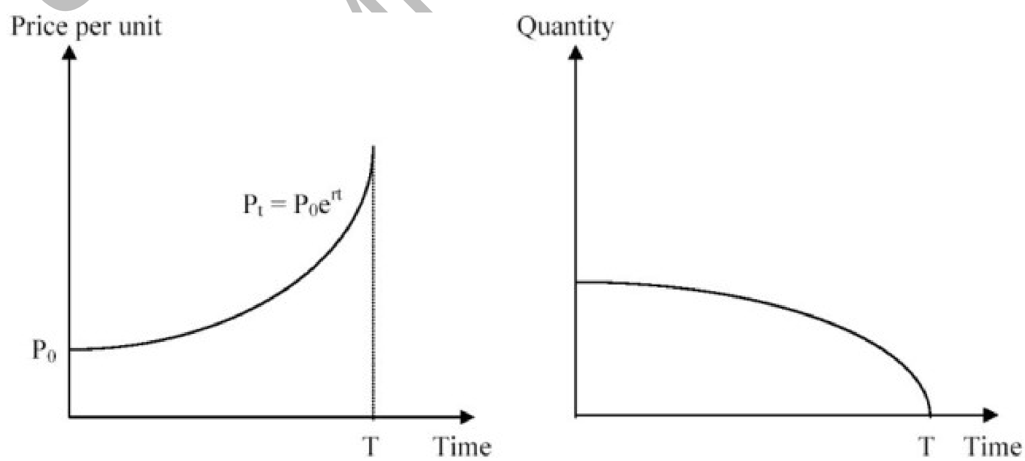


Figure 1: Equilibrium Price and Quantity Trajectories for a Non-Renewable Resource

This basic result that the price of a non-renewable resource in a competitive market would rise at the interest rate and that the production trajectory would be monotonically declining till the resource is exhausted was established by Harold Hotelling in his seminal 1931 article, “The Economics of Exhaustible Resources” published in the *Journal of Political Economy*. Few papers in economics have played such an important role in defining the field and contemporary research. It was the focus of Robert Solow’s 1973 Richard T. Ely lecture titled “The Economics of Resources or the Resources of Economics”. On the fiftieth anniversary of the publication of Hotelling’s paper, Shantayanan Devarajan and Anthony Fisher wrote a paper in the *Journal of Economic Literature* effectively showing that a large number of questions on the economics of non-renewable resources being asked today were first raised by Hotelling.

It can be shown that by following the above production trajectory the resource owner maximizes the present value of the flow of revenues from extraction over the time horizon from the present through the exhaustion of the resource. It can also be shown that the same production trajectory maximizes the discounted sum of producers’ and consumers’ surplus in a competitive market and is, therefore, Pareto optimal.

3. Variations on the Basic Hotelling Model

3.1. Extraction Costs

3.1.1. Exogenous Extraction Costs

Extraction costs per se do not change the fundamental logic of the above model. Suppose that the marginal extraction cost is slowly rising over time. This could be because a larger quantity of resources is being extracted in each period or due to more stringent environmental policies requiring more expensive extraction techniques, or both. Whatever the reason, so long as the marginal extraction cost is not determined directly by the *cumulative* amount of the resource extracted, the result would be that net price, i.e., price minus the marginal extraction cost, or scarcity rent, would rise exponentially at r per cent per year.

It is important to note that even though Hotelling’s model of resource depletion implies that net price would be rising exponentially at the interest rate, this does not mean that the market price (i.e., the price paid by the consumer) will follow this trajectory. The consumer price is the marginal extraction cost plus the scarcity rent. If extraction costs are falling, say due to technological improvements as in the case of the oil industry during the last decades of the twentieth century, then it is entirely possible that the market price is constant or even declining in the near term. So long as the downward pressure due to the falling marginal extraction cost outweighs the rising scarcity rent, the consumer price will be decreasing. Eventually, however, as the resource gets depleted and the scarcity rent rises rapidly and outweighs the marginal cost, the market price will rise.

When the marginal extraction cost is rising over time, the equilibrium production trajectory is monotonically declining, as in the simple case with no extraction cost. However, if the marginal extraction cost decreases with time, then it is also possible for the equilibrium quantity trajectory to increase in the near term. During this period, the downward pressure of the falling marginal cost more than offsets the rising user cost.

3.1.2. Reserve Dependent Costs

A more sophisticated theory of non-renewable resource depletion would link the marginal extraction cost directly to cumulative production or the remaining stock of the resource. These are referred to as "reserve dependent costs" in the literature. In this case, each unit of the resource extracted today is not only unavailable in the next period, but also increases future extraction costs by lowering the remaining reserves. The opportunity or user cost of extracting a finite stock of resources is now two fold: foregone interest income and higher extraction costs. In this case, the scarcity rent does not rise at the interest rate, but at the interest rate less the percentage increase in cost due to a marginal reduction in remaining reserves.

Note that the basic principle remains intact: at equilibrium, the marginal benefit from extraction must equal the marginal economic cost (defined as the sum of marginal extraction cost and the user cost). In the case with no or constant extraction costs, or with extraction costs that vary independently of cumulative production, the economic cost of extraction is just the foregone interest income. With reserve dependent costs, one must include the increase in marginal extraction cost that occurs as remaining reserves are drawn down by current extraction.

3.2. Monopoly

The fundamental results of the Hotelling model remain unchanged when the entire stock of the resource is owned by a single seller. In this case it is the marginal profit or the difference between the marginal revenue and marginal extraction cost that grows at r per cent per year. However, if in the presence of a static demand curve the price elasticity of demand decreases as the quantity extracted increases, the monopolist's production trajectory will be longer than that of the competitive resource owner when faced with identical costs, initial stock, and consumer demand. The monopolist takes advantage of the relatively lower price elasticity in the earlier periods to restrict output and charge a higher price than the perfectly competitive resource owner. The result is that the extraction path tends to get stretched out over time – that is, monopoly slows the depletion rate. This result has led to the adage, "a monopolist is a conservationist's best friend". The monopolistic and competitive price and quantity trajectories are compared in Figure 2, where T_c and T_m indicate exhaustion under competition and monopoly, respectively.

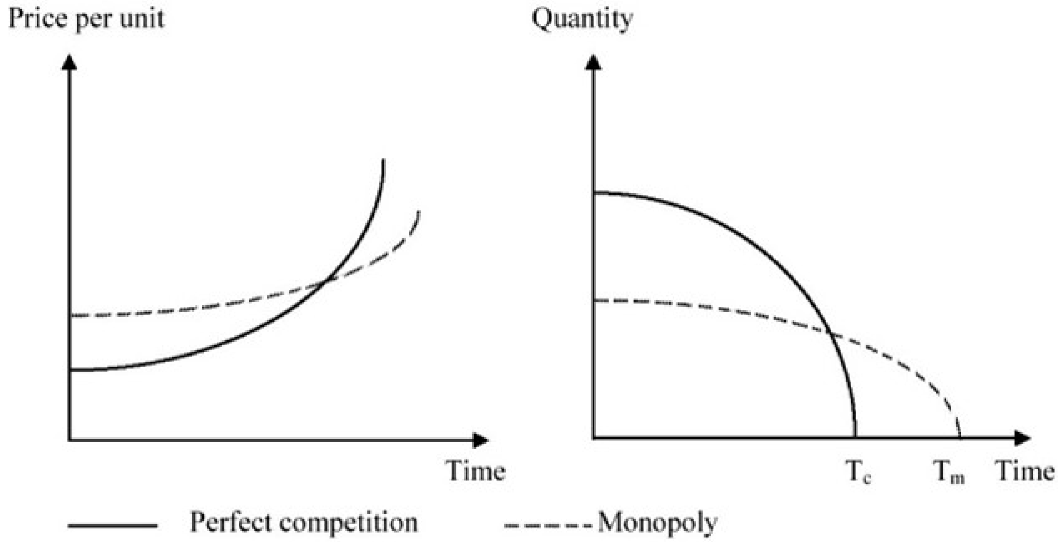


Figure 2: Monopoly vs. Competitive Equilibrium Price and Quantity Trajectories

One case where the competitive and monopoly equilibrium price and extraction paths are identical is when the resource owners face a constant elasticity demand curve that is unchanging over time, and when the extraction cost is independent of the quantity extracted in each period. The crucial feature of a constant elasticity demand curve, as opposed, say, to a linear demand curve, is that total revenue is the same at all points on the curve. No matter how much the monopolist raises the price of the resource, quantity demanded declines proportionately so that total revenue is constant. In this case, the monopolist cannot increase the present value of profits by restricting quantity and raising price in the earlier periods.

3.3. Multiple Sources of the Resource

Another interesting extension of the Hotelling model considers the situation where there are two sources of the same non-renewable resource but with different marginal extraction costs. An example would be the oil deposits in the Southwestern USA and Alaska, and the deposits in Saudi Arabia. Both regions are endowed with vast quantities of crude oil. However, mostly due to geological differences, per barrel extraction costs in Saudi Arabia are about a quarter of those in the US. In the abstract world of perfect competition, both sources would not supply the resource simultaneously. Like David Ricardo's argument for using higher grades of the resource – or in this case, lower cost mineral sources – first, production would begin in the low cost region. The net price would rise at the interest rate until it is exactly equal to the net price for the second, high cost source. At this point, the low cost resource is completely depleted and the high cost source supplies the entire market. Scarcity rent would rise again at the interest rate till the second resource is exhausted as well. This succession of equilibrium price trajectories is shown in Figure 3, where P_1 and P_2 indicate the net price of the resource from the first and second sources, respectively, and T_1 and T_2 indicate the exhaustion of the two sources.

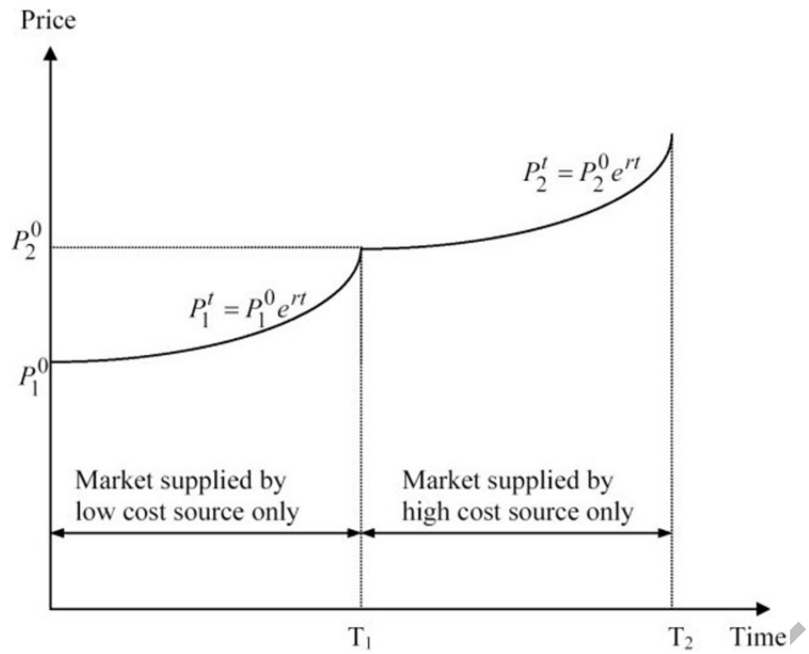


Figure 3: Equilibrium Price Trajectory with Multiple Sources

3.4. "Backstop" Resources

Suppose there is some other resource which is a perfect substitute for the non-renewable resource in question. Suppose also that this alternative, or "backstop" resource can be supplied at some high cost but in fairly large quantities so that it is inexhaustible for all practical purposes. Since the backstop has a virtually unlimited supply, its price will be just sufficient to cover its marginal extraction cost.

Implicitly, backstop technologies are assumed to be renewable. Ethanol fuel from renewable corn and sugar is frequently seen as a backstop for petroleum. However, a backstop technology can itself be non-renewable. For example, coal-based electric transportation would itself be a non-renewable backstop for finite petroleum resources. In the presence of a backstop, there is a ceiling on the net price of the non-renewable resource. In theory, as soon as the price of the non-renewable resource just exceeds the price of the backstop, the former will be priced out of the market and the demand would be entirely satisfied by the latter resource. (In effect, the price of the backstop is like the vertical intercept on the linear demand curve for the non-renewable resource.) The overall result is the same as in the case with multiple sources of the same non-renewable resource. The net price of the non-renewable resource will rise at the interest rate till it is completely exhausted. Exactly at that instant the net price would be equal to the price of the backstop and production would shift from the non-renewable to the backstop resource.

It is easy to argue that in the absences of a backstop, the non-renewable resource would be depleted at exactly the time when production shifts to the backstop. Suppose this is not the case and there are some remaining reserves of the non-renewable resource when its price rises to that of the backstop. Then the resource owner would be unable to sell the resource on the market since the net price necessary to cover the scarcity rent would exceed the price of a cheaper substitute. The only option is to sell the resource at an earlier and lower price. However, this would increase the supply in the market and the net price would fall. In fact, the price would decline to a level such that when it rises at the interest rate the resource is exhausted at the price of the backstop. A similar argument emerges in the situation where the resource is exhausted before its price reaches the ceiling set by the backstop. In this case, there is a large excess demand which would bid up the price for the resource. The profit-maximizing resource owner would then hold back some reserves to sell at the future higher price and the production horizon would be extended.

The above example assumes the supply curve for the backstop is horizontal at a price just sufficient to cover its marginal extraction cost. This assumption is not necessary. It is entirely possible that the price of the backstop is rising slowly. As long as the backstop price is rising slower than the interest rate, then the price trajectories for the non-renewable resource and the backstop will intersect at some point. At that point, the price of the backstop will become the ceiling for the price of the non-renewable resource and the latter resource will be completely depleted. From that point on, the market will be completely supplied by the backstop. Figure 4 shows the intersection of the price trajectories for the non-renewable and backstop resources, where P_{nr} indicates the price of the non-renewable resource, P_b indicates the price of the backstop, and T_{nr} indicates the depletion of the non-renewable resource.

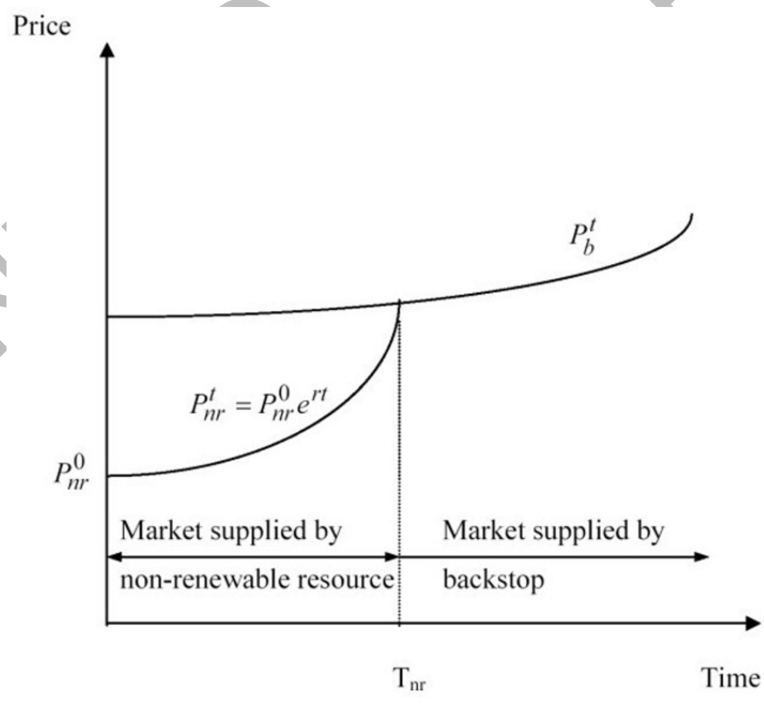


Figure 4: Impact of Backstop Resource

3.5. Growing Demand

In the entire analysis so far, we have assumed that the demand for the non-renewable resource is constant over time. This is not a very realistic assumption. Typically one would expect an increase in the market demand over time due to a growth in population as well as per capita income. Graphically, this would lead to a rightward shift of the demand curve from one period to the next.

This simple extension of the basic model introduces an element of realism into the model in two important ways. First, it reflects a realistic situation for non-renewable resources, such as oil, which have witnessed a rapid growth in their total demand. Second, *regardless of whether the marginal extraction cost is falling or rising over time*, the resulting equilibrium production trajectory may initially increase before eventually declining to exhaustion as illustrated in Figure 5 for both monopoly and competitive market structures. The logic is simple. In the first few years when the user cost of extraction is low, the positive impact of the income and population effects (and declining marginal extraction cost, if applicable) more than outweigh the negative impact of the substitution effect due to the rising economic cost of extraction. During this period, the extraction trajectory is rising. However, the user cost eventually dominates and equilibrium extraction begins to decline.

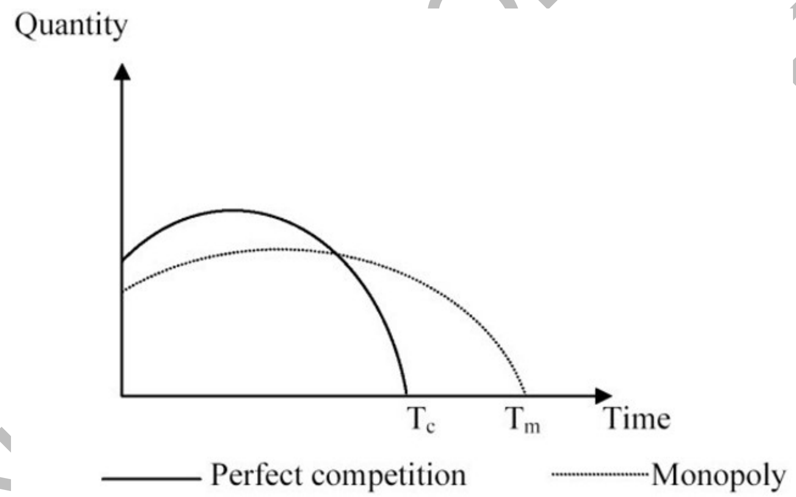


Figure 5: Equilibrium Production Trajectory with Growing Consumer Demand

Furthermore, as long as the marginal extraction costs are increasing over time, the market price trajectory will be monotonically increasing as in Figure 1. However, if marginal costs are declining over time, it is possible that the equilibrium net price would first decline and then increase. The exact solution depends on the interaction between growth in demand, the decline in extraction costs, and the increase in the scarcity rent. Other things being the same, the declining portion of the equilibrium price trajectory is longer in the presence of growing demand than when demand is intertemporally unchanging. Section 5 presents simulations for the global oil resource where alternative price trajectories may be obtained depending on the parameter values assumed.

4. On Discount Rates

It is clear from the preceding discussion that the interest rate plays a critical role in determining the equilibrium price and extraction trajectories, as well as the production horizon for a non-renewable resource. This interest rate, which reflects the return on the average investment in the economy, is typically referred to as the discount rate in the economics literature. The equilibrium production trajectory obtained under the Hotelling model and its several extensions is Pareto optimal only if the private discount rate being used by the resource owners is numerically identical to the social discount rate, also known as the social rate of time preference. (The private discount rate is the opportunity cost of private capital and reflects how an individual values future cash flows in terms of present dollars. The social discount rate, on the other hand, reflects the value that society as a whole places on future consumption.) Unfortunately, it is quite likely that this will not be the case. In fact, Robert Solow provides several reasons why the private discount rate might be systematically higher than the social rate of time preference. The result is that the scarcity rent would rise too rapidly, and the resource would be exhausted too soon compared to the socially optimal trajectory.

The discount rate issue does not end here. Even if the resource owner could somehow be coerced into using the social rate of time preference when making extraction decisions, there is no unambiguous numerical value for that parameter. The social rate of time preference is the sum of two parts: the pure rate of time preference, and a second piece which reflects the declining marginal utility of income that follows the growth of income over time. The second part is relatively uncontroversial. It implies that as a person gets richer the value of an additional dollar of income declines. The first part is fraught with debate. The pure rate of time preference reflects the psychological basis of intertemporal decision making, and captures the myopia in human decision-making processes. Some economists argue that the pure rate of time preference should be imputed from the historical savings record for an economy. They tend to be in favor of using a pure rate of time preference of about 3 per cent per year. Others focus on the ethical implications of discounting economic events simply because they occur at future points in time, especially if there are implications for future generations. This group argues that the pure rate of time preference should be zero or close to it. Still others argue that discount rate should be the rate of sustainable GDP growth.

The debate remains unresolved. Whatever the reader’s personal opinion on this issue, it is important to know that the higher the discount rate used, the higher the growth rate of prices, and the shorter the time interval to resource depletion.

5. Case Study – World Oil

How well has the Hotelling model done in terms of predicting the production and price trajectories of non-renewable resources in the real world? In the 1980s ecologist Paul Ehlrich and economist Julian Simon were engaged in a heated debate, and even a bet, on whether non-renewable resources were becoming more scarce, and whether their prices would rise as predicted by the Hotelling model. In 1990, Paul Ehlrich lost the bet. The inflation-adjusted prices of five previously agreed upon non-renewable resources – copper, chrome, nickel, tin, and tungsten – fell over the course of the decade.

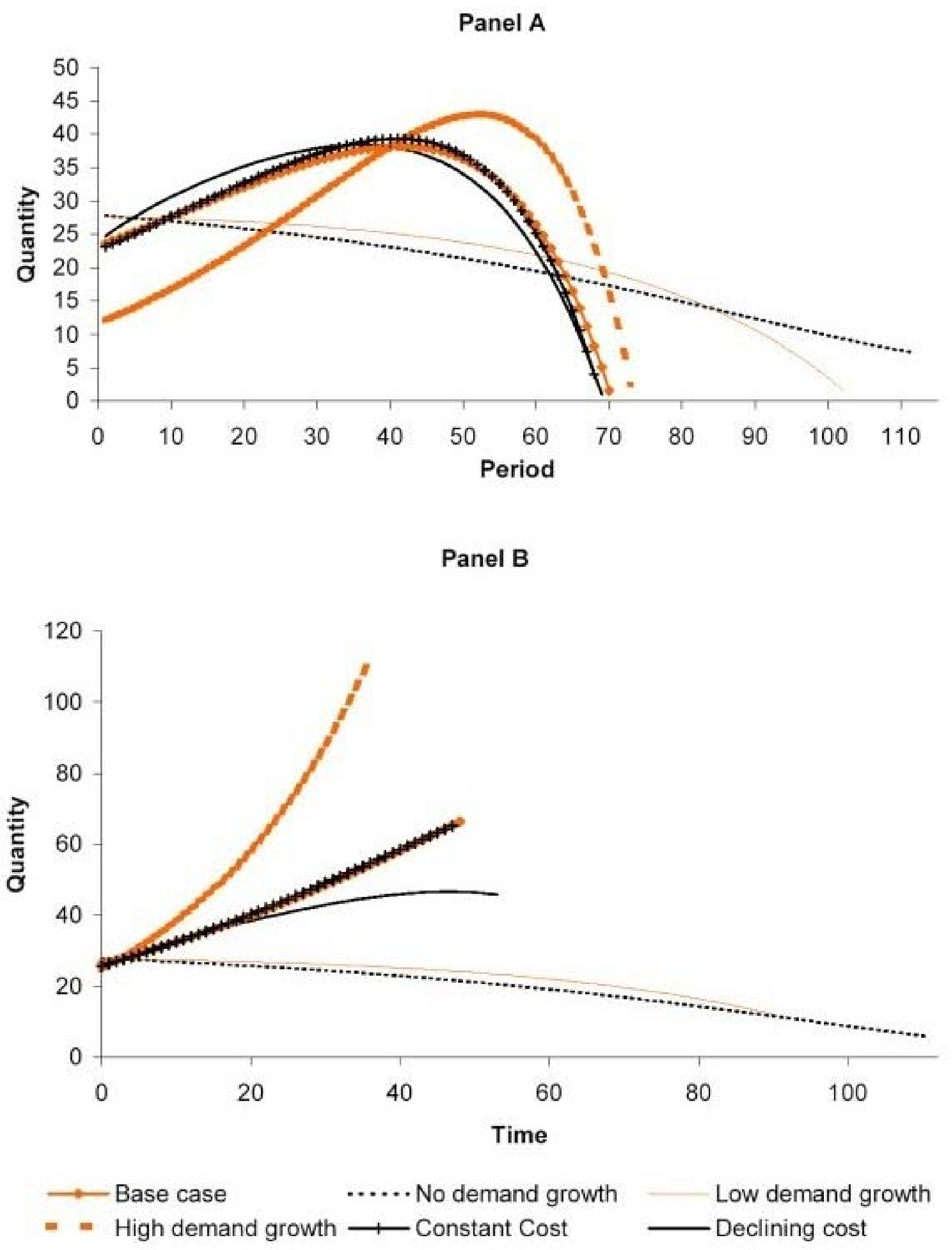
Even though Julian Simon won that bet and debate, it does not follow that declining prices are inconsistent with a resource-depletion model. The prediction that price would rise monotonically while quantity decreases is typically obtained under the assumptions that, over time, demand is constant, and marginal cost of extraction rises. An alternative version of the model can yield price and quantity trajectories that are much more consistent with the observed data.

To appreciate the power and versatility of the Hotelling model, consider the global oil market. There has been much debate over when the world will reach exhaustion of conventional petroleum resources. In the 1970s, following the first oil shock, some economists predicted that global oil supply would decline and that depletion would occur in the near future. However, with hindsight, one can argue that they had simply failed to recognize the monopoly power that the OPEC countries exercised over world oil production. Since then, there has been a general feeling of optimism among economists due to falling real oil prices. Some economists, and most geologists, however, continue to believe that the recent downward trend in oil prices was only temporary and that eventually there will be a more or less sustained upswing.

Presented below are simulations based on an extension of the Hotelling model that incorporates the increasing demand for oil due to the growth in world population and per capita income. In addition, the model allows for marginal extraction cost to change over time (increase or decrease), though independently of remaining oil stocks (i.e., extraction costs are not reserve-dependent). The discount rate is assumed to be five per cent per year, and remaining oil reserves are fixed at 2.1 trillion barrels. Six different scenarios are considered. In the base case, world demand is growing at about two per cent per year and the marginal cost of extraction grows at around 1.6 per cent per year. In the low demand growth case, the growth rate of demand falls to 0.2 per cent per year, whereas in the high demand growth scenario it is four per cent per year. In the penultimate scenario, world demand is growing at the rate assumed in the base case, but the marginal extraction cost is constant over time. This would reflect technological advancement such that the rise in extraction costs assumed in the base case is exactly offset over time. Finally, in the declining extraction cost case it is assumed that technical change is sufficiently rapid so that the marginal extraction cost falls at 7.5 per cent per year, while world demand is growing at 1.5 per cent per year. In the cases with a backstop resource, it is assumed that the resource is available at a price of $50/barrel.

As in the case of almost any economic model, the numerical results obtained under the different scenarios are sensitive to the parametric assumptions. However, the qualitative results are robust, and, therefore, the focus of our discussion here.

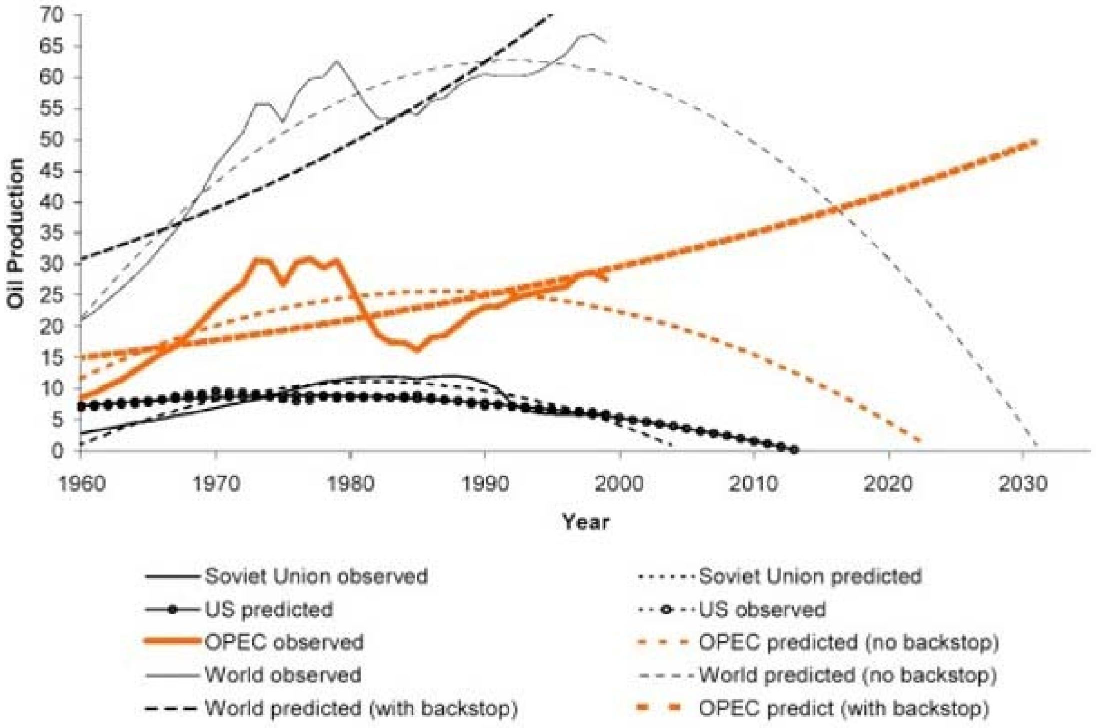
It is clear from panels A and B of Figure 6 that the Hotelling model can yield qualitatively different results for the equilibrium production trajectory depending on the scenario considered. In the no and low demand case, production declines monotonically – both with and without a backstop. In all other cases, production first rises and then declines to exhaustion in the absence of a backstop. It is also possible that production increases monotonically until the resource is exhausted, as in the base case, constant cost, and high demand growth scenarios, and in the presence of the backstop.



Note: Panel A assumes there is no backstop technology for oil; Panel B assumes a backstop resource is available at a constant price of $50/barrel.

Figure 6: Equilibrium Production Trajectories under Alternative Scenarios

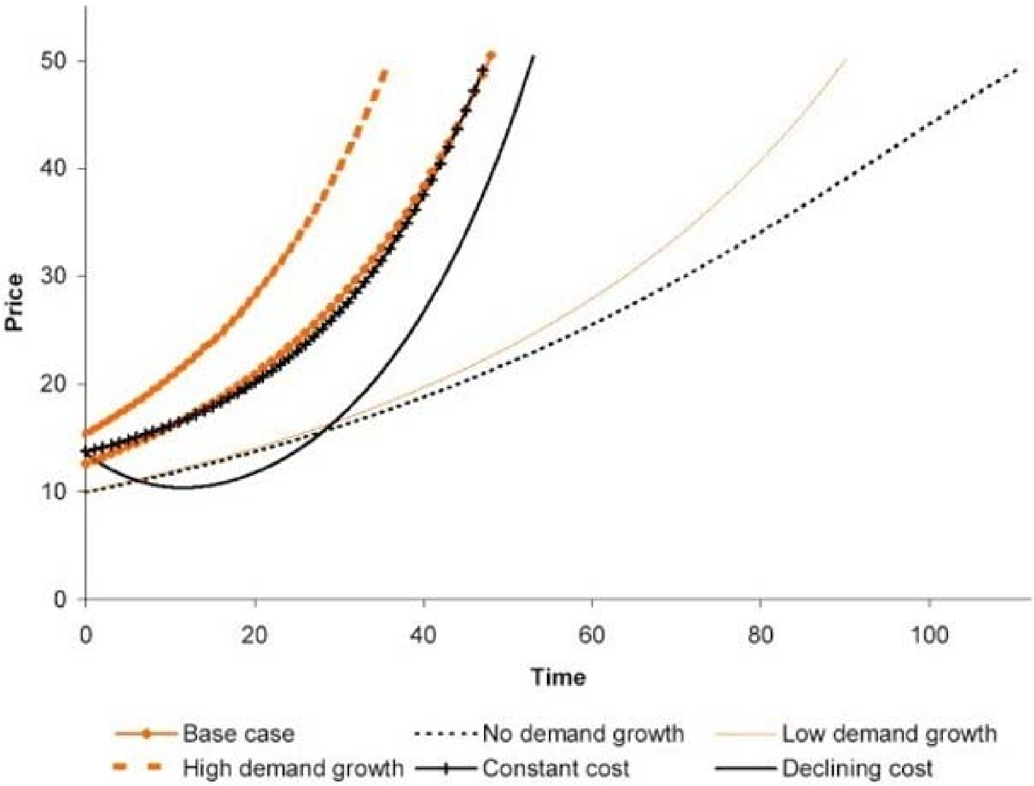
Figure 7 superimposes these stylistic results on observed oil production data. The Figure presents actual and predicted equilibrium quantity trajectories for 4 regions – OPEC, the former Soviet Union (and only Russia after 1991), USA, and the World. Clearly, the Hotelling model yields results that are consistent with observed reality. Furthermore, it is also able to reconcile the different positions of the optimists and the pessimists on the future scarcity of oil resources. If you believe that a credible backstop resource exists for oil, and that the marginal cost of extraction is likely to change only slowly, while world demand grows steadily at the historically observed rates, then you are likely to be optimistic about the future. Under these conditions, oil production is likely to rise monotonically until depletion. As oil prices rise, there would be a transition to the alternative fuel. There is no real economic scarcity in this case. Even if you are not convinced about the existence of a backstop, an optimistic viewpoint is not unreasonable. It is possible that world production will continue to increase for several years into the future before it peaks and eventually declines. The most pessimistic position would be one that believes that marginal extraction costs rise steadily while demand growth is sluggish. In this case, the equilibrium production trajectory declines monotonically until the resource is exhausted, regardless of whether there is a backstop or not.



Note: the predicted trajectories are stylistic representations of possible equilibrium trajectories obtained under the Hotelling model that are consistent with the observed data.

Figure 7: Observed and Predicted Oil Production (1960-2035): Selected Countries and Regions

Figure 8 illustrates that five out of the six scenarios considered yield a monotonically increasing price trajectory, assuming a backstop resource is available at $50/barrel. Qualitatively similar results are obtained in the absence of the backstop. The only case where oil prices are predicted to decline in the near term, under the assumed numerical values for the model parameters, is when the negative effect of the declining marginal extraction cost more than offsets the positive effects of rising demand and scarcity rent.



Note: this Figure assumes that a backstop resource is available at $50/barrel. Qualitatively similar results are obtained in the absence of the backstop.

Figure 8: Equilibrium Price Trajectories under Alternative Scenarios

The observed price trajectory (shown in Figure 9) can be hard to explain using any single economic model of non-renewable resources. The period from 1974 through 1985 saw unprecedented high prices. This is mostly explained by institutional and political factors that led OPEC countries to restrict production, and the Iran-Iraq war of the early 1980s during which period their combined production fell by about 70 per cent. If we ignore the decade from 1975 to1985, however, the trend in oil prices seems to be slightly downward. This is consistent with the results of the Hotelling model. During this period, marginal extraction costs declined, while the growth in world demand for crude oil was slower than expected due to the combined effect of several demand side management strategies that were put in place following the two oil shocks of the previous decade. The declining extraction cost scenario combines these two factors. The result is a declining price trajectory for several years in the near term followed by an upswing.

Figure 9: Oil Prices, 1949-1999

The production trajectory obtained in the declining extraction cost scenario is also interesting. Equilibrium oil production increases for several years, well into the period when oil prices are predicted to increase monotonically. It's only when depletion seems imminent and the user cost begins to rise sharply that production declines, or levels off. In the case of world oil, it is this author's considered opinion that world oil production is likely to continue to rise for several years before declining. As for prices, it is unlikely that the 12 to 15 $/barrel range observed in the past decade or so will return. Given recent developments, it is much more likely that the price of oil would hover around 25 to 30 $/barrel in the near future before beginning a slow and steady upswing.

6. Conclusions

The Hotelling model of resource depletion is the fundamental economic model used to analyze the issues relating to the use of non-renewable resources. Under this model, resource owners seek to maximize the present value of net benefits obtained from extracting the resource, given consumer demand for the resource and subject to the constraint that total extraction cannot exceed the initial resource stock. The optimal extraction decisions for each period in the production horizon are interdependent, since a unit of the resource extracted today is unavailable for extraction in the future. The key result obtained is that scarcity rent (the difference between marginal revenue and marginal cost, appropriately defined) increases at the rate of discount. The only exception to this "Hotelling rule" is the situation where the marginal extraction costs depend on cumulative extraction. In this case, the rent increases at a rate less than the discount rate. The difference is caused by the increase in marginal cost due to marginal reduction in remaining reserves. However, in all cases, the marginal benefit from extraction to the resource owner is exactly equal to the marginal economic cost at equilibrium.

The Hotelling rule holds regardless of whether the resource stock is owned by a monopoly or a perfectly competitive firm. The difference lies in the definition of scarcity rent. Under perfect competition, each of innumerable firms faces a perfectly elastic (horizontal) demand curve, and marginal revenue is identical to price. Here rent may be defined as the difference between price and marginal extraction cost. In the case of a monopoly, the firm faces a downward-sloping demand curve and price exceeds marginal revenue. Therefore, scarcity rent is the excess of marginal revenue above marginal cost.

Given the central role of the discount rate in the Hotelling rule, it is important to note that there is no sacrosanct numerical value for the social rate of time preference. Furthermore, the model assumes that private discount rates will be identical to the social discount rate. It is only under this assumption, in addition to the assumption of a perfectly competitive market structure, that the equilibrium price and production trajectories obtained are efficient.

The price to the consumer is defined by the sum of the marginal extraction cost and the scarcity rent. The shape and direction of the price trajectory, therefore, depends on the interaction between these two factors. So long as the marginal extraction cost is rising, prices will rise, though at a rate different from the discount rate. However, if the marginal cost is decreasing it is possible that this negative effect dominates the scarcity rent, at least in the near term. In this situation, the equilibrium price trajectory will initially decline but eventually rise as the increasing scarcity rent begins to outweigh the declining marginal cost.

The equilibrium production trajectory obtained under the Hotelling model varies according to the parametric specification. The trajectory may be monotonically increasing, monotonically decreasing, or may first increase, and then decrease. The result is determined by whether the growth in demand is stronger than the growth in the marginal economic cost of extraction.

Which of the production and price trajectories obtained under the Hotelling model of resource depletion will be observed in the future cannot be predicted with confidence. The final outcome would be determined by the interaction of several forces, including many non-economic factors that are hard to include in resource depletion models. But it is certainly the case that this model can predict an array of credible price and quantity trajectory for non-renewable resources under a reasonable set of assumptions. We now know that a period of declining prices (primarily induced by declining extraction cost) and accelerating consumption is theoretically consistent with the eventual exhaustion of a non-renewable resource.

Finally, it is important to mention a new dimension in the economics of non-renewable resources that has come into focus due to the environmental costs associated with the production (mainly extraction) and consumption of these resources. In terms of the theoretical framework examined here, these costs may substantially increase the marginal extraction (or economic) costs. In this case, it is possible that the marginal economic cost curve exceeds the vertical intercept of the linear demand curve. (If the demand curve is non-linear and asymptotic to the price intercept, the two curves would intersect at a very low quantity.) This would imply that the non-renewable resource may not be exhausted and some part of the remaining stock may remain unextracted.

Appendix

This appendix provides the mathematical and analytical basis for the main results discussed in the body of the paper. Section 1 derives the equilibrium quantity and price trajectories for a perfectly competitive market where extractions costs are independent of cumulative production. It begins with the most general case in which demand and marginal extraction cost may change over time. It then uses these results to discuss other scenarios as special cases. It ends with a brief discussion of the results obtained under monopoly. Section 2 considers reserve dependent extraction costs.

1. Exogenous Extraction Costs

1.1. Perfect Competition

Consider a perfectly competitive market for a non-renewable resource with a finite stock of remaining resources, *S*. Suppose a linear demand curve that shifts over time in response to a growing world population, L_t, rising per capita income, y_t. Suppose also, that firms face a marginal extraction cost, *C(t)* that varies over time. For mathematical simplicity, assume that *C(t)* is independent of the quantity of the resource extracted in each period. In other words, the marginal extraction cost curve is horizontal to the quantity axis but may shift up or down from one period to the next. Producers maximize the net present value (*NPV*) of profits by choosing the optimal duration of production, *T*, and the quantity produced in each time period, q_t, given the demand and cost schedules, and remaining resources. This can be written as:

Maximize *NPV* with respect to [q_t, *T*], where

$$NPV = \int_0^T \left(\int_0^{q_t} \left(P\left(q_t, L_t, y_t\right) - C(t)\right) dq \right) e^{-rt} dt$$

$$P_t = P(\bullet) = \beta_2 L_t^{\eta_1} y_t^{\eta_2} - \beta_1 q_t$$

$$C(t) \geq 0, \quad C'(t) \leq \geq 0 \qquad (1)$$

$$S \geq \int_0^T q_t \; dt$$

$$P_t, q_t, P_t - q_t \geq 0$$

and

P_t: price of the resource at time t

q_t: extraction at time t

$C(t)$: marginal extraction cost at time t

L_t: population at time t, $L'(t) > 0$

y_t: per capita income at time t, $y'(t) > 0$

$r > 0$: real risk-free interest rate

S: stock of remaining resources

$\beta_1 > 0$: slope of demand curve

β_2: calibration constant

$\eta_1 > 0$: population sensitivity parameter

$\eta_2 > 0$: income sensitivity parameter

Under perfectly competitive markets the Hamiltonian for the above problem is:

$$H = \frac{[P(\bullet) - C(t)]\, q_t}{e^{rt}} - \lambda_t q_t$$

$$\frac{\partial P_t}{\partial q_t} \equiv 0 \qquad (2)$$

where λ_t is the costate variable representing the change in the discounted *NPV* due to a small change in the quantity of remaining resources. In other words, it is the user cost associated with extraction. The optimal production trajectory, q_t^*, is found by solving the first order conditions and the constraints, simultaneously. The solution is:

$$q_t^* = \beta_3(t) + \frac{e^{rt}}{M(r)}(S - \beta_4)$$

where

$$\beta_3(\mathrm{t}) = \frac{\beta_2 L_t^{\eta_1} y_t^{\eta_2} - C(t)}{\beta_1}$$

$$\beta_4 = \int_0^{\mathrm{T}} \beta_3(t)\, dt$$

$$M(r) = \int_0^{\mathrm{T}} e^{rt}\, dt = \frac{e^{rT} - 1}{r} \qquad (3)$$

$$\lambda^* = \frac{\beta_1(\beta_4 - S)}{M(r)}$$

and $M(r)$ is the compound discount factor.

Note that $\beta_3(t)$ is the equilibrium production trajectory in the absence of a resource constraint. This is the solution that is obtained by equating price with marginal cost in each period as in any standard perfectly competitive market. In our case with the demand and marginal extraction cost curves shifting over time, $\beta_3(t)$ also defines the locus of the intersection points of these two curves. β_4 is the cumulative production over the entire horizon that would have been achieved in the absence of the resource constraint.[1] Thus, in the presence of a resource constraint the equilibrium production trajectory deviates from the standard one by a factor that reflects the user cost arising due to the finite stock of resources (based on the difference between remaining resources, S, and β_4). It is clear that in the initial periods (i.e., when t is small) the constrained and unconstrained production trajectories are very close. However, with time, the scarcity factor grows exponentially, creating a larger and larger gap between q_t^* and $\beta_3(t)$.

The equilibrium price trajectory, P_t^*, is obtained by substituting the expression for q_t^* in the demand function:

$$P_t^* = C(t) + \frac{e^{rt}}{M(r)} \beta_1(\beta_4 - S) \qquad (4)$$

The difference between price and marginal cost is scarcity rent. (Note, scarcity rent is mathematically equivalent to $e^{rt}\lambda^*$.) In the absence of a resource constraint, $\beta_4 = S$ and $P_t^* = C(t)$. As in the case of the quantity trajectory, the difference between the price trajectory with and without the resource constraint grows larger over time as the scarcity rent increases.

The optimal production horizon, T, is the minimum of T_1 and T_2:

$$\begin{aligned} T_1 &= T \ni \beta_2 L_T^{\eta_1} y_T^{\eta_2} = C(T) \\ T_2 &= T \ni Q_T = 0 \end{aligned} \qquad (5)$$

where T_1 is defined as the period when the marginal cost of extraction rises to the level of the intercept of the demand curve, and T_2 is the period when the equilibrium production trajectory falls to zero.

Taking the derivative of equation (3) with respect to time we get[2]:

1 Throughout the appendix, we assume an interior solution. So $\beta_4 > S$.

2 β_4 is independent of t. See footnote 3 for a simpler case.

$$\frac{\partial q_t^*}{\partial t} = \frac{\beta_2}{\beta_1}\left\{L_t^{\eta_1} y_t^{\eta_2}\left(\eta_1 \frac{L'(t)}{L(t)} + \eta_2 \frac{y'(t)}{y(t)}\right)\right\} - \frac{\beta_2}{\beta_1}C'(t) + \frac{re^{rt}}{M(r)}(S - \beta_4) \qquad (6)$$

$$< 0 \text{ if } C'(t) > 0 \text{ and } \frac{\beta_2}{\beta_1}\left\{L_t^{\eta_1} y_t^{\eta_2}\left(\eta_1 \frac{L'(t)}{L(t)} + \eta_2 \frac{y'(t)}{y(t)}\right)\right\} < \left|\frac{\beta_2}{\beta_1}C'(t) + \frac{re^{rt}}{M(r)}(S - \beta_4)\right|$$

$$> 0 \text{ if } C'(t) > 0 \text{ and } \frac{\beta_2}{\beta_1}\left\{L_t^{\eta_1} y_t^{\eta_2}\left(\eta_1 \frac{L'(t)}{L(t)} + \eta_2 \frac{y'(t)}{y(t)}\right)\right\} > \left|\frac{\beta_2}{\beta_1}C'(t) + \frac{re^{rt}}{M(r)}(S - \beta_4)\right|$$

$$< 0 \text{ if } C'(t) < 0 \text{ and } \frac{\beta_2}{\beta_1}\left\{L_t^{\eta_1} y_t^{\eta_2}\left(\eta_1 \frac{L'(t)}{L(t)} + \eta_2 \frac{y'(t)}{y(t)}\right)\right\} - \frac{\beta_2}{\beta_1}C'(t) < \left|\frac{re^{rt}}{M(r)}(S - \beta_4)\right|$$

$$> 0 \text{ if } C'(t) < 0 \text{ and } \frac{\beta_2}{\beta_1}\left\{L_t^{\eta_1} y_t^{\eta_2}\left(\eta_1 \frac{L'(t)}{L(t)} + \eta_2 \frac{y'(t)}{y(t)}\right)\right\} - \frac{\beta_2}{\beta_1}C'(t) > \left|\frac{re^{rt}}{M(r)}(S - \beta_4)\right|$$

Regardless of whether marginal extraction cost increases or decreases over time, the equilibrium production trajectory may first rise but then eventually falls. In the presence of increasing marginal costs, the growth in demand must outweigh the combined effect of the scarcity factor and extraction costs for the equilibrium quantity trajectory to decline. When the marginal cost is falling over time it reinforces the upward impact of the growing demand. In this case the equilibrium production trajectory will decline only when the absolute value of the scarcity factor exceeds the sum of these two factors.

The time derivative of the equilibrium price trajectory is given by:

$$\frac{\partial P_t^*}{\partial t} = C'(t) + \frac{re^{rt}}{M(r)}\beta_1(\beta_4 - S)$$

$$> 0 \text{ if } C'(t) > 0 \qquad (7)$$

$$< 0 \text{ if } |C'(t)| < 0 \text{ and } |C'(t)| > \left|\frac{re^{rt}}{M(r)}\beta_1(\beta_4 - S)\right|$$

$$> 0 \text{ if } |C'(t)| < 0 \text{ and } |C'(t)| < \left|\frac{re^{rt}}{M(r)}\beta_1(\beta_4 - S)\right|$$

It is clear from equations (4) and (7) that net price, P_t^* - $C(t)$, is growing at the interest rate. With marginal extraction cost rising over time, market price also rises monotonically (though at a faster rate). When the marginal extraction cost falls over time, the market price may initially decrease. However, over time, the scarcity rent grows rapidly (e^{rt} becomes large as t increases) and the market price will eventually increase.

Equations (3) and (4) provide the analytical expressions for the equilibrium quantity and price trajectories, respectively, in a general case: both demand and marginal extraction cost may vary over time. From these equations, it is possible to derive several the other results discussed in the paper as special cases. We consider these below.

Case A: Basic Hotelling model of resource depletion

In the most basic model of resource depletion, the marginal cost of extraction is zero and demand for the resource is constant over time. In terms of the model developed above this means that C(t) = C′(t) = *0* and $\frac{L'(t)}{L(t)} = \frac{y'(t)}{y(t)} = 0$. In other words, the intercept of the demand curve is constant at some level, say β2*. Substituting this into equations (3), (4), (6), and (7) we get the following solutions for the equilibrium quantity and price trajectories, respectively:

$$q_t^* = \beta_3 + \frac{e^{rt}}{M(r)}(S - \beta_4)$$

where

$$\beta_3 = \frac{\beta_2^*}{\beta_1} \qquad (8)$$

$$\beta_4 = \int_0^T \beta_3 dt = \int_0^T \frac{\beta_2^*}{\beta_1} dt = \frac{\beta_2^*}{\beta_1} T + const.$$

and

$$P_t^* = \frac{e^{rt}}{M(r)} \beta_1 (\beta_4 - S) \qquad (9)$$

with

$$\frac{\partial q_t^*}{\partial t} = r \frac{e^{rt}}{M(r)}(S - \beta_4) < 0 \quad for\ all\ t$$

$$(10)$$

$$\frac{\partial P_t^*}{\partial t} = r \frac{e^{rt}}{M(r)} \beta_1 (\beta_4 - S) > 0 \quad for\ all\ t$$

Since $\beta_4 > S$ (assuming interior solutions), the equilibrium quantity trajectory is monotonically declining and price is monotonically increasing. Furthermore, $\frac{P'(t)}{P(t)} = r$, that is, price grows at the interest rate.

Case B: Non-zero but intertemporally constant marginal extraction cost

In the slightly more general case where the marginal cost of extraction is some positive and constant value, *C*, rather than zero as assumed above, equations (8) and (9) are only slightly altered. The expression for equilibrium production remains the same, though β_3 would be redefined as $\beta_3 = \frac{\beta_2^* - C}{\beta_1}$ and price refers to net price, i.e., $P_t^* - C$.

Case C: Intertemporally changing marginal extraction costs

In this case, $C'(t) \neq 0$, i.e., the marginal extraction cost may change over time. However, for simplicity, the assumption that it is independent of the quantity extracted in any given period is retained (i.e., the marginal cost curve is horizontal to the quantity axis). In addition, assume that demand for the resource does not vary with changes in population and per capita income. Thus, as in case A, the demand intercept is constant at β_2^*. The equilibrium quantity and price trajectories are:

$$q_t^* = \beta_3 + \frac{e^{rt}}{M(r)}(S - \beta_4)$$

where

$$\beta_3 = \frac{\beta_2^* - C(t)}{\beta_1} \quad (11)$$

$$\beta_4 = \int_0^T \beta_3 dt$$

and

$$P_t^* = C(t) + \frac{e^{rt}}{M(r)}\beta_1(\beta_4 - S) \quad (12)$$

The equilibrium production trajectory is monotonically declining if marginal extraction cost is rising. In the case of falling marginal costs, it would increase and then decrease. This is apparent from equation (12):

$$\frac{\partial q_t^*}{\partial t} = -\frac{C'(t)}{\beta_1} + \frac{re^{rt}}{M(r)}(S - \beta_4)$$

$$
\begin{aligned}
&< 0 \text{ if } C'(t) > 0 \\
&> 0 \text{ if } C'(t) < 0 \text{ and } \left|\frac{C'(t)}{\beta_1}\right| > \left|\frac{re^{rt}}{M(r)}(S - \beta_4)\right| \\
&< 0 \text{ if } C'(t) < 0 \text{ and } \left|\frac{C'(t)}{\beta_1}\right| < \left|\frac{re^{rt}}{M(r)}(S - \beta_4)\right|
\end{aligned}
\qquad (13)[3]
$$

Consider the time derivative of the price trajectory:

$$\frac{\partial P_t^*}{\partial t} = C'(t) + \frac{re^{rt}}{M(r)}\beta_1(\beta_4 - S)$$

$$
\begin{aligned}
&> 0 \text{ if } C'(t) > 0 \\
&< 0 \text{ if } C'(t) < 0 \text{ and } \left|C'(t)\right| > \left|\frac{re^{rt}}{M(r)}\beta_1(\beta_4 - S)\right|
\end{aligned}
\qquad (14)
$$

So long as the marginal extraction cost increases over time, the consumer price follows suit. However, if marginal extraction cost is falling, then it is possible for consumer price to decrease initially while the declining *C(t)* more than offsets the scarcity rent.

1.2. Monopoly

Under a monopoly the basic economic problem is as defined in equation (1). Firms maximize the present value of profits subject to the constraint of the remaining resource stock, and given the demand and cost schedules. However, unlike perfectly competitive firms, a monopolist is a price setter. Therefore, the Hamiltonian for monopoly case is as shown in equation (2), but *without* the accompanying identity. The equilibrium quantity and price trajectories are:

---

[3] Here it is easy to see that β_4 is independent of *t*. $\beta_4 = \int_0^T \frac{\beta_2^* - C(t)}{\beta_1} dt = \frac{\beta_2^*}{\beta_1} T + \overline{C}$ where $\overline{C} = \int_0^T C(t)\, dt$ is a constant term representing the area under the marginal extraction cost curve over the entire production horizon.

$$q_t^m = \frac{1}{2}\beta_3(t) + \frac{e^{rt}}{M(r)}\left(\frac{1}{2}\beta_4 - S\right)$$

$$P_t^m = \frac{1}{2}\beta_2 L_t^{\eta_1} y_t^{\eta_2} + \frac{1}{2}C(t) + \beta_1 \frac{e^{rt}}{M(r)}\left(\frac{1}{2}\beta_4 - S\right) \qquad (15)$$

where β_3 and β_4 are as defined in equation (3), and the superscript m indicates monopoly. Equilibrium marginal revenue is:

$$MR_t^m = \beta_2 L_t^{\eta_1} y_t^{\eta_2} - 2\beta_1 q_t^m$$

$$= C(t) + \frac{e^{rt}}{M(r)}\left(\frac{1}{2}\beta_4 - S\right) \qquad (16)$$

Clearly, in this case, marginal profit, $MR_t^m - C(t)$, rather than price, P_t^m, grows at the interest rate.

2. Reserve Dependent Costs

A very simple, discrete time model may be used to derive the basic result. Consider a perfectly competitive market for a non-renewable resource with reserve-dependent costs. Extraction cost $C_t = C(q_t, R_t)$ is affected by the quantity of the resource extracted in each period, q_t, and by the level of remaining resources, R_t. Per unit prices, P_t, are known. Each firm aims to maximize the net present value of its profits over the production horizon. That is:

$$\text{Maximize} \quad \sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^t \left[P_t q_t - C_t\right]$$

$$\text{subject to} \quad R_{t+1} - R_t = -q_t \qquad (17)$$

where $\quad C_t = C(q_t, R_t)$ such that $\dfrac{\partial C_t}{\partial q_t} > 0$ and $\dfrac{\partial C_t}{\partial R_t} < 0$

and $\quad R_0$ and T are known.

The Lagrangian for the problem is:

$$L = \sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^t \left[P_t q_t - C(q_t, R_t) + \frac{\lambda_{t+1}}{1+r}\left\{-q_t + R_t - R_{t+1}\right\}\right] \qquad (18)$$

where λ_t is the Lagrange multiplier as usually defined and represents the user cost.

Solving the first order conditions for an interior solution yields:

$$\frac{\left(P_t^* - \frac{\partial C_t}{\partial q_t}\right) - \left(P_{t-1}^* - \frac{\partial C_{t-1}}{\partial q_{t-1}}\right)}{\left(P_{t-1}^* - \frac{\partial C_{t-1}}{\partial q_{t-1}}\right)} = r + \frac{\frac{\partial C_t}{\partial R_t}}{P_{t-1}^* - \frac{\partial C_{t-1}}{\partial q_{t-1}}} \qquad (19)$$

This shows that the growth rate in net price, $P_t^* - \frac{\partial C_t}{\partial q_t}$, is less than the interest r. Recall, $\frac{\partial C_t}{\partial R_t} < 0$, that is, extraction cost increases as the level of remaining resources decreases, or, equivalently, as cumulative extraction increases.

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Glossary

Non-renewable resources: resources with no significant natural re-growth over an economically relevant time horizon

User cost: marginal benefit of conserving a non-renewable resource; same as opportunity cost of extraction; based on physical scarcity of the resource relative to the demand for it

Marginal economic cost: sum of marginal extraction cost and user cost of extraction

Scarcity rent: excess of market price over marginal extraction cost; related to user cost; reflects physical scarcity of a non-renewable resource relative to its demand

Backstop: economically feasible substitute for a non-renewable resource; may be renewable or non-renewable

Remaining resources: total amount of resource available for extraction; may include probabilistic extrapolation of resource based on known scientific methods and information on natural processes

Net price: under perfect competition, same as scarcity rent

Marginal profit: excess of marginal revenue over marginal extraction cost; equivalent to scarcity rent under monopoly

Discount rate: inflation-adjusted interest rate used to deflate future monetary flows

Price elasticity of demand: percentage change in quantity demanded in response to a 1 percentage change in price, every thing else held constant. Income elasticity of demand is an analogous concept.