THE PHYSICS OF STARS

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Summary

In this chapter we describe the physics required to understand the structure and evolution of stars. It begins with a description of how we measure a star's luminosity and temperature from its brightness and color. The four basic equations of stellar structure that describe mass, hydrostatic equilibrium, heat transport and energy generation are then derived in one dimension. We consider the various contributions of gas, electron degeneracy and radiation, along with minor corrections to the equation of state. Radiation transport and convective transport of heat are examined in some detail and we explain the various sources of energy generated in a star, paying particular attention to nuclear burning. The stellar atmosphere as a boundary condition to our interior models is briefly discussed and then we go on to describe how stars evolve. Finally we discuss white dwarfs, neutron stars and black holes, along with supernovae as the end points of stellar evolution.

1. Introduction

The physics of stars is in one sense simple because the high interior temperatures preclude the complexities of chemistry. On the other hand every aspect of physics from high energy nuclear to solid state is involved as a star lives out its complex and varied life. Stars have fascinated mankind since ancient times but it is only in the last century that we have begun to fully understand them and only in the last fifty years that we have been able to model their evolution in quantitative detail. Indeed there are many aspects of their evolution that still await elucidation. Here we give a comprehensive overview of the basic physics needed to make stellar models and describe the structure and evolution of typical stars.

2. Observable Quantities

When we look at stars in the night sky they have two apparent properties: they vary in brightness and color. The brightness is assessed in terms of magnitudes. Historically, and we are going back to the ancient Greeks here, stars fall into six magnitude classes. The brightest stars are of first magnitude and the faintest stars visible to the naked eye are sixth magnitude, though these are rarely visible with today's city lights. The eye measures brightness logarithmically so that a star of magnitude 6.0 turns out to be one hundred times fainter than a star of magnitude 1.0. Modern photometry can measure the magnitude of stars extremely accurately and in different wavelength ranges.

But these magnitudes are only apparent. A star can vary in brightness for two reasons. First it may be brighter because it is intrinsically more luminous. Alternatively it might just be brighter because it is close to us. Indeed, in the eighteenth century, the astronomer Sir William Herschel, like Sir Isaac Newton before him, hoped that all stars were of similar intrinsic luminosity so that he might map the Galaxy by taking variations in brightness to indicate variations in distance. By the turn of the century he had recognized that some double stars of very different brightness are gravitationally bound and therefore near neighbors. These he called binary systems. Indeed the Revd. John Michel had already noted that this must be the case simply because there are more close pairs of stars on the sky than would be expected in a random distribution.

Today we know that binary stars are very common. Our nearest neighbor, α-Centauri, is a binary star and has even a third fainter companion known as Proxima Centauri that currently lies on the nearside of the system. In fact the Sun is relatively unusual in not having a companion. About nine out of ten stars have stellar companions. Half of these are close enough to affect one another by tides at some point in their lives. Some, closer still, end up transferring matter between them but this is all another fascinating story. First we must understand how stars evolve in isolation. Today the distances to nearby stars can be determined by accurate trigonometric parallaxes. The motion of the star is measured against the background of distant, apparently immovable, stars and galaxies as the Earth moves around its orbit. Once the distance is known an absolute magnitude can be calculated from the observed apparent magnitude and from this we get an estimate of the luminosity of the star.

The second observable quantity is a star's color. Some stars, such as Betelgeuse, appear red while others, like Sirius, are distinctly blue. The color of a star is related to its surface temperature. The apparent surface or photosphere of a star is the locus of points at which the majority of photons were last emitted or scattered before they began their journey through space to the Earth. Typically the spectrum of radiation emitted by a star is close to that of a black body. The hotter the black body the bluer is the peak in its spectrum.

Thus blue stars are hot while red stars are relatively cool. Another way to determine the surface temperature of a star is to look at the dark lines in its spectrum. These generally occur at wavelengths where an atomic transition of an electron makes the absorption of a photon particularly favorable. The light traveling from the photosphere directly towards us is absorbed and re-emitted in all directions so that the line appears dark against the uninhibited background. Historically spectra were classified by the strength of their hydrogen lines. Those with the strongest hydrogen lines are of type A while those with the weakest are of type M. Hydrogen ionizes at about 10,000K and it is stars of this temperature that have the most prominent hydrogen lines. As the temperature rises fewer and fewer atoms have bound electrons and the lines disappear from the spectra. As the temperature falls the electrons around the hydrogen nuclei become more and more energetically confined to the ground state orbits. This in turn leads to fewer weaker hydrogen lines in the spectra. However lines from the more weakly bound electrons of other atoms and molecular rotation and vibration bands become more prominent. So it is easy to distinguish the very hot O stars from the relatively very cool M stars. The sequence of spectral types from the hottest to the coolest normal stars follows

O B A F G K M.

Once we know the temperature and nature of a star's atmosphere we can relate its absolute magnitude to a bolometric luminosity. This bolometric luminosity L is the total energy radiated by the star per unit time.

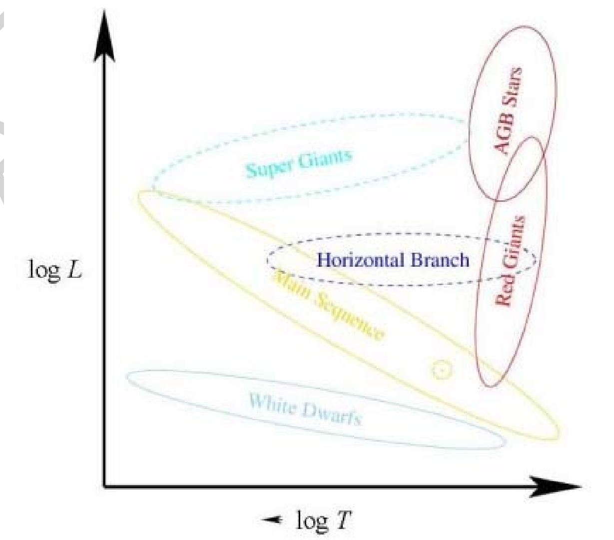

Figure 1. A schematic Hertzsprung-Russell diagram showing the position of stars in a surface temperature-luminosity or color magnitude diagram. Temperature increases

from right to left along the horizontal axis. Color changes from blue to red from left to right. Most stars, like the Sun, lie along the main sequence but other distinct groups of stars are visible, particularly when the diagrams are plotted for clusters of stars.

In the early years of the twentieth century Henry Norris Russell, an American who worked partly in Cambridge at the time, and the Danish astronomer and chemist Ejnar Hertzsprung examined the correlations of these two quantities with each other. The resulting Hertzsprung-Russell diagram has become the major tool for describing the evolution of stars over their lifetimes. Rather than populating the whole of such a diagram we find that most of the stars lie on a band running from hot bright stars to cool faint stars (Fig. 1). The Sun takes its place in one of the most populated parts of this band which is called the main sequence. Because the radiation from stars is very close to a black body the temperature of the photosphere is close to the effective temperature T_{eff} .

The total luminosity L is the energy flux, that depends only on the effective temperature for a blackbody, multiplied by the area of the star.

$$L = 4\pi\sigma R^2 T_{\mathrm{eff}}^4, \qquad (1)$$

where σ is the Stefan-Boltzmann constant and R is the radius of the photosphere. So the loci of stars of constant radius are straight lines of slope -4 in the Hertzsprung-Russell diagram. This means that stars at the top left of the main sequence are blue giants or supergiants while those at the bottom right are red dwarfs. In a diagram of the brightest stars another region to the right and nearly vertically upwards from the main sequence is prominent. These are the red giants. In diagrams of globular clusters this giant branch splits into two distinct parts, the normal red giants and the asymptotic giant branch. We shall see later how these are populated by stars in quite distinct evolutionary phases. In Hertzsprung-Russell diagrams of nearby stars the fainter but relatively common white dwarfs appear in a band below the main sequence. Also discernible as separate, though not so distinct regions, are the supergiants from blue to red across the very top of the diagram and the subgiants between the main sequence and the true red giants.

Many stars are actually members of clusters. Often all the stars in the cluster formed together and so today they have the same age. There are open clusters, such as the Pleiades and Hyades, that are easily found in the northern sky. These tend to contain thousands of stars relatively loosely bound and tend to be rather young. Their age can be determined by the extent of the main sequence because the more massive stars, which are bluer and brighter, use up their energy first and then evolve away (Section 9). Globular clusters, such as 47 Tucanae and the more unusual ω Centauri easily found in the Southern hemisphere, or M13 in the constellation of Hercules in the northern, are much denser and older. They have the advantage that the stars, numbering millions, all lie at approximately the same distance so that relatively, though not absolutely, the errors associated with distance measurements are significantly reduced. Today some very beautiful Hertzsprung-Russell diagrams of globular clusters have been plotted with data from large telescopes and these reveal all sorts of interesting details. Of particular note is the horizontal branch at relatively constant luminosity extending from red to blue across from the red giants. The structure and population of this feature varies considerably from cluster to cluster and contains clues to the age and initial chemical composition of the constituent stars. Because the Sun lies right in the middle of the most populated part of the main sequence we can deduce that it is typical of the majority of stars. In the next sections we shall investigate the physics and the mathematical models that have allowed us to unravel the life of a star as it moves about the Hertzsprung-Russell diagram from the main sequence to the red giant branch, perhaps to the horizontal branch or back to the subgiant area, then on to the asymptotic giant branch and, in the case of the Sun, finally to a white dwarf (Section 11). A white dwarf remnant is left whenever the star has lost enough mass to avoid a supernova explosion (Section 12). More massive stars end their lives as the even more exotic neutron stars or black holes.

3. Structural Equations

The structure of a star can be described remarkably well by a one dimensional model with spherical symmetry. At a radius r from the centre the properties of the star are identical over the surface of the sphere described by r = const. This structure can be described in essence with four differential equations. Two of these, that describe the variation of mass and pressure with radius, can be called the structural equations. They are the subject of this section. With any equation of state that can relate pressure to density they form the building blocks of a stellar model.

The first equation is easily derived by considering a thin shell of mass δm and thickness δr at radius r in the star (Fig. 2). The mass in the shell is just its volume multiplied by the local density $\rho(r)$ and when we take the limit as δr tends to zero we obtain

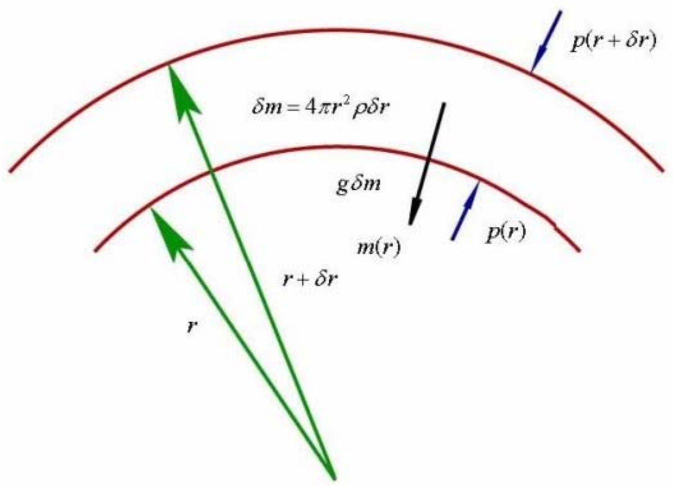

Figure 2. The structure of a fluid sphere. The mass enclosed by a spherical surface of radius r is $m(r)$. A shell of mass δm and thickness δr at this radius is supported against gravity by a pressure gradient.

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{2}$$

the mass equation in which the mass $m = m(r)$ is the mass interior to the sphere of radius r.

The mass inside this shell exerts on it an attractive radial force of magnitude $g\delta m = 4\pi r^2 \rho g \delta r$, where $g(r) = Gm/r^2$ is the local gravitational acceleration. This must be balanced by the difference in the force of the pressure P on either side of the shell $4\pi r^2 \left(P(r+\delta r) - P(r)\right)$. Again in the limit as δr tends to zero we obtain

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}. \quad (3)$$

This is the equation of hydrostatic equilibrium. Equations (2) and (3) are special spherically symmetric cases of the more general equations of mass conservation and the Euler momentum equation of fluid dynamics when the velocity in the fluid is everywhere zero and remains zero.

If we can write P explicitly as a function of ρ only we can obtain a full solution to the structure of the star. The simplest boundary conditions to apply are, at $r = 0$,

$$m(0) = 0 \quad (4)$$

and, because the gravity vanishes at the origin by symmetry,

$$\frac{dP}{dr} = 0 \quad (5)$$

and, at $r = R$,

$$m(R) = M \qquad \rho(R) = 0, \quad (6)$$

where M is the total mass of the star. It turns out that the equation of state of very degenerate matter takes just such a form and white dwarfs (Section 11) can be modeled with just these equations.

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Biographical Sketch

Christopher Tout was born in Billericay, England on 25th April 1964. He was educated at Brentwood School and Emmanuel College, Cambridge. He has a BA (1985), MA (1989) and PhD (1989) in astronomy from the University of Cambridge. His major field of study is stellar evolution.

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Dr Tout is a Fellow of the Royal Astronomical Society. He is a keen cyclist with a general love of the outdoors and the natural world.