EVOLUTION AND LEARNING IN GAMES

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Summary

This article surveys the main developments of Evolutionary Game Theory, a field which originated as a useful tool in the analysis of biological phenomena and then became widely used to model bounded rationality and learning in socioeconomic environments. The discussion first centers on the static concepts, next turns to deterministic dynamics, and finally introduces noise to arrive at stochastic evolutionary processes. In every case, one of the leading concerns is to assess the extent to which evolutionary forces may lead to as-if rational behavior, the latter understood in a variety of more or less stringent senses. In addition, another objective is to shed light on the important issue of equilibrium selection in games; a problem that can be successfully tackled in some interesting cases (e.g. coordination games).

1. Introduction

During the last fifty years, Game Theory has risen to be one of the most versatile and widely used tools in the study of social, economic, or political phenomena. Until quite recently, however, the vastly predominant approach to the analysis of strategic situations (i.e. what could be labeled the "classical" field of Game Theory) has embodied a strong postulate on agents' rationality. Specifically, it has assumed, either implicitly or explicitly, the following two-fold premise:

(i) The context of interaction is perfectly defined in players' minds (albeit, possibly, subject to some exogenous uncertainty).
(ii) Players display virtually unlimited capabilities in reasoning about the situation at

hand. (See *Foundations of Noncooperative Games.*)

However, this stylized theoretical premise is far from representing a suitable description of the way in which much of the social, economic, or political interaction takes place in modern societies. This is certainly so concerning some of their most interesting phenomena, such as learning, technological progress, or institutional change. Nevertheless, one could still defend the methodological approach of classical Game Theory on the basis that it represents a useful (if not realistic) way of predicting or reproducing some of the key features characterizing, say, economic reality. (The classical defense of such an "as if" methodology for economic analysis was undertaken by Milton Friedman in a celebrated essay published in 1953.) Unfortunately, the key (and, arguably, irremediable) problem of such a standpoint is that, by its very nature, it abstracts from those features which are at the core of those phenomena, namely, their complexity and unpredictable variability. The need to open new methodological routes to studying them is thus apparent. (These considerations notwithstanding, it has to be stressed that classical Game Theory has been a very powerful tool to understand many social and economic problems where complexity and ever-present change do not play an essential role. Any modern textbook in the field of Industrial Organization is a forceful proof of this fact.)

A proposal in this vein, which is gaining much strength, goes under the label of *Evolutionary Game Theory*. This theoretical framework for the analysis of games has some old precursors in nineteenth century Economics. (For example, one can already find in Alfred Marshall's *Principles* a quite convincing elaboration on the important analogies between biological and economic processes.) It holds the view that many social and economic processes share with (evolutionary) biological ones some analogies (but only that, analogies) whose suitable exploitation may render useful insights and modeling paradigms. A brief outline of the developments and research agenda of this surging literature is the main objective of the present article.

The presentation is structured as follows. First, I focus on the initial developments of Evolutionary Game Theory, as they were originally motivated and implemented for the study of biological setups. Next, I summarize the different routes pursued by the game-theoretic literature in extending and adapting those original contributions to non-biological contexts. These extensions have led to a substantial enrichment of the biologically induced models at different levels: more general interaction structures, selection dynamics displaying higher flexibility, alternative forms of population renewal, or the introduction of experimentation. Finally, the article closes with a brief description of some of the most recent contributions and a final reassessment of the general methodological point that motivates the socioevolutionary literature.

2. Biological Contexts: A Static Approach

Because of its special simplicity, our discussion starts by focusing on the paradigmatic scenario where a very large number of individuals (say, those belonging to certain biological species) are randomly matched in pairs to play a certain bilateral game. More precisely, we posit that a continuum population (say, with individuals indexed by the points of the interval [0,1]) are randomly matched in pairs to play a symmetric bilateral game with (square) payoff matrix A. In this matrix, the entry a_{ij} ($i, j = 1, 2, ..., n$) simply

reflects the payoffs earned by the individual who adopts the pure strategy s_i when her opponent plays s_j. Later on, this scenario with random pairwise matching, will be extended in two different directions. In Section 5, the context interaction will involve a *finite* population (still matched randomly in pairs). Then, in Section 6, individuals will be supposed to *play the field*, i.e. interact jointly through a single common game.

In biological setups, the different strategies should be conceived as reflecting alternative *inherent* features of the individuals, i.e. they are *not* variable objects of choice. (Naturally, this viewpoint will be modified when concerned with social environments below.) In general, however, we shall admit the possibility that individuals may display mixed strategies. In this case, the interpretation is simply that the underlying (say, behavioral) features are not deterministically fixed but are ex-ante random. Then, the (von Neumann-Morgenstern) expected payoff of an individual who plays the mixed strategy $\sigma = (\sigma_1, \sigma_2, ..., \sigma_n) \in \Delta^{n-1}$ against any other that adopts σ' is given by: (For simplicity, we dispense with any notation for matrix transposition (for example, in s) when no ambiguity can arise.)

$$\sigma A \sigma' = \sum_{i=1}^{n} \sigma_i \left[\sum_{j=1}^{n} a_{ij} \sigma_j' \right] = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \sigma_i \sigma_j', \tag{1}$$

i.e. the probability that any particular payoff a_{ij} materializes for player i is given by the product of the (independent) probabilities with which she chooses si and the opponent chooses s_j.

The central equilibrium concept considered in the evolutionary biological literature is that of Evolutionary Stable Strategy (ESS) proposed by Maynard Smith and Price. (This concept can be readily adapted to contexts where individuals play the field or the population is finite. However, these alternative scenarios pose interesting questions of their own.) Formally, it is defined as follows:

Definition 1

A (mixed) strategy $\sigma \in \Delta^{n-1}$ is an ESS if $\forall \sigma' \neq \sigma$, $\exists \bar{\varepsilon} > 0$ such that if $0 < \varepsilon \leq \bar{\varepsilon}$, $\sigma A [(1 - \varepsilon)\sigma + \varepsilon\sigma'] > \sigma' A [(1 - \varepsilon)\sigma + \varepsilon\sigma']$.

The above definition has a clear biological interpretation. Consider a population originally playing strategy σ in a homogenous fashion (such a population is typically called *monomophic*). The question that underlies the ESS concept can be posed as follows: Can the originally monomorphic population be "invaded" (permanently disturbed) by a small fraction of mutants adopting a different strategy $\sigma' \neq \sigma$? If one identifies the ability to invade with obtaining higher relative payoffs (and therefore, it is assumed, a higher ability to survive and reproduce), the strategy σ is said to be evolutionarily stable if the former question is answered in the negative for any alternative σ' and some maximum threshold $\bar{\varepsilon}$. (Note that, at the time of the attempted invasion, the population configuration confronted by each individual, mutant or not, is $x' = (1 - \varepsilon)\sigma + \varepsilon\sigma'$).

What is the relationship between an ESS and the traditional equilibrium concepts of Game Theory? An instructive answer to this question is derived from the following characterization result.

Proposition 1

Let σ be an ESS. Then:

(i) $\sigma A \sigma \geq \sigma' A \sigma, \forall \sigma' \in \Sigma$;
(ii) $\forall \sigma' \in \Sigma$, $\sigma A \sigma = \sigma' A \sigma \Rightarrow \sigma A \sigma' > \sigma' A \sigma'$.

Reciprocally, if a certain strategy $\sigma \in \Sigma$ satisfies (i) and (ii), then it is an ESS.

By part (i) of the former result, every ESS induces a symmetric Nash equilibrium of the bilateral game with payoff table A. Its part (ii), however, indicates that *not* every symmetric Nash equilibrium induces an ESS. Thus, in this light, one can view the ESS notion as a refinement of Nash Equilibrium.

Let us now consider a simple, and by now classic, illustration of these matters: the *Hawk-Dove game*. There is a large population of a certain species competing for some scarce and indivisible resource (food, territory, etc.) in bilateral encounters. The individuals are matched in pairs

and, in each of these encounters, they can display one of the following two types of behavior (i.e. they must choose one of the following strategies):

*   Aggressive behavior (the "Hawk" strategy H);
*   Peaceful behavior (the "Dove" strategy D).

If the strategy H is identified with the first row (and column) and the strategy D is identified with the second one, the following payoff matrix is postulated:

$$\begin{pmatrix} \frac{V-C}{2} & V \\ 0 & \frac{V}{2} \end{pmatrix} \tag{2}$$

where V is the "fitness" value of the resource, and C is the cost of being defeated in a fight (if both individuals behave aggressively). (As explained below, biological fitness is to be identified with the ability of producing viable o/spring - thus, its magnitude coincides precisely with the number of such o/spring born by the individual in question.)

The interpretation of these payoff magnitudes is as follows. If both individuals adopt H (and thus enter into a fight), both enjoy the same probability (i.e. 1/2) of overcoming the opponent (and therefore obtaining a payoff of V) or being defeated (which involves a negative payoff of $-C$). If both instead adopt D, again the probability of securing the resource is the same for each but, unlike before, the one that is left without it does not incur any cost. Finally, if one of the individuals adopts H whereas the other chooses D, the

former is certain to obtain the resource whereas the latter experiences no cost whatsoever. The most interesting situation arises when the parameters of the situation are such that $V < C$. In this case, no *pure* strategy can be ESS. However, if we extend our consideration to mixed strategies, it is easy to check that

$$\sigma = (\sigma_1, \sigma_2) = (V/C, 1 - V/C) \tag{3}$$

is an ESS (in fact, the unique one). That is, a "probabilistic mixture" between aggressive and peaceful behavior (the later being more likely the higher is the cost *C*) turns out to be evolutionarily stable in the sense of Definition 1.

Despite its intuitive motivation, the ESS concept is afflicted by a number of important problems. The first, very basic one is that no ESS exists in a wide variety of non-pathological contexts. A second problem is of a more conceptual nature: by definition, the ESS concept restricts to *monomorphic* configurations where every individual adopts the same strategy (possibly mixed). That is, it does not contemplate the possibility that a truly polymorphic population might provide the heterogeneous balances required for an evolutionary stable situation to prevail. In view of these two significant drawbacks, theoretical biologists have often chosen to forgo the *static* (i.e. "equilibrium") analysis implicitly reflected by the ESS concept. The alternative must then be an explicitly dynamic analysis, as presented in the next section.

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Biographical Sketch

Fernando Vega-Redondo is Professor of Economics at the University of Alicante (Spain) and at the University of Essex (United Kingdom). He obtained his Ph.D from the University of Minnesota in 1984. Since then, he has held visiting positions at different universities, including the Hebrew University at Jerusalem, Harvard University, and the University of California at San Diego. His recent research has focused on models of learning, bounded rationality, and technological change. He has published a number of books and about fifty professional articles on these and other issues.