MATHEMATICAL MODELS IN INPUT-OUTPUT ECONOMICS

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Contents

Summary

This chapter describes the mathematical basis for input-output economics, the major types of models, and the underlying economic theory. The features of these models that make them especially well suited for understanding the connections between the economy and the environment are emphasized throughout. These include the dual physical and price representations and the representation of resource inputs as factors of production, whether they are priced or not.

The basic static physical and price models are described, along with their major properties and associated databases. The most important approaches to analysis involve multipliers, decomposition, and scenario analysis. Going beyond the basic static framework requires the progressive closure of the model by making exogenous variables endogenous while maintaining simplicity, transparency, and the distinctive feature of an input-output model: the simultaneous determination of solutions at the sectoral level and the economy-wide level. Closures for household activities and for investment are described by way of example.

The major extensions of the basic model accommodate the representation of pollutant emissions and policies for constraining them, dynamic models, and multi-regional models, the latter including a new version of a world model that solves for bilateral trade flows and region-specific prices based on comparative advantage with factor constraints. The concluding section describes the challenges currently being addressed within the field. An annotated bibliography provides references for further reading and includes both classic articles and active research areas.

1. The Basic Static Input-Output Model

1.1. Introduction

Economists are concerned with promoting innovation, and achieving efficiency by reducing production costs, in order to maximize the prospects for growth, profits, and increased consumption. They portray the economy in terms of a circular flow of income between producers and consumers: producers pay incomes to workers, and workers use their income to buy goods and services. Goods are assumed to move around the circle in the opposite direction from the money flows.

While stylized, this image accurately conveys the duality between the systems of physical flows and of money values that constitute an economy. However, while the money flows indeed stay within the economic system, producing the physical flows requires inputs of resources from the environment and discharges wastes into the environment. Typically the role of the environment is ignored by economists: resources are treated as "free gifts of nature," and wastes are called "externalities," meaning that they are external to the economic system.

Input-output economics shares with other conceptual approaches to economics a concern for achieving efficiency in the use of resources to produce goods. However, input-output economics also accommodates other objectives, not only conceptually but also operationally: it differs from other schools of thought in not *imposing* growth and efficiency objectives on its mathematical models. Both resource inputs and wastes generated are standard variables in an input-output analysis that are explicitly represented in equations, whether they have prices or not. These features, while not always fully exploited, make input-output economics especially suitable for environmental analysis.

In the 1930s Wassily Leontief published a pair of articles that laid the groundwork for input-output economics. The first article described the design and construction of what he called a Tableau Economique of the United States for the year 1919, the direct precursor of today's input-output table with its distinctive focus on the inter-industry transactions until then essentially ignored by economists. The second article was in two parts. First came the theoretical framework, describing the interdependence of the different parts of an economy by a set of mathematical equations intended for manipulating this new kind of table, followed by an empirical implementation of the mathematical model. While the table had been compiled in terms of 41 sectors, it was for practical reasons aggregated to only 10 and, even so, the results had to be approximated because it was not then feasible to compute the inverse of a (10x10) matrix. The close integration of model and database remains a defining characteristic of input-output economics, even as both have been extended into new domains.

The order of the publications emphasized the fundamental role accorded to the table, comprised as it still is today of a square inter-industry portion, recording the flows of deliveries from each industry to every other one, supplemented by additional rows representing other inputs, namely labor and capital or, on occasion, resources such as water and land. The latter are designated factors of production, or factor inputs, to indicate that unlike other inputs they are not produced in any industry (or, in the case of capital goods, their capacity cannot be expanded in a single production period). The table is completed by additional columns that represent deliveries from industries to households and other final users; these are called final deliveries or final demand. The resulting table is rectangular, since the number of final demand categories is in general not equal to the number of factors of production. This rectangular table records all transactions taking place in the economy in a specific period of time. The capacity to absorb a substantial level of detail, and the conceptual simplicity and transparency of the framework, make the input-output table and the models that manipulate it well suited to evaluating strategies for sustainable development.

The reference in Leontief's article is to the 18th century *Tableau Economique* of François Quesnay, which depicted the flows of income among landlords, manufacturers (artisans), farmers and other agricultural workers, and merchants derived from the sale of agricultural goods and fabricated products. By contrast, a contemporary input-output flow table distinguishes dozens if not hundreds of sectors producing goods and services in a modern economy.

Input-output tables are compiled in many countries by official statistical offices, specialized official or semi-official institutes such as national banks or universities, private companies, or individual researchers. They use several methods to build input-output tables, all of which share characteristics associated with the System of National Accounts. These guidelines guarantee internal consistency of the tables, consistency with widely used national aggregates such as gross national product, and comparability among tables representing different economies. International organizations such as the UN, OECD (previously OEEC) or Eurostat have played an important role in issuing guidelines for the national accounts, including for input-output tables.

The square portion of the input-output table has n rows and n columns, and the figure in the i-th row and j-th column represents the amount of product from industry i delivered to industry j in a particular calendar year. The result of dividing that quantity by the total output of industry j is a coefficient measuring input per unit of output. In this way the $n \times n$ portion of the flow table is converted to an $n \times n$ matrix of coefficients, of which the entries in the j-th column include (when supplemented by the j-th column of factor inputs per unit of output) all inputs needed to produce one unit of output of industry j. This column of coefficients is said to represent the average technology in use in industry j. For simplicity it is assumed that every industry, or sector, is associated with a single characteristic output produced using a single average technology.

Call the $(n \times n)$ matrix of inter-industry coefficients $\mathbf{A}$, the $(n \times 1)$ vector of outputs $\mathbf{x}$, and the likewise $(n \times 1)$ vector of final deliveries $\mathbf{y}$, while $\mathbf{F}$ is the $k \times n$ matrix of factor inputs per unit of output (one row for each of k factors) and total factor use is the vector $\mathbf{f}$. Then the basic static input-output model states that:

$(\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{y}$, or

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} \text{ and} \qquad (1)$$

$$\mathbf{f} = \mathbf{F}\mathbf{x} \qquad (2)$$

where the inverse matrix $(\mathbf{I} - \mathbf{A})^{-1}$ has been called the Leontief inverse. It is also known as the multiplier matrix or matrix of multipliers. (Because the economy needs to produce a larger amount of a specific good than the amount of final demand for that good, final demand $\mathbf{y}$ needs to be ‘multiplied’ to obtain $\mathbf{x}$.)

Equations (1) and (2) are called a quantity input-output model, and the corresponding parameters (the coefficients in the $\mathbf{A}$ and $\mathbf{F}$ matrices) are ratios of physical units such as tons of plastic per computer (dollars’ worth being one special case of a quantity). If $\mathbf{y}$ is given, the solution vector $\mathbf{x}$ represents the quantities of sectoral outputs. Equations (1) and (2) comprise the basic static input-output model. The following sections of this chapter describe extensions to that model and the kinds of questions the extended models have been designed to answer.

1.2. Reasons for Popularity

By the end of the 20th century, the basic static input-output model had been used extensively in empirical economic analysis. More recently, the special appeal of this approach for environmental analysis has become apparent for reasons that will be discussed below. In the typical case, a rectangular input-output flow table compiled by a statistical office for some past year is available as the starting point for deriving the coefficient matrices, $\mathbf{A}$ and $\mathbf{F}$, and Eqs. (1) and (2) are used to compute the impact on outputs ($\mathbf{x}$) or on employment (a component of $\mathbf{f}$), of alternative hypothetical assumptions about changes in input coefficients ($\mathbf{A}$ and $\mathbf{F}$) or in final deliveries ($\mathbf{y}$). When there is no table, the matrices can be estimated directly from technological information or by starting from matrices for a similar economy. The fact that input-output tables are widely available makes the model easy to implement and, in turn, the popularity of the model has encouraged the production of tables that are increasingly detailed and frequent as well as more comprehensive in their coverage, notably including environmental data classified in categories that are compatible with input-output accounts.

Two important assumptions underlie the model given by Eqs. (1) and (2). First, output is a linear function of final demand. Second, in the absence of exogenous assumptions to the contrary, the input coefficients of matrices $\mathbf{A}$ and $\mathbf{F}$ remain constant for variations in $\mathbf{y}$. As a consequence of these assumptions, if final demand for all commodities increased by 10%, total required outputs from each sector would also increase by this same percentage. A rationale for the latter, so-called fixed coefficients assumption is that the columns of $\mathbf{A}$ (and the corresponding columns of $\mathbf{F}$) represent the most efficient technologies (or production functions) available to produce each good, and they are assumed to remain the optimal ones even if there are variations in the composition of final demand. A more realistic rationale is that, while these technologies are not necessarily optimal, they are the ones effectively in place and cannot be quickly changed given the existing stock of fixed capital. In either case, the time period during which real-world technologies will in fact remain the same is limited because new technologies may become available over time and new fixed capital can be put in place. However, for a certain period of time the coefficients can be expected to remain more or less unchanged, and the model can be used to compute the required change in $\mathbf{x}$ even if $\mathbf{y}$ changes. (We shall come back to the subjects of linearity and fixed coefficients, starting with Sections 1.2 and 1.5).

The power of this simple model resides in the fact that, while an entry of $\mathbf{A}$ quantifies a relation between only two industries, each element of the inverse matrix reflects the interdependencies among all industries comprising the economic system. The solution to the basic static model is obtained by deriving the inverse matrix and applying it to the vector of final deliveries. Thus if one element of final deliveries changes, say final demand for cars is cut in half, the model can compute the implications for not only the output of cars but also for the outputs of all other industries, namely steel to make the cars, coal to produce the steel, energy to extract the coal, and so on. This ability to capture indirect effects is one reason for the model's popularity and for the fact that it is incorporated in virtually all empirical economic models that distinguish a sectoral level of detail, including in particular the more elaborate input-output models that will be described below.

For empirical applications, the input-output database is as important as the mathematical model. Most analyses start from tables in money units and devote a great deal of effort to making sure that the total money value of each row is equal to that of the corresponding column before deriving a coefficient matrix. By contrast, Leontief stressed the technological interpretation of each column of coefficients and urged collaboration of economists with engineers and other technological experts to project, column by column, coefficient matrices representing hypothetical changes in technologies in different industries based on information in physical units, such as that developed for the use phase of life-cycle engineering studies. The episodic collaboration of input-output economists with engineers is of long standing, but it has surged and matured dramatically, and is being fostered by several interdisciplinary professional societies, as concerns about the environment have deepened and industrial ecologists and other environmental scientists and engineers have sought to evaluate not only the direct effects but the full, economy-wide impact of alternative technologies governing the use of energy and materials. Input-output models manipulate data in both physical and money units. They capture the direct as well as the indirect environmental impacts of alternative products or processes on the basis of the explicit representation of physical stocks and flows of energy and materials, measured in physical units, through an entire economic system. This is the reason for their value for industrial ecologists.

1.3. Physical Model and Price Model

The equation $\left(\mathbf{I}-\mathbf{A}\right)\mathbf{x}=\mathbf{y}$, with all variables measured in money units, is used in the vast majority of empirical input-output studies. However, industrial ecologists have incorporated into the input-output database data in physical units representing material flows, inputs and outputs of manufacturing processes, and product life cycles. To fully exploit the power of the model coupled with such a database for evaluating environmental impacts, it is vital to sharpen a few distinctions that often remain unclear. Goods and services were distinguished from resource inputs in Eqs. (1) and (2), respectively. In this section, quantities of goods and services are distinguished from their unit prices, and prices of goods and services from prices of resources and other factors of production. This framework separately tracks all of these quantities and accommodates resource commodities that are priced, either in market prices or through legislation such as carbon fees, as well as inputs or outputs that are not priced, such as fresh water or the discharge of pollutants, as seen below.

Assume in Eq. (1) that each industry's output is measured in a unit appropriate for that sector, such as steel and plastics in tons, electricity in kWh, and computers and automobiles in numbers of standard units (e.g., number of computers of average capability). Some service sector output may be measured in a physical unit, such as the number of visits to a doctor's office; but other sectors may be measured in the money value of output, say dollars' worth of business services. A mixed-unit flow table accommodates variables measured in different units and can be constructed with no conceptual difficulty. In the coefficient matrix $\mathbf{A}$ derived from such a flow table, the ij-th element is equal to the ij-th element of the flow table divided by the j-th row total. (Note that it makes no sense to calculate the j-th column total in a mixed-unit table.) A mixed-unit $\mathbf{A}$ matrix may instead be constructed directly as columns of coefficients.

Equations (1) and (2) comprise an abbreviated form of the basic input-output model. The simplest form of the full model involves two additional equations (where (.)′ indicates transposition):

$$\mathbf{p}'(\mathbf{I} - \mathbf{A}) = \mathbf{v}' = \boldsymbol{\pi}' = \boldsymbol{\pi}'\mathbf{F}, \text{ or}$$

$$\mathbf{p}' = \mathbf{v}'(\mathbf{I} - \mathbf{A})^{-1} = \boldsymbol{\pi}'\mathbf{F}(\mathbf{I} - \mathbf{A})^{-1} \tag{3}$$

$$\mathbf{p}'\mathbf{y} = \mathbf{v}'\mathbf{x} = \boldsymbol{\pi}'\mathbf{F}\mathbf{x} \tag{4}$$

In Eqs. (3) and (4), $\mathbf{p}$ is the vector of unit prices and $\mathbf{v}$ is value-added, that is, the total money value of factor inputs per unit of output. If inputs of individual factors are measured in physical units in the corresponding rows of the $\mathbf{F}$ matrix, such as number of workers or hectares of land per unit of sectoral output, and the unit factor prices are specified by the vector $\boldsymbol{\pi}'$, then value added, $\mathbf{v}'$, can be substituted by $\boldsymbol{\pi}'\mathbf{F}$ in Eqs. (3) and (4).

Equation (3) is the basic static input-output price model, and the components of the vector of unit prices are price per ton of plastic, price per computer, etc. Equation (3) shows the unit price of a good as the sum of the amounts paid out to each one of the factors of production. For a sector whose output is measured in dollars in Eq. (1), for example financial services, the corresponding unit price is simply 1.0. With this equation one can compute the impact on prices of changes in technical coefficients ($\mathbf{A}$) or in the quantity or price of factors ($\mathbf{F}$ or $\boldsymbol{\pi}'$), or value-added ($\mathbf{v}'$), per unit of output. Finally, Eq. (4), called the income equation, is derived from Eqs. (1) and (3): this identity (the GDP identity) assures that the value of final deliveries is equal to total value-added (the value of all factor inputs), not only in the base-year situation for which the data have been compiled but also under scenarios where values of parameters and exogenous variables are changed.

It generally escapes notice that Eqs. (1) and (2) have the attributes of a quantity model when, as is most frequently the case, the outputs of all sectors and even the quantities of factor inputs are measured in money units. One component of the output vector, for example, would be the value of the output of plastic or steel, each figure being the implicit product of a quantity and a unit price, but with inadequate information to distinguish the quantity from the price. Under these circumstances, there is no perceived benefit from a separate price model: all elements of the price vector in Eq. (3) would be 1.0, and the price model is therefore deemed to be trivial. This is an incorrect assessment, however, since the price model can be used to calculate changes in prices associated with changes in $\mathbf{A}$ or $\mathbf{v}'$ (e.g., improved efficiency of energy use might result in an 8% reduction in the price of a dollar's worth of business services from 1.0 to 0.92). When the variables of the quantity model are measured in physical units, the calculated prices are in money values per physical unit.

1.4. Properties of Nonnegative Matrices

To realize the full potential of input-output analysis, insight into the properties of the model given by equations (1) to (4) is needed, in particular into their economic interpretation. The properties of the input coefficients in the $\mathbf{A}$ matrix play a central role. Consider the case where $\mathbf{A}$ has been derived from a flow table by dividing the elements in each column by the appropriate total output. Clearly, if all data in the square part of the table are nonnegative, the result is a square, nonnegative matrix where all column sums are smaller than unity. But suppose $\mathbf{A}$ was obtained from a compilation of engineering data in physical units. Which properties does that matrix need to possess in order to satisfy the requirements of an input-output coefficient matrix? What is needed is a general theory for the equation system (1) to (4). The theory of nonnegative matrices provides the needed foundation.

Much attention has been given to conditions that guarantee that the multiplier matrix (or Leontief inverse) is strictly positive, i.e., that each element is positive. Such conditions make sense because basic economic logic would require that an increase $\Delta\mathbf{y} > \mathbf{0}$ in final demand in Eq. (1) should result in an increase $\Delta\mathbf{x} > \mathbf{0}$ in total output. If the matrix $(\mathbf{I}-\mathbf{A})^{-1}$ were not strictly positive, this logic could be violated. We can also formulate this result differently, i.e., as an answer to the question whether Eq. (1) always has a solution $\mathbf{x} > \mathbf{0}$ for $\mathbf{y} > \mathbf{0}$. In fact, the study of Eq. (1) has led to a number of equivalent statements about $\mathbf{A}$, of which:

1. $(\mathbf{I}-\mathbf{A})^{-1} > \mathbf{0}$.

2. $(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\ldots$ (That is, the series $\sum \mathbf{A}^k$ is convergent.)

3. All successive principal minors of $(\mathbf{I}-\mathbf{A})^{-1}$ are positive.

4. There exists a choice of units such that all row sums or all column sums of $\mathbf{A}$ are smaller than unity.

5. A has a dominant eigenvalue λ where $0 < \lambda < 1$.

1. A related result is:

6. The dominant eigenvalue λ of $\mathbf{A}$ gets larger if one element of $\mathbf{A}$ is increased, and λ gets smaller if one element of $\mathbf{A}$ is decreased.

Statement 2 is important for distinguishing the industries contributing output in different phases of production. It says that output $\mathbf{x}=\mathbf{y}+\mathbf{Ay}+\mathbf{A}(\mathbf{Ay})+\ldots$. So the quantity y must be produced, plus Ay which is the vector of input to produce y, etc. Statement 3 is the well-known Hawkins-Simon condition, which assures that each subsystem is productive; that is, each subgroup of industries $i, j, k, \ldots$ requires less input from the economic system than it produces in terms of outputs. According to statement 4, the

Brauer-Solow condition, value-added in each sector is positive in coefficient matrices derived from input-output tables in (nominal) money values. That is, units for measuring physical output are such that each one costs one monetary unit (thus, if the output unit is dollars, the unit price is 1.0 by definition). Assuming that the matrix describes a viable economy, this property assures that if output is measured in any chosen physical units, there exists a set of prices such that each industry has a positive value-added (i.e., revenue left to pay for factor inputs).

The dominant eigenvalue λ is a measure of the size of the intermediate outputs produced in the economy in relation to total production. That is, λ indicates the net surplus of an economy in the sense that the larger λ (within the bounds described by statement 5), the smaller the net output. The surplus so defined can be consumed, invested for growth, devoted to environmental protection, etc. Statement 6 is useful for interpreting the role of technological change. For example, a technological innovation that reduces the need for certain intermediate inputs results in a lower dominant eigenvalue for the new coefficient matrix, leaving more surplus. Innovations that are not cost-reducing, on the other hand, will result in a larger λ. An example might be more secure disposal of hazardous wastes; see also Section 3.1. Input-output analysis can effectively identify those industries where increased technological efficiency would have a significant economy-wide impact. Thus, λ is a kind of efficiency indicator in that of two matrices describing two different economies, the one with a larger dominant eigenvalue represents the economy that is less efficient economically although it may have other desirable features. Eigenvalues also play an important role in dynamic models, where they have an interpretation in terms of rates of growth or contraction and profit rates (see Section 3.2).

If the economy does not produce a surplus (i.e., $\mathbf{y} = \mathbf{0}$ in Eq. (1)), we are dealing with a closed model of the following form,

$$\mathbf{x} = \mathbf{M}\mathbf{x} \qquad (5)$$

In this special case, $\mathbf{M}$ has a dominant eigenvalue equal to unity, and total output $\mathbf{x}$ is the Perron-Frobenius eigenvector of $\mathbf{M}$. Solving the model for $\mathbf{x}$ thus means solving for this eigenvector. The solution provides only the production proportions; the scale has to be determined in other ways, such as external knowledge about the size of certain elements of $\mathbf{x}$.

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Biographical Sketches

Faye Duchin has been active for many years in developing alternative scenarios about plausible ways to address global social and environmental problems and models and databases for analyzing them. The scenarios are mainly about technological options and household lifestyle decisions. Modeling results describe outcomes including resource use, pollution emissions, incomes and income distribution, and patterns of international trade. Most of this work requires a team of researchers, and she is engaged increasingly with interdisciplinary teams that are geographically dispersed.

Faye Duchin is a professor of economics at Rensselaer Polytechnic Institute in Troy, New York, and was previously at New York University, where she collaborated for many years with Wassily Leontief and succeeded him as Director of the Institute for Economic Analysis. She is involved in building new research networks through leadership positions in several interdisciplinary professional societies and scholarly journals. At the present time she is especially active in the integration of input-output economics and industrial ecology to address challenges of sustainable development, mainly input-output models of economies and engineering-based life cycle analysis. She is a Visiting Professor in the Program in Industrial Ecology at the Norwegian University of Science and Technology.

Albert E. (Bert) Steenge studied economics and econometrics at the University of Groningen, the Netherlands. After graduating in 1970, he became a staff member at the Econometric Institute of this University. During that time he became familiar with input-output (I-O) analysis. It somehow seemed to offer the 'right' level of abstraction for policy and structure to meet. In addition, the long intellectual history of I-O provided much insight and inspiration.

In 1986, he was appointed Professor of Economics at the University of Twente in Enschede (Netherlands), teaching a wide range of courses. In addition, he published extensively on mathematical aspects of I-O analysis, stability issues, environmental economics, and various special topics. Lately he has been working on the introduction of institutional aspects of production and consumption in a multi-sectoral setting. Other recent work includes the economics of catastrophic flooding and scenario analysis. He had the opportunity to edit or co-edit special issues of economic journals or books, including acting as co-editor of the joint German-Dutch series "Wirtschaft: Forschung und Wissenschaft," in which the proceedings of a special German-Dutch workshop are published annually. He is on the Board of Managing Editors of Structural Change and Economic Dynamics and a member of the Editorial Board of Economic Systems Research.

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