AN INTRODUCTION TO QUANTUM GRAVITY

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Contents

Summary

Quantum gravity was born as that branch of modern theoretical physics that tries to unify its guiding principles, i.e., quantum mechanics and general relativity. Nowadays it is providing new insight into the unification of all fundamental interactions, while giving rise to new developments in mathematics. The various competing theories, e.g. string theory and loop quantum gravity, have still to be checked against observations. We review the classical and quantum foundations necessary to study field-theory approaches to quantum gravity, the passage from old to new unification in quantum field theory, canonical quantum gravity, the use of functional integrals, the properties of gravitational instantons, the use of spectral zeta-functions in the quantum theory of the universe, Hawking radiation, some theoretical achievements and some key experimental issues.

1. Introduction

The aim of theoretical physics is to provide a clear conceptual framework for the wide variety of natural phenomena, so that not only are we able to make accurate predictions to be checked against observations, but the underlying mathematical structures of the world we live in can also become sufficiently well understood by the scientific community. What are therefore the key elements of a mathematical description of the physical world? Can we derive all basic equations of theoretical physics from a set of symmetry principles? What do they tell us about the origin and evolution of the universe? Why is gravitation so peculiar with respect to all other fundamental interactions?

The above questions have received careful consideration and have led, in particular, to several approaches to a theory aimed at achieving a synthesis of quantum physics on the one hand, and general relativity on the other. This remains, possibly, the most important task of theoretical physics. In the early work of the 1930s, Rosenfeld [131,132] computed the gravitational self-energy of a photon in the lowest order of perturbation theory, and obtained a quadratically divergent result. With hindsight, one can say that Rosenfeld's result implies merely a renormalization of charge rather than a non-vanishing photon mass [40]. A few years after Rosenfeld's papers [131,132], Bronstein realized that the limitation posed by general relativity on the mass density radically distinguishes the theory from quantum electrodynamics and would ultimately lead to the need to reject Riemannian geometry and perhaps also to reject our ordinary concepts of space and time [20,135].

Indeed, since the merging of quantum theory and special relativity has given rise to quantum field theory in Minkowski spacetime, while quantum field theory and classical general relativity, taken without modifications, have given rise to an incomplete scheme such as quantum field theory in curved spacetime [65], which however predicts substantially novel features like Hawking radiation [87,88], here outlined in Section 7, one is led to ask what would result from the "unification" of quantum field theory and gravitation, despite the lack of a quantum gravity phenomenology in earth-based laboratories. The resulting theory is expected to suffer from ultraviolet divergences [157], and the 1-loop [94] and 2-loop [74] calculations for pure gravity which are outstanding pieces of work. As is well described in Ref. [157], if the coupling constant of a field theory has dimension mass^d in $\hbar = c = 1$ units, then the integral for a Feynman diagram of order N behaves at large momenta like $\int p^{A-Nd} dp$, where A depends on the physical process considered but not on the order N. Thus, the "harmful" interactions are those having negative values of d, which is precisely the case for Newton's constant G, where $d = -2$, since $G = 6.67 \times 10^{-39} \text{GeV}^{-2}$ in $\hbar = c = 1$ units. More precisely, since the scalar curvature contains second derivatives of the metric, the corresponding momentum-space vertex functions behave like p^2, and the propagator like p^{-2}. In d dimensions each loop integral contributes p^d, so that with L loops, V vertices and P internal lines, the superficial degree D of divergence of a Feynman diagram is given by [53]

$$D = dL + 2V - 2P. \tag{1}$$

Moreover, a topological relation holds:

$$L = 1 - V + P, \tag{2}$$

which leads to [53]

$$D = (d-2)L + 2. \tag{3}$$

In other words, D increases with increasing loop order for $d > 2$, so that it clearly leads to a non-renormalizable theory.

A quantum theory of gravity is expected, for example, to shed new light on singularities in classical cosmology. More precisely, the singularity theorems prove that the Einstein theory of general relativity leads to the occurrence of spacetime singularities in a generic way [86]. At first sight one might be tempted to conclude that a breakdown of all physical laws occurred in the past, or that general relativity is severely incomplete, being unable to predict what came out of a singularity. It has been therefore pointed out that all these pathological features result from the attempt of using the Einstein theory well beyond its limit of validity, i.e. at energy scales where the fundamental theory is definitely more involved. General relativity might be therefore viewed as a low-energy limit of a richer theory, which achieves the synthesis of both the basic principles of modern physics and the fundamental interactions in the form currently known.

So far, no less than 16 major approaches to quantum gravity have been proposed in the literature. Some of them make a direct or indirect use of the action functional to develop a Lagrangian or Hamiltonian framework. They are as follows.

1. Canonical quantum gravity [16,17,43,44,32,99,100,6,54,144].
2. Manifestly covariant quantization [116, 33, 94, 74, 7, 152, 21, 103].
3. Euclidean quantum gravity [68, 90].
4. R-squared gravity [142].

5. Supergravity [64,148].
6. String and brane theory [162, 98, 10].
7. Renormalization group and Weinberg's asymptotic safety [129,106].
8. Non-commutative geometry [26, 75].

   Among these 8 approaches, string theory is peculiar because it is not field-theoretic, spacetime points being replaced by extended structures such as strings.

   A second set of approaches relies instead upon different mathematical structures with a more substantial (but not complete) departure from conventional pictures, i.e.

9. Twistor theory [122,123].
10. Asymptotic quantization [67, 5].
11. Lattice formulation [114, 22].
12. Loop space representation [133,134,136,145,154].
13. Quantum topology [101], motivated by Wheeler's quantum geometrodynamics [159].
14. Simplicial quantum gravity [72, 1, 109, 2] and null-strut calculus [102].
15. Condensed-matter view: the universe in a helium droplet [155].
16. Affine quantum gravity [105].

After such a concise list of a broad range of ideas, we hereafter focus on the presentation of some very basic properties which underlie whatever treatment of classical and quantum gravity, and are therefore of interest for the general reader rather than (just) the specialist. He or she should revert to the above list only after having gone through the material in Sections 2–7.

2. Classical and Quantum Foundations

Before any attempt to quantize gravity we should spell out how classical gravity can be described in modern language. This is done in the subsection below.

2.1. Lorentzian Spacetime and Gravity

In modern physics, thanks to the work of Einstein [51], space and time are unified into the spacetime manifold (M, g), where the metric g is a real-valued symmetric bilinear map

$$g : T_p(M) \times T_p(M) \to \mathbf{R}$$

of Lorentzian signature. The latter feature gives rise to the light-cone structure of spacetime, with vectors being divided into timelike, null or spacelike depending on whether $g(X, X)$ is negative, vanishing or positive, respectively. The classical laws of nature are written in tensor language, and *gravity is the curvature of spacetime*. In the theory of general relativity, gravity couples to the energy-momentum tensor of matter through the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}. \qquad (4)$$

The Einstein–Hilbert action functional for gravity, giving rise to Eq. (4), is diffeomorphism-invariant, and hence general relativity belongs actually to the general set of theories ruled by an infinite-dimensional [31] invariance group (or pseudo-group). With hindsight, following DeWitt [39], one can say that general relativity was actually the first example of a non-Abelian gauge theory (about 38 years before Yang–Mills theory [164]).

Note that the spacetime manifold is actually an equivalence class of pairs (M, g), where two metrics are viewed as equivalent if one can be obtained from the other through the action of the diffeomorphism group $\mathrm{Diff}(M)$. The metric is an additional geometric structure that does not necessarily solve any field equation.

2.2. From Schrödinger to Feynman

Quantum mechanics deals instead, mainly, with a probabilistic description of the world on atomic or sub-atomic scale. It tells us that, on such scales, the world can be described by a Hilbert space structure, or suitable generalizations. Even in the relatively simple case of the hydrogen atom, the appropriate Hilbert space is infinite-dimensional, but finite-dimensional Hilbert spaces play a role as well. For example, the space of spin-states of a spin-s particle is $\mathbf{C}^{2s+1}$ and is therefore finite-dimensional. Various pictures or formulations of quantum mechanics have been developed over the years, and their key elements can be summarized as follows:

(i) In the *Schrödinger picture*, one deals with wave functions evolving in time according to a first-order equation. More precisely, in an abstract Hilbert space $\mathcal{H}$, one studies the Schrödinger equation

$$\mathrm{i}\hbar \frac{d\psi}{dt} = \hat{H}\psi, \qquad (5)$$

where the state vector ψ belongs to $\mathcal{H}$, while $\hat{H}$ is the Hamiltonian operator. In wave mechanics, the emphasis is more immediately put on partial differential equations, with the wave function viewed as a complex-valued map $\psi : (x,t) \rightarrow \mathbf{C}$ obeying the equation

$$\mathrm{i}\hbar \frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + V \right)\psi, \qquad (6)$$

where $-\Delta$ is the Laplacian in Cartesian coordinates on $\mathbf{R}^3$ (with this sign convention, its symbol is positive-definite).

(ii) In the *Heisenberg picture*, what evolves in time is instead the operators, according to the first-order equation

$$\mathrm{i}\hbar \frac{d\hat{A}}{dt} = [\hat{A}, \hat{H}]. \qquad (7)$$

Heisenberg performed a quantum mechanical re-interpretation of kinematic and mechanical relations [93] because he wanted to formulate quantum theory in terms of observables only.

(iii) In the *Dirac quantization*, from an assessment of the Heisenberg approach and of Poisson brackets [41], one discovers that quantum mechanics can be made to rely upon the basic commutation relations involving position and momentum operators:

$$[\hat{q}^j, \hat{q}^k] = [\hat{p}_j, \hat{p}_k] = 0, \tag{8}$$

$$[\hat{q}^j, \hat{p}_k] = \mathrm{i}\hbar \delta^j_k. \tag{9}$$

For generic operators depending on $\hat{q}, \hat{p}$ variables, their formal Taylor series, jointly with application of (8) and (9), should yield their commutator.

(iv) *Weyl quantization*. The operators satisfying the canonical commutation relations (9) cannot be both bounded [57], whereas it would be nice to have quantization rules not involving unbounded operators and domain problems. For this purpose, one can consider the strongly continuous 1-parameter unitary groups having position and momentum as their infinitesimal generators. These read as $V(t) \equiv \mathrm{e}^{\mathrm{i}t\hat{q}}$, $U(s) \equiv \mathrm{e}^{\mathrm{i}s\hat{p}}$, and satisfy the Weyl form of canonical commutation relations, which is given by

$$U(s)V(t) = \mathrm{e}^{\mathrm{i}st\hbar} V(t)U(s). \tag{10}$$

Here the emphasis was, for the first time, on group-theoretical methods, with a substantial departure from the historical development, that relied instead heavily on quantum commutators and their relation with classical Poisson brackets.

(v) *Feynman quantization* (i.e., Lagrangian approach). The Weyl approach is very elegant and far-sighted, with several modern applications [57], but still has to do with a more rigorous way of doing canonical quantization, which is not suitable for an inclusion of relativity. A spacetime approach to ordinary quantum mechanics was instead devised by Feynman [62] (and partly Dirac himself [42]), who proposed to express the Green kernel of the Schrödinger equation in the form

$$G[x_f, t_f; x_i, t_i] = \int_{\text{all paths}} \mathrm{e}^{\mathrm{i}S/\hbar} d\mu, \tag{11}$$

where $d\mu$ is a suitable (putative) measure on the set of all spacetime paths (including continuous, piecewise continuous, or even discontinuous paths) matching the initial and final conditions. This point of view has enormous potentialities in the quantization of field theories, since it preserves manifest covariance and the full symmetry group, being derived from a Lagrangian.

It should be stressed that quantum mechanics regards wave functions only as a technical tool to study bound states (corresponding to the discrete spectrum of the Hamiltonian operator $\hat{H}$), scattering states (corresponding instead to the continuous spectrum of $\hat{H}$), and to evaluate probabilities (of finding the values taken by the observables of the theory). Moreover, it is meaningless to talk about an elementary phenomenon on atomic (or sub-atomic) scale unless it is registered [160], and quantum mechanics in the laboratory needs also an external observer and assumes the so-called reduction of the wave packet (see [57] and references therein). There exist indeed different interpretations of quantum mechanics, e.g. Copenhagen [160], hidden variables [15], many worlds [60, 35].

2.3. Spacetime Singularities

Now we revert to the geometric side. In Riemannian or pseudo-Riemannian geometry, geodesics are curves whose tangent vector X moves by parallel transport [85], so that eventually

$$\frac{dX^{\lambda}}{ds}+\Gamma^{\lambda}_{\mu\nu}X^{\mu}X^{\nu}=0, \tag{12}$$

where s is the affine parameter and $\Gamma^{\lambda}_{\mu\nu}$ are the connection coefficients. In general relativity, timelike geodesics correspond to the trajectories of freely moving observers, while null geodesics describe the trajectories of zero-rest-mass particles (Section 8.1 of Ref. [85]). Moreover, a spacetime (M,g) is said to be singularity-free if *all timelike and null geodesics can be extended to arbitrary values of their affine parameter*. At a spacetime singularity in general relativity, all laws of classical physics would break down, because one would witness very pathological events such as the sudden disappearance of freely moving observers, and one would be completely unable to predict what came out of the singularity. In the 1960s, Penrose [121] proved first an important theorem on the occurrence of singularities in gravitational collapse (e.g. formation of black holes). Subsequent work by Hawking [79, 80, 81, 82, 83], Geroch [66], Ellis and Hawking [84, 52], Hawking and Penrose [86] proved that spacetime singularities are generic properties of general relativity, provided that physically realistic energy conditions hold. Very little analytic use of the Einstein equations is made, whereas the key role emerges of topological and global methods in general relativity.

On the side of singularity theory in classical cosmology, explicit mention should be made of the work in Ref. [14], since it has led to significant progress by Damour et al. [27], despite having failed to prove singularity avoidance in classical cosmology. As pointed out in Ref. [27], the work by Belinsky et al. is remarkable because it gives a description of the generic asymptotic behaviour of the gravitational field in 4-dimensional spacetime in the vicinity of a spacelike singularity. Interestingly, near the singularity the spatial points essentially decouple, i.e. the evolution of the spatial metric at each spatial point is asymptotically governed by a set of second-order, non-linear ordinary differential equations in the time variable [14]. Moreover, the use of qualitative Hamiltonian methods leads naturally to a billiard description of the asymptotic evolution, where the logarithms of spatial scale factors define a geodesic motion in a region of the Lobachevskii plane, interrupted by geometric reflections against the walls bounding this region. Chaos follows because the Bianchi IX billiard has finite volume [27]. A self-contained derivation of the billiard picture for inhomogeneous solutions in D dimensions, with dilaton and p-form gauge fields, has been obtained in Ref. [27].

2.4. Unification of All Fundamental Interactions

The fully established unifications of modern physics are as follows.

(i) *Maxwell*: electricity and magnetism are unified into electromagnetism. All related phenomena can be described by an antisymmetric rank-two tensor field, and derived from a 1-form, called the potential.

(ii) *Einstein*: space and time are unified into the spacetime manifold. Moreover, inertial and gravitational mass, conceptually different, are actually unified as well.

(iii) *Standard model of particle physics*: electromagnetic, weak and strong forces are unified by a non-Abelian gauge theory, normally considered in Minkowski spacetime (this being the base space in fibre-bundle language).

The physics community is now familiar with a picture relying upon four fundamental interactions: electromagnetic, weak, strong and gravitational. The large-scale structure of the universe, however, is ruled by gravity only. All unifications beyond Maxwell involve non-Abelian gauge groups (either Yang–Mills or Diffeomorphism group). At least three extreme views have been developed along the years, i.e.,

(i) Gravity arose first, temporally, in the very early Universe, then all other fundamental interactions.

(ii) Gravity might result from Quantum Field Theory (this was the Sakharov idea [139]).

(iii) The vacuum of particle physics is regarded as a cold quantum liquid in equilibrium. Protons, gravitons and gluons are viewed as collective excitations of this liquid [155].

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Biographical Sketch

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His original contributions are mainly devoted to quantum gravity and quantum field theory on manifolds with boundary (1-loop conformal anomalies, mixed and diff-invariant boundary conditions, heat-kernel asymptotics, Casimir effect), spontaneous symmetry breaking in the early universe, scattering from singular potentials in quantum mechanics. He has received 1831 citations for his papers until 8 September 2011 according to the SLAC database criteria, including 7 top-cited papers (2 very well-known and 5 well-known) and 45 known papers. This yields an h-index for his work equal to 23, i.e. well above the minimal h-value of 18 for promotion to full professor position (see Hirsch in quant-ph/0508025).

He is author or co-author of 180 publications, including, in particular, 27 papers in Classical and Quantum Gravity, 19 papers in Physical Review D15, papers in various other journals including Communications in Mathematical Physics, Journal of High Energy Physics, Nuclear Physics B and Physical Review Letters, and 6 monographs: "Quantum Gravity, Quantum Cosmology and Lorentzian Geometries" (Springer-Verlag 1992 and 1994), "Complex General Relativity" (Kluwer 1995), "Euclidean Quantum Gravity on Manifolds with Boundary" with A.Yu. Kamenshchik and G. Pollifrone (Kluwer 1997), "Dirac Operators and Spectral Geometry" (Cambridge University Press 1998), "Quantum Gravity in Four Dimensions" (Nova Science 2001), "From Classical to Quantum Mechanics" with G. Marmo and G. Sudarshan (Cambridge University Press 2004).

He has supervised 13 theses and has been among the Editors of "Quantum Gravity and Spectral Geometry", Nucl. Phys. B Proc. Suppl. Vol. 104 (2002); "General Relativity and Gravitational Physics", AIP Conf. Proc. Vol. 751 (2005); "New Paths Towards Quantum Gravity", Springer Lecture Notes in Physics Vol. 807 (2010).

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