NONLINEAR ELECTRO- AND MAGNETOELASTIC INTERACTIONS

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Summary

This article begins by providing a development of the basic principles of the classical theory of electromagnetism, from the fundamental notions of point charges and magnetic dipoles through to distributions of charge and current in a non-deformable continuum, time-dependent electromagnetic fields and Maxwell's equations. The modifications of the theory required to account for the deformability of material media are then summarized. In a specialization of the theory details are developed of the constitutive structure for static magnetoelastic interactions in a deformable material, and a review of the relevant continuum mechanics is included. The constitutive equations are presented first in the Eulerian description and then an alternative formulation of the equations based on a Lagrangian approach is adopted, which leads to an elegant and relatively simple structure of the governing equations. The theory is specialized further to the case of an isotropic magnetoelastic material and representative prototype boundary-value problems are formulated and then solved using a simple model constitutive law in order to illustrate the magnetoelastic coupling.

1. Introduction

Anyone who has ever played with a permanent magnet has been intrigued by how metal objects are attracted by the magnetic force. The force acts not only on the object as a whole, but on each bit of material, inducing a change in shape and/or size of the object commonly known as magnetostriction. For typical metals this change is very small and the associated variations in the magnetic and mechanical properties of the material can be neglected. The magnetism of transition metals comes hand-in-hand with mechanical stiffness, a duality that has come to be understood at a deep level in terms of the quantum mechanics of the electronic bands. Only recently, however, have researchers come to appreciate the profound potential of multi-functional compliant magneto-sensitive materials as new materials have been synthesized. These mechanically soft materials are capable of large elastic deformations under the influence of an external magnetic field, much larger than in conventional magnetostriction. To put this advance in perspective, the new materials represent a step change by several orders of magnitude compared to conventional magnetic metals in both high magneto-mechanical compliance and large elastic deformability. The new materials are highly deformable and magnetizable polymers composed of a rubber-like base matrix embedded with micron-sized magneto-active particles. Like a typical rubber, they have low mechanical stiffness and are very compliant, especially in low-dimensional structures such as membranes and rods, while demonstrating good magnetic susceptibility. The small particle size ensures that the materials are effectively homogeneous, and the material processing has already been advanced to the point where robust material characteristics can be developed.

The transformative concepts of nonlinearity in the response and the magneto-mechanical coupling of these materials open the door for many new devices, impacting a range of applications that could not be addressed with previously available materials. The nonlinearity is the key. In his classic textbook on electrodynamics J. D. Jackson states that "*In substances other than ferromagnets, for weak enough fields the presence of an applied magnetic field induces a magnetization proportional to the magnitude of the applied field. We then say that the response of the medium is linear*". In other words, the linear electromagnetic theory, applicable to infinitesimal deformations and weak fields, neglects the magneto-mechanical coupling in the sense that there is no change in mechanical properties due to the applied magnetic field and no change in the magnetic properties due to mechanical deformations. The availability of materials that can operate in a highly nonlinear magneto-mechanical regime offers very exciting possibilities and challenges from the perspectives of device design, materials science, constitutive modeling and magnetomechanical theory.

At present the influence of magnetic fields on the behavior of magneto-sensitive materials in the highly nonlinear regime is not well understood and the development of an appropriate theoretical framework is essential to further that understanding. While the extension of the theory of the magnetism of continuous media to highly deformable systems seems natural from an academic point of view, it has languished undeveloped because there has been no practical motivation hitherto. The materials did not exist! Recently we have begun to make progress in constructing a theoretical framework for the analysis of these materials, as we describe in the latter sections of this article. We need to take this theory further so as to describe accurately the nonlinear magneto-mechanical coupling when large deformations are involved. The theory of large deformations is of fundamental interest, both in terms of the unique properties offered by magneto-sensitive elastomers and in terms of potential applications to, for example, sensors and controllable devices.

We begin this chapter by first providing an overview of the fundamental principles of the classical theory of electromagnetism. Starting from the concepts of point and distributed electric charges in Section 2 we define the Lorentz force and the time independent electric field associated with a point charge and then establish Gauss's theorem for a distribution of electrostatic charge. In Section 3, using the idealization of a magnetic dipole (equivalent to a current loop), we define the magnetostatic field and the magnetic potential and show how the magnetic field is connected to the current density associated with moving charge by Ampère's Circuital Law. An explicit formula for the magnetic induction in terms of the current density is provided by the Biot-Savart Law.

In Section 4 the interconnection between time-dependent electric and magnetic fields is quantified. In particular, Faraday's Law is developed as a mathematical formulation expressing the association between time-varying magnetic and electric fields. Then, in Section 5, the full set of Maxwell's equations governing the electric and magnetic fields for a known charge density and current distribution are collected together. The electric displacement vector and the magnetic field vector are introduced and the notions of polarization and magnetization in material media are discussed with particular reference to a linear (non-deformable) electromagnetic material. The continuity conditions across a material boundary for the electric and magnetic field vectors are summarized in Section 6.

The development next takes account of the deformability of material media. To describe the nonlinear magnetoelastic interactions in a deformable material, a review of continuum kinematics is necessary and this is provided in Section 7. Electromagnetic field variables and boundary conditions, which, in general, are defined with respect to the current configuration, are re-cast in Lagrangian form and the Lagrangian forms of the field equations are derived. From this point on we specialize to the case of magnetostatics in order to illustrate the application of the theory. In Section 8, we summarize in a simple form the equilibrium equations for a highly deformable magnetoelastic material whose mechanical properties can be changed significantly by the application of a magnetic field. We assume stationary conditions, neglect all electric fields, and consider the nonlinear magnetoelastic coupling only. An overview of different ways in which the equations of mechanical equilibrium can be written in the presence of magneto-mechanical interactions is provided. In addition, we list some of the many possible definitions of stress tensor that can be included in the equilibrium equations along with the associated magnetic body force terms.

The general constitutive law for a nonlinear magnetoelastic material is derived and expressed in a compact form, with either the magnetic field or the magnetic induction as the independent magnetic variable. Here we consider an isotropic magnetoelastic material for which the constitutive equations can be expressed in terms of six invariants involving the deformation and a magnetic vector, which reduce to five for an incompressible material, as is appropriate for elastomers. These equations are used in Section 9, for an incompressible material, in the solution of two representative boundary-value problems involving circular cylindrical geometry, specifically the helical shear of a circular cylindrical tube with an axial magnetic field and the extension and inflation of a circular cylindrical tube with a circumferential magnetic field. For each problem a general formulation is developed without specialization of the (isotropic) constitutive law, and then specific results are discussed briefly for a special choice of such a law. It is noted, in particular, that certain restrictions may be placed on the class of constitutive laws for a considered combination of deformation and magnetic field to be admitted.

Because of space limitations only partial coverage of the vast subject of electromagnetic effects in deformable media can be provided in this chapter, and many interesting phenomena are not included. For example, we do not include discussion of dissipative effects such as those arising in electrically conducting materials or of the different types of magnetic properties of materials. Moreover, the applications considered in Sections 8 and 9 are purely static, while there are many applications that involve dynamic couplings which are not treated herein. For pointers to other applications and for broader perspectives on both the mathematical and physical modeling of complex electro-magneto-mechanical couplings the reader is referred to the monographs cited in the Bibliography.

2. Electrostatics

It is convenient to begin by introducing a time-independent distribution of charges and the associated electromagnetic interactions. Historically, electromagnetic theory has been defined as a macroscopic phenomenon, the concept of distributed charges and corresponding interactions being an idealization that admits a mathematical description of the experimentally observed phenomena. The charged particles in a distribution interact by generating forces on one another, and the total force on any one particle results from the presence of all the others. The force acting on a (test) particle with point charge e is a vector function and identifies the electric field at this point. For time-independent phenomena a particle carrying charge e which is at rest at location $\mathbf{x}$ is subject to a force $\mathbf{f}$ given by

$$\mathbf{f} = e\mathbf{E}, \tag{1}$$

which defines the electric field vector $\mathbf{E}$ at $\mathbf{x}$, i.e. the force per unit charge on a stationary particle. If, now, the particle moves with constant velocity $\mathbf{v}$ in a magnetic field, it experiences an additional force perpendicular to its direction of motion and proportional to the magnitude of $\mathbf{v}$. The total electromagnetic force on the particle is then given by

$$\mathbf{f} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{2}$$

which is known as the *Lorentz force* and which identifies the *magnetic induction vector* $\mathbf{B}$.

2.1. Coulomb's Law

Coulomb, based on experimental data, showed that the electric field $\mathbf{E}$ due to an isolated and stationary particle is proportional to its charge e and varies inversely with the square of the distance from the particle. The electric field at the point $\mathbf{x}$ due to a point charge e located at the origin therefore has the form

$$\mathbf{E}(\mathbf{x}) = k\,e\frac{\mathbf{x}}{r^3} = k\,e\frac{\hat{\mathbf{x}}}{r^2}, \tag{3}$$

where $r = |\mathbf{x}|$, $\hat{\mathbf{x}} = \mathbf{x}/r$ is a unit vector and k is a constant of proportionality that depends on the units used. If the particle is located at the fixed point $\mathbf{x}'$ instead of the origin then Eq. (3) is replaced by

$$\mathbf{E}(\mathbf{x}) = k\,e\frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}. \tag{4}$$

In addition, Coulomb was able to quantify the force of interaction between two charged particles at rest. If the two particles have charges e_1 and e_2 and are placed at locations $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively, the interaction force is given by *Coulomb's Law*

$$\mathbf{f} = k\, e_1 e_2 \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|^3}, \tag{5}$$

which is attractive if the charges are of the opposite type, repulsive otherwise. This law is exact for static (point) particles.

In a similar way, if we consider a point charge e moving with uniform velocity, the resulting magnetic induction $\mathbf{B}$ at position $\mathbf{x}$ relative to the point charge is proportional to

$$\mathbf{B} = k' e \frac{\mathbf{v} \times \hat{\mathbf{x}}}{r^2}, \tag{6}$$

where the constant k' again depends on the system of units used. Unlike Coulomb's law this is an approximation in the sense that it is only valid in the non-relativistic situation (when $|\mathbf{v}|$ is much smaller than the speed of light and the acceleration is negligible). Relativistic effects are not considered in the present work.

2.1.1. Units

In the SI system, the unit of electric charge is the Coulomb (C), the electric current is given in Ampères (A), the force in Newtons (N) and the length in meters (m). The electric charge of an electron, for example, is $e = -1.602 \times 10^{-19}\ \mathrm{C}$. The unit of the electric field $\mathbf{E}$ is Volt per meter (V m^{-1}) and the magnetic induction $\mathbf{B}$ has units of Newton per Ampère meter (N A^{-1} m^{-1}). The constants of proportionality k and k' introduced in (5) and (6) are chosen such that the electric field and magnetic induction are given respectively by

$$\mathbf{E} = \frac{e}{4\pi\varepsilon_0} \frac{\hat{\mathbf{x}}}{r^2}, \quad \mathbf{B} = \frac{\mu_0 e}{4\pi} \frac{\mathbf{v} \times \hat{\mathbf{x}}}{r^2}, \tag{7}$$

where $\varepsilon_0 \approx 8.854 \times 10^{-12}\, \mathrm{C}^2\, \mathrm{N}^{-1}\, \mathrm{m}^{-2}$ is the *permittivity of free space* and μ_0, which is equal to $4\pi \times 10^{-7}\, \mathrm{N\, A}^{-2}$, is the magnetic *permeability of free space* . It turns out that for SI units, the speed c of propagation of electromagnetic effects (the speed of light) in free space is given by

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}, \tag{8}$$

as will be seen later.

2.2. Charge Conservation

The definition of the electric field up to this point assumes the existence of a set of discrete point charges. We now expand this concept to include a charge distributed over a certain region in space. Consider an infinitesimal element of volume $\mathrm{d}V$ and let $\rho_{\mathrm{e}}\,\mathrm{d}V$ be the total charge within this element. Then ρ_{e} is the *charge density*, which may be positive or negative and depends, in general, on the position $\mathbf{x}$ and time t, i.e. $\rho_{\mathrm{e}} = \rho_{\mathrm{e}}(\mathbf{x}, t)$.

If $\mathbf{v}$ is the mean velocity of the individual charges in $\mathrm{d}V$, then

$$\mathbf{J} = \rho_{\mathrm{e}}\mathbf{v}, \tag{9}$$

defines the *current density*. The Lorentz force for a discrete point charge subject to electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ has been defined in Eq. (2). For a distribution with charge density ρ_{e} and current density $\mathbf{J}$, the Lorentz force per unit volume is given by

$$\mathbf{f} = \rho_{\mathrm{e}}\mathbf{E} + \mathbf{J} \times \mathbf{B}. \tag{10}$$

Consider a *fixed* volume in space V bounded by a surface S with unit outward normal $\mathbf{n}$. The charge density per unit volume within V is ρ_{e} and the rate at which charge flows across S is given by $\mathbf{J} \cdot \mathbf{n}$ per unit area. The rate of increase of charge within V must arise from the influx. Thus,

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho_e \mathrm{d}V = -\int_S \mathbf{J} \cdot \mathbf{n}\,\mathrm{d}S = -\int_V \operatorname{div}\mathbf{J}\,\mathrm{d}V, \tag{11}$$

where the divergence theorem has been used to convert the surface integral to an integral over the volume V. It follows that

$$\int_V \left(\frac{\partial \rho_{\mathrm{e}}}{\partial t} + \operatorname{div}\mathbf{J} \right) \mathrm{d}V = 0, \tag{12}$$

which must hold for arbitrary V. Provided the integrand in (12) is continuous we may deduce the local form of the *charge conservation equation* as

$$\frac{\partial \rho_{\mathrm{e}}}{\partial t} + \operatorname{div}\mathbf{J} = 0, \tag{13}$$

where the partial derivative indicates that the charge density ρ_{e} may also depend on the location $\mathbf{x}$ in V. In a steady state situation (no time dependence) we have $\partial\rho_{\mathrm{e}}/\partial t = 0$ and Eq. (13) reduces to

$$\operatorname{div}\mathbf{J} = 0. \tag{14}$$

The corresponding integral form is

$$\int_S \mathbf{J}\cdot\mathrm{d}\mathbf{S} = 0 \tag{15}$$

for arbitrary closed surfaces S.

2.3. The Field of a Static Charge Distribution

As we have seen, the electric field at a location $\mathbf{x}$ due to an isolated point charge e located at the origin is given by Eq. $(7)_1$. Equivalently, this can be written as

$$\mathbf{E}(\mathbf{x}) = \frac{e}{4\pi\varepsilon_0}\frac{\hat{\mathbf{x}}}{r^2} = -\frac{e}{4\pi\varepsilon_0}\operatorname{grad}\left(\frac{1}{r}\right). \tag{16}$$

When the point charge is placed at the position $\mathbf{x}'$, the electric field is given by Eq. (4) or, alternatively, by

$$\mathbf{E}(\mathbf{x}) = \frac{e}{4\pi\varepsilon_0}\frac{\mathbf{R}}{R^3} = -\frac{e}{4\pi\varepsilon_0}\operatorname{grad}\left(\frac{1}{R}\right), \tag{17}$$

where $R = |\mathbf{R}|$ and $\mathbf{R} = \mathbf{x} - \mathbf{x}'$.

If we consider the charge within the volume V to be continuously distributed, the point charge e can be replaced by the charge $\rho_\mathrm{e}\,\mathrm{d}V$ in the volume element $\mathrm{d}V$. If $\rho_\mathrm{e} = 0$ outside the specified volume V, then the electric field at $\mathbf{x}$ is given by

$$\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int_V \rho_\mathrm{e}(\mathbf{x}')\frac{\mathbf{R}}{R^3}\mathrm{d}V(\mathbf{x}') = -\frac{1}{4\pi\varepsilon_0}\int_V \rho_\mathrm{e}(\mathbf{x}')\operatorname{grad}\left(\frac{1}{R}\right)\mathrm{d}V(\mathbf{x}'), \tag{18}$$

where the integration is with respect to the $\mathbf{x}'$ variable, but the grad operator is with respect to **x** and can therefore be taken outside the integral. Thus,

$$\mathbf{E}(\mathbf{x}) = -\frac{1}{4\pi\varepsilon_0}\operatorname{grad}\int_V \frac{\rho_\mathrm{e}(\mathbf{x}')}{R}\,\mathrm{d}V(\mathbf{x}'). \tag{19}$$

The gradient operator in the above equation acts on a scalar function. It is therefore convenient to formalize this process by explicitly introducing a scalar potential function ϕ, known as the *electrostatic potential*. Equation (19) is then written compactly as

$$\mathbf{E}(\mathbf{x}) = -\operatorname{grad}\phi(\mathbf{x}), \tag{20}$$

where the scalar potential ϕ depends on the charge density function ρ_e and is given by

$$\phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_\mathrm{e}(\mathbf{x}')}{R}\mathrm{d}V(\mathbf{x}'). \tag{21}$$

Since $\mathrm{curl}(\mathrm{grad}\,\phi) \equiv \mathbf{0}$ for any scalar function ϕ, we obtain the *first equation of electrostatics*

$$\mathrm{curl}\,\mathbf{E} = \mathbf{0}. \tag{22}$$

Far from the charge distribution the field is approximately that of a point charge situated at the origin with a charge equal to the total charge within the distribution. In this case we have $1/R \approx 1/r$ and the electrostatic potential (21) can be approximated by

$$\phi(\mathbf{x}) \approx \frac{e}{4\pi\varepsilon_0 r}, \tag{23}$$

where

$$e = \int_V \rho_\mathrm{e}(\mathbf{x}')\mathrm{d}V(\mathbf{x}') \tag{24}$$

is now the total charge in V.

Note that the work done in moving a particle of charge e from $\mathbf{x}_0$ to $\mathbf{x}$ in the presence of an electric field $\mathbf{E}$ is given by

$$W = -\int_{\mathbf{x}_0}^{\mathbf{x}} \mathbf{f}\cdot\mathrm{d}\mathbf{x} = -e\int_{\mathbf{x}_0}^{\mathbf{x}} \mathbf{E}\cdot\mathrm{d}\mathbf{x}, \tag{25}$$

where we used the definition of the electric field in Eq. (1), with the sign reversed since we are considering the work done *against* the electric field. Using the electrostatic potential ϕ introduced in (20), we have

$$W = e\int_{\mathbf{x}_0}^{\mathbf{x}} \mathrm{grad}\,\phi\cdot\mathrm{d}\mathbf{x} = e\left[\phi(\mathbf{x}) - \phi(\mathbf{x}_0)\right], \tag{26}$$

where $\phi(\mathbf{x})$ can be seen as the potential energy of a particle with unit charge. Equation (26) shows that the work done in moving the particle from $\mathbf{x}_0$ to $\mathbf{x}$ is independent of the actual path and depends only on the initial and final locations. If the path is closed, i.e. the initial and final locations coincide, we have

$$\oint \mathbf{E}\cdot\mathrm{d}\mathbf{x} = 0 \tag{27}$$

provided ϕ is a single-valued function. Note that by Stokes' theorem Eq. (27) can be written

$$\int_S \operatorname{curl}\mathbf{E}\cdot\mathbf{n}\,\mathrm{d}S = 0\,, \tag{28}$$

where S is an arbitrary oriented open surface. The local form of (28) is (22).

2.3.1. The Field of a Dipole

Consider now a distribution of charge with density $\rho_\mathrm{e}(\mathbf{x}')$ confined to a finite volume V, where $\mathbf{x}'$ is the position vector of a typical point in V relative to an origin O located within V and $\rho_\mathrm{e} = 0$ outside V. Let $\mathbf{x}$ be the position vector of a point P far from V at which the electrostatic field is to be calculated (see Figure 1).

Then $|\mathbf{x}'| \ll |\mathbf{x}|$ for all $\mathbf{x}'$ in V, and we may use the Taylor expansion to obtain the approximation

$$\frac{1}{R} \equiv \frac{1}{|\mathbf{x}-\mathbf{x}'|} \approx \frac{1}{r} - \mathbf{x}'\cdot\operatorname{grad}\left(\frac{1}{r}\right), \tag{29}$$

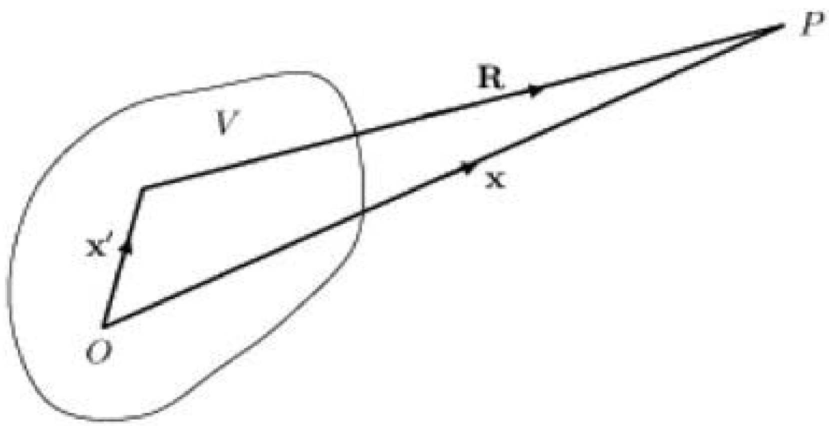

Figure 1. Volume V containing a charge distribution with density $\rho_\mathrm{e}(\mathbf{x}')$ such that $\rho_\mathrm{e} = 0$ outside V, showing field point P having position vector $\mathbf{x}$ relative to origin O in V and $\mathbf{R} = \mathbf{x} - \mathbf{x}'$.

recalling that $r = |\mathbf{x}|$. Hence, from (21), the electrostatic potential at $\mathbf{x}$ is approximated as

$$\phi(\mathbf{x}) \approx \frac{e}{4\pi\varepsilon_0 r} - \frac{1}{4\pi\varepsilon_0}\boldsymbol{\mu}\cdot\operatorname{grad}\left(\frac{1}{r}\right), \tag{30}$$

where e is the total charge in V given by the formula (24) and $\boldsymbol{\mu}$ is defined by

$$\boldsymbol{\mu} = \int_V \rho_\mathrm{e}(\mathbf{x}')\mathbf{x}'\mathrm{d}V(\mathbf{x}'). \tag{31}$$

If $e \neq 0$ then the origin can be translated to the center of charge (analogous to the center of mass in mechanics) so that $\boldsymbol{\mu} = \mathbf{0}$, in which case

$$\phi(\mathbf{x}) \approx \frac{e}{4\pi\varepsilon_0 r}, \tag{32}$$

which is the field of a point charge e located at the origin. Thus, the field of a charge distribution at a large distance is indistinguishable from that of a point charge. On the other hand, if $e = 0$ and $\boldsymbol{\mu} \neq \mathbf{0}$ we have

$$\phi(\mathbf{x}) \approx -\frac{1}{4\pi\varepsilon_0}\boldsymbol{\mu}\cdot\operatorname{grad}\left(\frac{1}{r}\right) = \frac{\boldsymbol{\mu}\cdot\mathbf{x}}{4\pi\varepsilon_0 r^3}. \tag{33}$$

This is the potential due to an *electric dipole* of strength $\boldsymbol{\mu}$ situated at the origin. This is equivalent to having two charges of equal and opposite signs very close together. The potential of such a combination is given by (33).

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Biographical Sketches

Luis Dorfmann received his undergraduate degree from the University of Padova in Italy and his Master and PhD degrees from the University of California at Los Angeles in 1989 and 1992. After working in industry from 1992 to 1997, he worked at the University of Vienna from 1997-2004, spent one year at the University of Glasgow and moved to Tufts University, USA, in 2005. Since 1999 he has been conducting research on the mechanics of soft materials, in which the main focus has been the development of constitutive equations to describe the mechanical properties of natural polymeric compounds and of magneto- and electro-active elastomers. Another theme of his research is the nonlinear mechano-electrical interaction in biological tissue and work on the development and analysis of constitutive laws for anisotropic solids.

Ray Ogden gained his first degree in Mathematics at the University of Cambridge, where he also completed his PhD in Applied Mathematics, specializing in theoretical solid mechanics. He has been George Sinclair Professor of Mathematics at the University of Glasgow since 1984. His main research interests are in nonlinear elasticity theory and its applications, including applications to the mechanics of rubber-like solids, the mechanics of soft biological tissues and to electro- and magneto-mechanical interactions. He has published extensively in international journals on these topics and his monograph “Non-Linear Elastic Deformations”, published in 1984, has become a classic text. In 2005 he was awarded the Koiter Medal of the American Society of Mechanical Engineers and in 2006 was elected a Fellow of the Royal Society of London.