SYSTEM IDENTIFICATION AND CONTROL IN STRUCTURAL ENGINEERING

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Summary

There has been tremendous effort in the development of smart structures in the past

decades due to the rapid development of sensor and actuator technology. A smart structure has essentially two main systems, namely health monitoring system and vibration control system, in addition to the concerned civil engineering structure. Health monitoring system is used to detect any possible damage and/or deterioration while vibration control system is used to suppress the vibration of a structure for safety and serviceability consideration. Health monitoring system includes data acquisition system, identification algorithm, diagnosis and prognosis system. Data acquisition system records structural response (usually acceleration) for the identification algorithm to estimate some key parameters, such as modal frequencies or stiffnesses of the structure. By using this result, the diagnosis system determines any possible damage, its location and severity. Finally, the prognosis system estimates the possible consequences of the identified damage. On the other hand, there are several types of vibration control systems: passive, active and semi-active control system. Passive control system suppresses structural vibration by base isolation or energy dissipating mechanism without using any sensory system. Active or semi-active control system includes data acquisition system and controller algorithm. The measured response is used to compute the feedback by the controller. An active control system applies feedback force through an actuator system while a semi-active control system applies feedback to adjust in a real time manner the variable damping and/or stiffness properties of some advanced devices installed in a structure.

In this chapter, we focus on the algorithms for both health monitoring and vibration control system. First, we present the fundamental concepts of system identification, including definition of input-output relationship, modal identification, model updating and model identifiability, etc. Then, we introduce a number of well-known parametric identification methods using measured response and they are categorized into non-Bayesian and Bayesian types. Next, an iterative model updating procedure using identified modal parameters of a structure will be presented. Afterwards, we will introduce another level of system identification problem, which is the selection of a suitable model class for parametric identification. Three well-known methods are presented: Akaike information criterion, Bayesian information criterion and Bayesian asymptotic expansion. In the second half of this chapter, we will focus on vibration control for civil engineering structures. Passive control, active control and semi-active control strategy will be introduced. Finally, two popular control algorithms, namely the linear quadratic Gaussian regulator and the sliding mode control, will be introduced. Their application in conjunction with the clipped optimal controller for semi-active control is also presented.

1. Introduction

To fully exploit new technologies for response mitigation and structural health monitoring, improved design methodologies are desirable (Kozin and Natke 1986; Unbehauen and Rao 1987; Natke 1988; Farrar and Doebling 1997; Doebling et al. 1998; Ivanović et al. 2000; Chang et al. 2003; Sohn et al. 2003; Kerschen et al. 2006; Kołakowski 2007). The design of smart structures involves system identification and vibration control. In this chapter, we will introduce fundamental concepts and some of the well-known algorithms for these two areas. First, we present the fundamental concepts of system identification, including definition of input-output relationship, modal identification, model updating and model identifiability, etc. Then, a number of well-known parametric identification methods are introduced using measured response and they are categorized into non-Bayesian and Bayesian type. Next, an iterative model updating procedure using identified modal parameters of a structure is introduced. Afterwards, we will introduce another level of system identification problem, which is the selection of a suitable model class for parametric identification. Three well-known methods are presented: Akaike information criterion, Bayesian information criterion and Bayesian asymptotic expansion. Then, the second part of this article will be focused on structural vibration control. The basic concepts of passive, active and semi-active control will be introduced. Finally, the well-known linear quadratic Gaussian regulator and the sliding mode control algorithm will be derived. The application with the clipped optimal controller for semi-active control system is also introduced.

2. System Identification in Structural Engineering

Figure 1 shows the general relationship of different structural dynamics problems. In structural engineering, our concern is structural systems, such as buildings, bridges and towers. To estimate the performance of a structure, we need to construct a mathematical model, e.g., the mass, damping and stiffness matrices in the linear case. From this mathematical model, one can proceed with an *eigenvalue problem* to compute the natural frequencies and mode shapes of the structure. On the other hand, one can proceed with *response calculation* or *random vibration analysis* to assess the performance of the structural design. Furthermore, one can go from eigenvalues and eigenvectors for response calculation or random vibration analysis and this is called modal analysis. These are forward problems in structural dynamics. On the other hand, the backward or *inverse problems* receive more and more attention in recent decades. By using the measured structural response, one can estimate the modal frequencies and mode shapes and this process is called *modal identification*. One can also estimate the model parameters in the structural model using measured structural response and/or identified modal parameters and this process is called *model updating*.

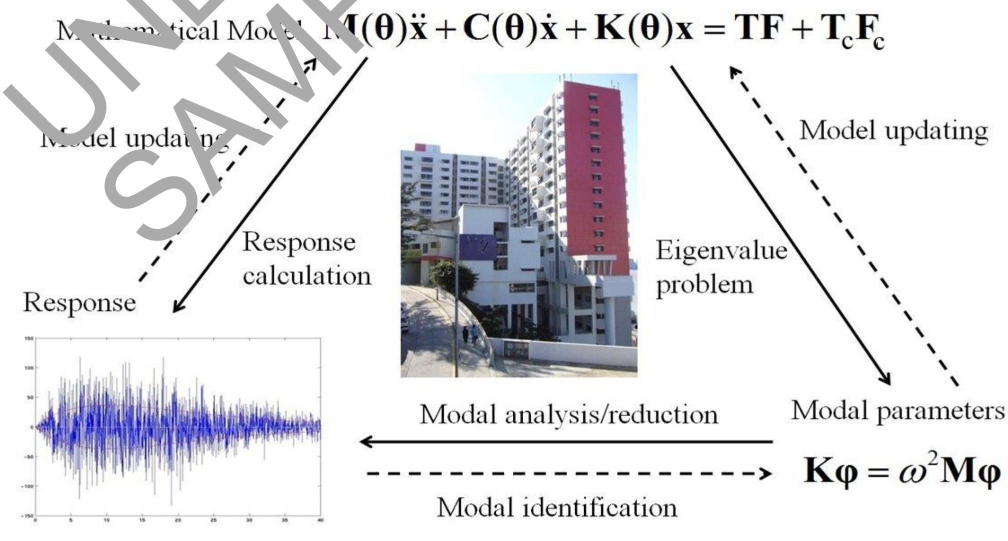

Figure 1. Relationship among different structural dynamics problems

2.1. Basic Concepts

2.1.1. System

Figure 2 shows the block diagram of a system and it consists of the input, output and the plant. In structural identification, the input usually is the excitation to the structure and the output is the structural response. The plant is the structure of concern. There are two levels of system identification problems, namely *parametric identification* and *model class selection*. The parametric identification problem is to identify unknown parameters given a class of mathematical models for a particular structural system. The second level deals with the selection of a suitable class of mathematical models for parametric identification. The second level is significantly more difficult but also more crucial than the first level since parametric identification results will be by no means meaningful if one fails to obtain a suitable class of models. However, due to the difficulty of this problem, it is usually determined by user's judgment.

Figure 2. Schematic diagram of systems

Development of system identification techniques began earlier in aerospace engineering and electrical engineering (Eykhoff 1974; Ljung 1977; Peterka 1981; Soderstrom and Stoica 1989; Unbehauen and Rao 1990; Peeters and De Roeck 2001). Some of the methods were migrated to structural engineering problems but it is not a straight-forward exercise due to some unique features in civil engineering structural systems. One main difficulty comes from the large scale of civil engineering structures. Furthermore, constitutive relationship of some materials, such as concrete or soil, can be very complex. Therefore, modeling error is large when comparing with aerospace engineering, electrical or mechanical engineering problems. As a result, model class selection in civil engineering problems will be more crucial compared with other engineering or science disciplines. Furthermore, due to the large scale, there are usually a large number of uncertain parameters to be identified. In this case, well-posedness will be an issue of concern. In other words, there may be multiple (finite or infinite) optimal solutions.

Another difficulty is due to the fact that the input is usually unknown. In the context of structural dynamics, the input is the excitation that includes self weight of the structure, ground motion, wind pressure field and other moving loads (e.g., force generated by moving people or vehicles). Except for the self weight and ground motion, the others are difficult to measure. Therefore, system identification problems in structural dynamics usually require treatment of unmeasured input. This is in contrast to some other disciplines that the input can be measured or even be controlled by the user.

2.1.2. Model Identifiability

System identification is an inverse problem so ill conditioning is inevitably an important issue for consideration. In parametric identification, there may exist one, multiple (but finite) or infinite values of the model parameters to give identical system output. Therefore, given one set of measured system output, it is not necessary to give the unique solution of the model parameters. This issue was discussed in Ljung and Glad (1994) and Katafygiotis and Beck (1998). This is important especially for large number of uncertain parameters because it is difficult to visualize. Given a set of input-output measurements of the underlying system *D*, use $S_{\text{mod}}(\boldsymbol{\theta}_0;D)$ to denote the set of all possible model parameters which give the same model output as the model associated with $\boldsymbol{\theta}_0$.

A parameter θ_l of $\boldsymbol{\theta}$ is *model-identifiable* at $\boldsymbol{\theta}^*$ for model class *C* if there exists a positive number ε_l such that

$$\boldsymbol{\theta} \in S_{\text{mod}}(\boldsymbol{\theta}_0;D) \Leftrightarrow \left|\theta_l - \theta_l^*\right| < \varepsilon_l \text{ or } \theta_l = \theta_l^* \qquad (1)$$

In other words, θ_l^* is uniquely specified within a neighborhood of each of its possible values by *D*. There are three main categories of identifiability:

1. A parameter θ_l of $\boldsymbol{\theta}$ is *globally model-identifiable* at $\boldsymbol{\theta}^*$ for model class *C* if

$$\boldsymbol{\theta} \in S_{\text{mod}}(\boldsymbol{\theta}_0;D) \Leftrightarrow \theta_l = \theta_l^* \qquad (2)$$

In other words, θ_l^* is uniquely specified by *D*. If θ_l is globally model-identifiable at $\boldsymbol{\theta}^*$, then it is also model-identifiable at $\boldsymbol{\theta}^*$.

2. A parameter θ_l of $\boldsymbol{\theta}$ is *locally model-identifiable* at $\boldsymbol{\theta}^*$ for model class *C* if it is model-identifiable but not globally model-identifiable.

3. A parameter θ_l of $\boldsymbol{\theta}$ is *model-unidentifiable* if it is not model-identifiable.

2.2. Some Well-Known Parametric Identification Methods Using Measured Response

Parametric identification of civil engineering structures is a challenging task that has attracted extensive research efforts over the latest decades (Goodwin and Payne 1977; Ljung 1987; Imai et al. 1989; Soderstrom and Stoica 1989; Sinha and Rao 1991; Johansson 1993; Ghanem and Shinozuka 1995; Alvin et al. 2003; Kijewski-Correa et al. 2008). Comprehensive literature reviews (Bekey 1970; Astrom and Eykhoff 1971; Peeters and DeRoeck 2001; Deistler 2002; Gevers 2006; Kerschen et al. 2006) studied the development of this flourishing research area. Numerous methods have been proposed for parametric identification using measured response (Kozin and Natke 1986; Lew et al. 1993; Doebling et al. 1998; Petsounis and Fassois 2001; Maia and Silva 2001; Soderstrom 2003; Giraldo et al. 2009). In this section, we focus on non-Bayesian parametric identification techniques. Three representative methods, including the recursive least squares approach, the extended Kalman filter (EKF), and the eigensystem realization algorithm (ERA), are presented in the following subsections.

2.2.1. Recursive Least Squares Approach

Recursive least squares approach is an iterative algorithm to minimize the squared residual between the measurements and the model outputs (Ljung 1977; Soderstrom et al. 1978; Ljung and Soderstrom 1983; Solo 1980; Zhou and Cluett 1996; Sharia 1998; Young 2011). It is an extension of the ordinary least squares approach in the sense that the solutions are obtained in an efficient iterative manner (Durbin and Watson 1950; Ljung and Soderstrom 1983). Nevertheless, in contrast with the ordinary least squares approach, the recursive least squares approach is an online estimation technique and does not require to store or reprocess the entire set of data at every time instant. Due to its computation efficiency and simplicity, the recursive least squares approach is a popular parametric identification technique in the 20th century (Bekey 1970; Astrom and Eykhoff 1971; Caravani et al. 1977; Young 1984). In the following, its identification procedure is introduced.

Consider a dynamical system that is parameterized by N_θ model parameters $\boldsymbol{\theta} = [\theta_1, ..., \theta_{N_\theta}]^{\mathrm{T}}$. The objective here is to use discrete response measurement for the identification of these model parameters. Assume that there exists a contaminated linear relationship between the measurement $\mathbf{y}_n \in \mathbf{R}^{N_o}$ and the model parameters:

$$\mathbf{y}_n = \mathbf{P}_n \boldsymbol{\theta} + \boldsymbol{\varepsilon}_n, n = 1, 2, ..., N \tag{3}$$

The measurement noise $\boldsymbol{\varepsilon}_n \in \mathbf{R}^{N_o}$ is modeled as zero-mean discrete Gaussian white noise with covariance matrix $E[\boldsymbol{\varepsilon}_n \boldsymbol{\varepsilon}_n^{\mathrm{T}}] = \boldsymbol{\Sigma}_{\varepsilon_n}$. The transformation matrix $\mathbf{P}_n \in \mathbf{R}^{N_o \times N_\theta}$ is used to describe this relationship between the measurement $\mathbf{y}_n$ and the model parameter vector $\boldsymbol{\theta}$. For example, consider an autoregressive (AR) model: $x_n = a_1 x_{n-1} + a_2 x_{n-2} + \varepsilon_n$. In this case, the measurement is $\mathbf{y}_n = x_n$, the transformation matrix is $\mathbf{P}_n = [x_{n-1}, x_{n-2}]$ and the model parameter vector is $\boldsymbol{\theta} = [a_1, a_2]^{\mathrm{T}}$.

The recursive least squares algorithm identifies the model parameters by minimizing a weighted sum of squared residuals between the measurements and corresponding prediction by the model. This algorithm identifies the optimal parameter vector $\boldsymbol{\theta}_n$ based on measurements up to the n^{th} time step in a recursive manner. The *cost/objective function* can be written for the n^{th} time step in the following form:

$$J(\boldsymbol{\theta}_n) = \sum_{k=1}^{n} \mu_{n,k} \left(\mathbf{y}_k - \mathbf{P}_k \boldsymbol{\theta}_n \right)^{\mathrm{T}} \boldsymbol{\Sigma}_{\varepsilon_k}^{-1} \left(\mathbf{y}_k - \mathbf{P}_k \boldsymbol{\theta}_n \right) \tag{4}$$

where the variables $\mu_{n,k}, k = 1, 2, ..., n,$ are used to assign differential weighting to different data points. The idea is to gradually fade out the contribution of data points far away from the current time step. One popular choice is given as follows (Ljung and Soderstrom 1983; Lozano 1983; Kulhavy and Zarrop 1993):

$$\mu_{n,k} = \begin{cases} \eta_n \eta_{n-1} \cdots \eta_{k+1}, & 1 \le k < n \\ 1, & k = n \end{cases} \tag{5}$$

where $\eta_n \le 1$ is called the *forgetting factor* so it is clearly that the weightings $\mu_{n,k}$ decrease as $n-k$ increases. Selection of the forgetting factor, and thus the weightings, is a trade-off between the parameter tracking capability and the robustness against noise of the algorithm. One popular choice is the exponential weighting function $\mu_{n,k} = e^{(n-k)\log \eta_n}$ with $0 < \eta_n \le 1$ for all n (Johnstone et al. 1982). Another widely used form is to set the forgetting factor as a constant with value between 0 and 1 (Zarrop 1983). Hence, the weighting function is expressed as $\mu_{n,k} = \eta_0^{n-k}$ so the weightings are reduced by a factor of η_0 in each time step.

The optimal model parameter vector at the n^{th} time step $\hat{\boldsymbol{\theta}}_n$ can be determined by minimizing the objective function in Eq. (4) with respect to $\boldsymbol{\theta}_n$:

$$\hat{\boldsymbol{\theta}}_n = \arg\min_{\boldsymbol{\theta}_n} J = \mathbf{R}_n^{-1} \sum_{k=1}^{n} \mu_{n,k} \mathbf{P}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{\varepsilon_k}^{-1} \mathbf{y}_k \tag{6}$$

where the matrix $\mathbf{R}_n$ is given by:

$$\mathbf{R}_n = \sum_{k=1}^{n} \mu_{n,k} \mathbf{P}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{\varepsilon_k}^{-1} \mathbf{P}_k = \eta_n \mathbf{R}_{n-1} + \mathbf{P}_n^{\mathrm{T}} \boldsymbol{\Sigma}_{\varepsilon_n}^{-1} \mathbf{P}_n \tag{7}$$

By using Eqs. (5) - (7), the following recursive formula can be obtained to update the model parameters at each time step:

$$\hat{\boldsymbol{\theta}}_n = \mathbf{R}_n^{-1} \left[\left(\mathbf{R}_n - \mathbf{P}_n^{\mathrm{T}} \boldsymbol{\Sigma}_{\varepsilon_n}^{-1} \mathbf{P}_n \right) \boldsymbol{\theta}_{n-1} + \mathbf{P}_n^{\mathrm{T}} \boldsymbol{\Sigma}_{\varepsilon_n}^{-1} \mathbf{y}_n \right] = \hat{\boldsymbol{\theta}}_{n-1} + \mathbf{G}_n \left(\mathbf{y}_n - \mathbf{P}_n \boldsymbol{\theta}_{n-1} \right) \tag{8}$$

where the *estimator gain matrix* $\mathbf{G}_n$ is given by:

$$\mathbf{G}_n = \mathbf{R}_n^{-1} \mathbf{P}_n^{\mathrm{T}} \boldsymbol{\Sigma}_{\varepsilon_n}^{-1} \tag{9}$$

Finally, in order to avoid direct computation of the inverse $\mathbf{R}_n^{-1}$, the matrix inversion lemma $\left(\mathbf{A} + \mathbf{BCD}\right)^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{C}^{-1} + \mathbf{DA}^{-1}\mathbf{B}\right)^{-1} \mathbf{DA}^{-1}$ is utilized to obtain the following recursive formula for $\mathbf{R}_n^{-1}$:

$$\mathbf{R}_n^{-1} = \left(\eta_n \mathbf{R}_{n-1} + \mathbf{P}_n^{\mathrm{T}} \mathbf{\Sigma}_{\varepsilon_n}^{-1} \mathbf{P}_n\right)^{-1} = \eta_n^{-1}\left[\mathbf{I} - \mathbf{R}_{n-1}^{-1}\mathbf{P}_n^{\mathrm{T}}\left(\eta_n \mathbf{\Sigma}_{\varepsilon_n} + \mathbf{P}_n \mathbf{R}_{n-1}^{-1}\mathbf{P}_n^{\mathrm{T}}\right)^{-1}\mathbf{P}_n\right]\mathbf{R}_{n-1}^{-1} \tag{10}$$

The recursive least squares parametric identification procedure can be summarized as follows:

(1) Start with an initial model parameter vector $\hat{\boldsymbol{\theta}}_0$ and matrix $\mathbf{R}_0^{-1}$;

(2) Calculate $\mathbf{R}_n^{-1}$ by Eq. (10);

(3) Calculate the estimator gain matrix $\mathbf{G}_n$ by Eq. (9);

(4) Compute the optimal model parameter vector $\hat{\boldsymbol{\theta}}_n$ by Eq. (8);

(5) Repeat step (2) to (4) for the next time step.

2.2.2. Extended Kalman Filter (EKF)

Extended Kalman filter (EKF) (or *Kalman-Schmidt filter*) was developed on the foundation of *Kalman filter* for the parametric identification of dynamical systems (Bellantoni and Dodge 1967; Jazwinski 1970; Schmidt 1981; Grewal and Andrews 1993; Brown and Hwang 1997; Simon 2006). Kalman filter was developed to estimate the state vector of linear systems (Kalman 1960; Kalman and Bucy 1961; Sorenson 1985; Ruymgaart and Soong 1988). It propagates the first two statistical moments of the state vector by *predicting* and *filtering* alternately at each time step. Kalman filter is the optimal filter for state estimation of linear systems subjected to Gaussian excitation. The EKF extends the Kalman filter to handle also slightly nonlinear systems. Furthermore, an augmented state vector can be defined to extend the state vector to include also the model parameters. In such a way, the model parameters can be identified with the state estimation process of Kalman filter. Recognizing the power of EKF on parametric identification, it has been widely used in many different disciplines (Hoshiya and Saito 1984; Dhaouadi et al. 1991; Lin and Zhang 1994; Brown and Hwang 1997; Yun and Lee 1997; Einicke and White 1999; Chui and Chen 2009; Grewal and Andrews 2010; Hoi et al. 2010).

Some literatures categorized the EKF as a Bayesian updating process (Jazwinski 1970; Chen 2003; Yuen 2010a) because the algorithm can be derived under the Bayesian probabilistic framework. In addition, the EKF shared a remarkable feature with Bayesian approaches that they can determine the optimal values of the model parameters as well as their associated uncertainties. In this section, we follow the original derivation (Kalman 1960; Kalman and Bucy 1961) which is formulated without adopting the Bayesian perspective. The identification procedure is presented as follows.

Use $\mathbf{X} = [\mathbf{x}^{\mathrm{T}}, \dot{\mathbf{x}}^{\mathrm{T}}]^{\mathrm{T}}$ to denote the state vector that consists of the generalized displacement and velocity vector. Then, the well known state-space representation of an N_{d} degrees of freedom (DOFs) linear dynamical system can be written as follows:

$$\mathbf{X}_{n+1} = \mathbf{A}_{\mathrm{d}}\mathbf{X}_n + \mathbf{B}_{\mathrm{d}}\mathbf{F}_n$$

$$\mathbf{y}_n = \mathbf{C}\mathbf{X}_n + \boldsymbol{\varepsilon}_n \tag{11}$$

where $\mathbf{A}_\mathrm{d} \in \mathbf{R}^{2N_\mathrm{d} \times 2N_\mathrm{d}}$ is the state matrix; $\mathbf{B}_\mathrm{d} \in \mathbf{R}^{2N_\mathrm{d} \times N_\mathrm{F}}$ is the force distributing matrix; $\mathbf{C} \in \mathbf{R}^{N_\mathrm{o} \times 2N_\mathrm{d}}$ is the observation matrix; $\mathbf{X}_n \in \mathbf{R}^{2N_\mathrm{d}}$ is the state vector at the n^{th} time step; $\mathbf{F}_n \in \mathbf{R}^{N_\mathrm{F}}$ is the input excitation; $\mathbf{y}_n \in \mathbf{R}^{N_\mathrm{o}}$ is the measured model output and $\boldsymbol{\varepsilon}_n \in \mathbf{R}^{N_\mathrm{o}}$ is the measurement noise. The excitation $\mathbf{F}$ and measurement noise $\boldsymbol{\varepsilon}$ are modeled as independent discrete Gaussian white noise with zero mean. Their covariance matrices satisfy:

$$E\left[\mathbf{F}_n \mathbf{F}_{n'}^\mathrm{T}\right] = \boldsymbol{\Sigma}_F \delta_{nn'}\ ,\ E\left[\boldsymbol{\varepsilon}_n \boldsymbol{\varepsilon}_{n'}^\mathrm{T}\right] = \boldsymbol{\Sigma}_\varepsilon \delta_{nn'} \text{ and } E\left[\mathbf{F}_n \boldsymbol{\varepsilon}_{n'}^\mathrm{T}\right] = \mathbf{0} \tag{12}$$

where $\delta_{nn'}$ denotes the Kronecker delta.

Kalman filter propagates in estimating the state vector by predicting and filtering alternately at each time step. Given the measurement set $D_n = \{\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_n\}$, the predicted state vector can be calculated by:

$$\hat{\mathbf{X}}_{n+1|n} \equiv E\left[\mathbf{X}_{n+1} \mid D_n\right] = E\left[\mathbf{A}_\mathrm{d}\mathbf{X}_n + \mathbf{B}_\mathrm{d}\mathbf{F}_n \mid D_n\right] = \mathbf{A}_\mathrm{d}\hat{\mathbf{X}}_{n|n} \tag{13}$$

where the symbol $\hat{\mathbf{X}}_{m|n} \equiv E\left[\mathbf{X}_m \mid D_n\right]$ is defined for notation convenience only. Based on Eqs. (11) and (13), the covariance matrix of the prediction error can be determined:

$$\hat{\boldsymbol{\Sigma}}_{n+1|n} \equiv E\left[\left(\mathbf{X}_{n+1} - \hat{\mathbf{X}}_{n+1|n}\right)\left(\mathbf{X}_{n+1} - \hat{\mathbf{X}}_{n+1|n}\right)^\mathrm{T} \mid D_n\right] = \mathbf{A}_\mathrm{d}\hat{\boldsymbol{\Sigma}}_{n|n}\mathbf{A}_\mathrm{d}^\mathrm{T} + \mathbf{B}_\mathrm{d}\boldsymbol{\Sigma}_F\mathbf{B}_\mathrm{d}^\mathrm{T} \tag{14}$$

Again, the symbol $\hat{\boldsymbol{\Sigma}}_{m|n} \equiv E\left[\left(\mathbf{X}_m - \hat{\mathbf{X}}_{m|n}\right)\left(\mathbf{X}_m - \hat{\mathbf{X}}_{m|n}\right)^\mathrm{T} \mid D_n\right]$ is defined for notation convenience only. When a new data point $\mathbf{y}_{n+1}$ is available, the data set is enlarged to $D_{n+1} = \{\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_{n+1}\}$ and the state vector can be filtered with the information carried by the new data point. The filtered state vector is given by (Kalman 1960; Jazwinski 1970):

$$\hat{\mathbf{X}}_{n+1|n+1} \equiv E\left[\mathbf{X}_{n+1} \mid D_{n+1}\right] = \hat{\boldsymbol{\Sigma}}_{n+1|n+1}\left(\hat{\boldsymbol{\Sigma}}_{n+1|n}^{-1}\hat{\mathbf{X}}_{n+1|n} + \mathbf{C}^\mathrm{T}\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{y}_{n+1}\right) \tag{15}$$

where the associated uncertainty of the filtering error $\left(\mathbf{X}_{n+1} - \hat{\mathbf{X}}_{n+1|n+1}\right)$ has the following form:

$$\hat{\boldsymbol{\Sigma}}_{n+1|n+1} \equiv E\left[\left(\mathbf{X}_{n+1} - \hat{\mathbf{X}}_{n+1|n+1}\right)\left(\mathbf{X}_{n+1} - \hat{\mathbf{X}}_{n+1|n+1}\right)^\mathrm{T} \mid D_{n+1}\right] = \left(\hat{\boldsymbol{\Sigma}}_{n+1|n}^{-1} + \mathbf{C}^\mathrm{T}\boldsymbol{\Sigma}_\varepsilon\mathbf{C}\right)^{-1} \tag{16}$$

By using Eqs. (15) and (16), the filtered state vector expression can be rewritten in the following form (Jazwinski 1970; Simon 2006):

$$\hat{\mathbf{X}}_{n+1|n+1} = \hat{\mathbf{X}}_{n+1|n} + \mathbf{G}_{n+1}\left(\mathbf{y}_{n+1} - \mathbf{C}\hat{\mathbf{X}}_{n+1|n}\right) \tag{17}$$

where the *Kalman gain matrix* at the $(n+1)^{th}$ time step is given by:

$$\mathbf{G}_{n+1} = \hat{\mathbf{\Sigma}}_{n+1|n+1}\mathbf{C}^{\mathrm{T}}\mathbf{\Sigma}_{\varepsilon}^{-1} \tag{18}$$

Equation (17) provides a similar form as Eq. (8) in the recursive least squares approach.

The Kalman filter estimation process starts from an initial prescribed state vector $\hat{\mathbf{X}}_{0|0}$ (e.g., zero vector) and covariance matrix $\hat{\mathbf{\Sigma}}_{0|0}$, which is usually a diagonal matrix with large diagonal elements. The predicted state vector $\hat{\mathbf{X}}_{1|0}$ can be determined by Eq. (13) and the covariance matrix of the prediction error $\hat{\mathbf{\Sigma}}_{1|0}$ can be calculated by Eq. (14). When the first data point $\mathbf{y}_1$ is available, the filtered state vector $\hat{\mathbf{X}}_{1|1}$ as well as its associated covariance matrix $\hat{\mathbf{\Sigma}}_{1|1}$ can be obtained by Eq. (17) and Eq. (16), respectively. This finishes one cycle of the predicting and filtering process. Then, the process will be repeated for the subsequent time steps. It can be seen explicitly from the estimation equations that the noise covariance matrices affect the performance of the algorithm. Previous studies demonstrated that arbitrary choice of the noise characteristics may lead to biased estimation (Fitzgerald 1971; Reif et al. 1999). To tackle with this problem, Ljung (1979), Valappil and Georgakis (2000) and Yuen et al. (2007a) proposed computational strategies for proper selection of the noise parameters.

The extended Kalman filter (EKF) starts with defining the augmented state vector, which extends the state vector to include also the model parameters:

$$\boldsymbol{\chi} = [\mathbf{x}^{\mathrm{T}}, \dot{\mathbf{x}}^{\mathrm{T}}, \boldsymbol{\theta}^{\mathrm{T}}]^{\mathrm{T}} \tag{19}$$

where the model parameter vector $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_{N_\theta}]^{\mathrm{T}}$ contains N_θ variables to govern the dynamical system. A state-space representation for general linear/nonlinear systems can be written as:

$$\begin{aligned} \boldsymbol{\chi}_{n+1} &= \boldsymbol{p}\left(\boldsymbol{\chi}_n, \mathbf{F}_n\right) \\ \mathbf{y}_n &= \boldsymbol{q}\left(\boldsymbol{\chi}_n\right) + \boldsymbol{\varepsilon}_n \end{aligned} \tag{20}$$

where $\boldsymbol{p}(\cdot)$ and $\boldsymbol{q}(\cdot)$ are vector functions with dimension $2N_{\mathrm{d}} + N_\theta$ and N_{o}, respectively. The excitation $\mathbf{F}$ and measurement noise $\boldsymbol{\varepsilon}$ are modeled as zero-mean discrete Gaussian white noise with covariance matrices $E\left[\mathbf{F}_n\mathbf{F}_{n'}^{\mathrm{T}}\right] = \mathbf{\Sigma}_F\delta_{nn'}$ and

$E\left[\boldsymbol{\varepsilon}_n\boldsymbol{\varepsilon}_{n'}^{\mathrm{T}}\right]=\boldsymbol{\Sigma}_{\varepsilon}\delta_{nn'}$, respectively. Furthermore, the excitation and measurement noise are assumed to be statistically independent.

The dynamical system in Eq. (20) can be linearized locally utilizing Taylor expansion:

$$\begin{aligned}\boldsymbol{\chi}_{n+1} &\approx \tilde{\mathbf{A}}_{\mathrm{d},n}\boldsymbol{\chi}_n+\tilde{\mathbf{B}}_{\mathrm{d},n}\mathbf{F}_n+\tilde{\boldsymbol{p}}_n\\ \mathbf{y}_n &\approx \tilde{\mathbf{C}}_n\boldsymbol{\chi}_n+\tilde{\boldsymbol{q}}_n+\boldsymbol{\varepsilon}_n\end{aligned} \tag{21}$$

where the state, force distributing and observation matrices are given by $\tilde{\mathbf{A}}_{\mathrm{d},n}=\left.\dfrac{\partial\boldsymbol{p}\left(\boldsymbol{\chi}_n,\mathbf{F}_n\right)}{\partial\boldsymbol{\chi}_n}\right|_{\boldsymbol{\chi}_n=\hat{\boldsymbol{\chi}}_{n|n},\mathbf{F}_n=\mathbf{0}}$, $\tilde{\mathbf{B}}_{\mathrm{d},n}=\left.\dfrac{\partial\boldsymbol{p}\left(\boldsymbol{\chi}_n,\mathbf{F}_n\right)}{\partial\mathbf{F}_n}\right|_{\boldsymbol{\chi}_n=\hat{\boldsymbol{\chi}}_{n|n},\mathbf{F}_n=\mathbf{0}}$ and $\tilde{\mathbf{C}}_n=\left.\dfrac{\partial\boldsymbol{q}\left(\boldsymbol{\chi}_n\right)}{\partial\boldsymbol{\chi}_n}\right|_{\boldsymbol{\chi}_n=\hat{\boldsymbol{\chi}}_{n|n-1}}$, respectively. Furthermore, the vector functions $\tilde{\boldsymbol{p}}_n$ and $\tilde{\boldsymbol{q}}_n$ are defined to compensate the linearization error: $\tilde{\boldsymbol{p}}_n=\boldsymbol{p}\left(\hat{\boldsymbol{\chi}}_{n|n},\mathbf{0}\right)-\tilde{\mathbf{A}}_{d,n}\hat{\boldsymbol{\chi}}_{n|n}$ and $\tilde{\boldsymbol{q}}_n=\boldsymbol{q}\left(\hat{\boldsymbol{\chi}}_{n|n-1}\right)-\tilde{\mathbf{C}}_n\hat{\boldsymbol{\chi}}_{n|n-1}$.

The prediction and filtering equations for EKF are given in analogy to Eqs. (14)-(18), as follows:

$$\begin{aligned}\hat{\boldsymbol{\chi}}_{n+1|n} &\equiv E\left[\boldsymbol{\chi}_{n+1}\mid D_n\right]=\tilde{\mathbf{A}}_{\mathrm{d},n}\hat{\boldsymbol{\chi}}_{n|n}+\tilde{\boldsymbol{p}}_n\\ \hat{\boldsymbol{\Sigma}}_{n+1|n} &=\tilde{\mathbf{A}}_{\mathrm{d},n}\hat{\boldsymbol{\Sigma}}_{n|n}\tilde{\mathbf{A}}_{\mathrm{d},n}^{\mathrm{T}}+\tilde{\mathbf{B}}_{\mathrm{d},n}\boldsymbol{\Sigma}_F\tilde{\mathbf{B}}_{\mathrm{d},n}^{\mathrm{T}}\\ \hat{\boldsymbol{\chi}}_{n+1|n+1} &=\hat{\boldsymbol{\chi}}_{n+1|n}+\tilde{\mathbf{G}}_{n+1}\left(\mathbf{y}_{n+1}-\tilde{\boldsymbol{q}}_{n+1}-\tilde{\mathbf{C}}_{n+1}\hat{\boldsymbol{\chi}}_{n+1|n}\right)\\ \hat{\boldsymbol{\Sigma}}_{n+1|n+1} &=\left(\hat{\boldsymbol{\Sigma}}_{n+1|n}^{-1}+\tilde{\mathbf{C}}_{n+1}^{\mathrm{T}}\boldsymbol{\Sigma}_{\varepsilon}^{-1}\tilde{\mathbf{C}}_{n+1}\right)^{-1}\\ \tilde{\mathbf{G}}_{n+1} &=\hat{\boldsymbol{\Sigma}}_{n+1|n+1}\tilde{\mathbf{C}}_{n+1}^{\mathrm{T}}\boldsymbol{\Sigma}_{\varepsilon}^{-1}\end{aligned} \tag{22}$$

Following the same estimation procedure as the Kalman filter, the augmented state vector and its associated covariance matrix can be obtained. Consequently, the model parameters and its associated uncertainty can be determined as part of the augmented state vector.

2.2.3. Eigensystem Realization Algorithm (ERA)

Eigensystem realization algorithm (ERA) identifies the minimal *state-space realization* of a system using pulse response measurement (Silverman 1971; Juang and Pappa 1985). It was developed under the realization theory (Ho and Kalman 1966; De Schutter 2000). Using pulse response measurements, the Markov parameters of the system can be calculated and hence the Hankel matrix can be constructed. The Hankel matrix is factorized via singular value decomposition and the minimal state-space realization can be determined. This algorithm has been widely applied to system identification with field test data. Successful applications demonstrated its efficacy (Pappa and Juang 1988; Lus et al. 1999; Qin et al. 2001; Lus et al. 2002; Brownjohn 2003; Siringoringo and Fujino 2008; Caicedo 2011). In the following, the key identification procedure of ERA is presented.

Consider the state-space representation of an N_d DOFs linear dynamical system with N_o DOFs observation:

$$\begin{aligned}\mathbf{X}_{n+1} &= \mathbf{A}_\mathrm{d}\mathbf{X}_n + \mathbf{B}_\mathrm{d}\mathbf{F}_n \\ \mathbf{y}_n &= \mathbf{C}\mathbf{X}_n + \mathbf{D}\mathbf{F}_n\end{aligned} \tag{23}$$

where the state vector $\mathbf{X}_n = [\mathbf{x}_n^\mathrm{T}, \dot{\mathbf{x}}_n^\mathrm{T}]^\mathrm{T} \in \mathbf{R}^{2N_\mathrm{d}}$ includes the displacement and velocity vector at the n^th time step; $\mathbf{F}_n \in \mathbf{R}^{N_\mathrm{F}}$ is the excitation vector at the n^th time step; and $\mathbf{y}_n \in \mathbf{R}^{N_\mathrm{o}}$ is the model output vector at the n^th time sep. The state-space model matrices $\mathbf{A}_\mathrm{d}$, $\mathbf{B}_\mathrm{d}$, $\mathbf{C}$, $\mathbf{D}$ are the system, force distributing, observation and direct transmission matrix, respectively. The quadruple set $(\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}, \mathbf{D})$ is called the state-space realization of the system and the objective of ERA is to determine the minimal state-space realization.

By using Eq. (23), the model output can be rewritten as:

$$\mathbf{y}_n = \mathbf{C}\mathbf{A}_\mathrm{d}^n\mathbf{X}_0 + \sum_{k=0}^{n-1}\mathbf{C}\mathbf{A}_\mathrm{d}^{n-k-1}\mathbf{B}_\mathrm{d}\mathbf{F}_k + \mathbf{D}\mathbf{F}_n \tag{24}$$

Define the response matrix as follows:

$$\mathbf{Y}_n = \left[\mathbf{y}_n^{(1)}, \mathbf{y}_n^{(2)}, .., \mathbf{y}_n^{(N_F)}\right], \qquad n = 0,1,2,... \tag{25}$$

where $\mathbf{y}_n^{(i)}$ is the model output at n^th time step subjected to excitation $\mathbf{F}_0 = [0,...,0,1,0,...,0]^\mathrm{T}$ (unity at the i^th component) and $\mathbf{F}_n = \mathbf{0}$, $n > 0$ with zero initial condition $\mathbf{X}_0 = \mathbf{0}$. Then, Eq. (24) gives the following relationship:

$$\begin{aligned}\mathbf{Y}_0 &= \mathbf{D} \\ \mathbf{Y}_{n+1} &= \mathbf{M}_n = \mathbf{C}\mathbf{A}_\mathrm{d}^n\mathbf{B}_\mathrm{d}, \quad n = 0,1,2,...\end{aligned} \tag{26}$$

where $\mathbf{M}_n = \mathbf{C}\mathbf{A}^n\mathbf{B}_\mathrm{d}, n = 0,1,2,...,$ are called the *Markov parameters*. Then the *Hankel matrix* can be constructed as follows:

$$\mathbf{H}(n) = \begin{bmatrix} \mathbf{M}_n & \mathbf{M}_{n+1} & \cdots & \mathbf{M}_{n+s_2-1} \\ \mathbf{M}_{n+1} & \mathbf{M}_{n+2} & \cdots & \mathbf{M}_{n+s_2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_{n+s_1-1} & \mathbf{M}_{n+s_1} & \cdots & \mathbf{M}_{n+s_1+s_2-2} \end{bmatrix} \tag{27}$$

where the choice of the values of s_1 and s_2 depends on the number of significant modes contributing to the structural response. Details can be found in Juang and Pappa (1985) and Dohner (1994). For identification purpose, this matrix can be estimated using the measured pulse response due to Eq. (26):

$$\mathbf{H}(n) \approx \begin{bmatrix} \mathbf{Y}_{n+1} & \mathbf{Y}_{n+2} & \cdots & \mathbf{Y}_{n+s_2} \\ \mathbf{Y}_{n+2} & \mathbf{Y}_{n+3} & \cdots & \mathbf{Y}_{n+s_2+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{n+s_1} & \mathbf{Y}_{n+s_1+1} & \cdots & \mathbf{Y}_{n+s_1+s_2-1} \end{bmatrix}, \quad n \geq 0 \tag{28}$$

Substituting Eq. (26) to this equation, the Hankel matrix can be factorized as follows:

$$\mathbf{H}(n) = \mathbf{H}_{\mathrm{L}} \mathbf{A}_{\mathrm{d}}^{n} \mathbf{H}_{\mathrm{R}}, n \geq 0 \tag{29}$$

where $\mathbf{H}_{\mathrm{L}}$ and $\mathbf{H}_{\mathrm{R}}$ are the *observability matrix* and *controllability matrix* (Juang et al. 1992):

$$\mathbf{H}_{\mathrm{L}} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{A}_{\mathrm{d}} \\ \vdots \\ \mathbf{C}\mathbf{A}_{\mathrm{d}}^{s_1-1} \end{bmatrix} \text{ and } \mathbf{H}_{\mathrm{R}} = \begin{bmatrix} \mathbf{B}_{\mathrm{d}} & \mathbf{A}_{\mathrm{d}}\mathbf{B}_{\mathrm{d}} & \cdots & \mathbf{A}_{\mathrm{d}}^{s_2-1}\mathbf{B}_{\mathrm{d}} \end{bmatrix} \tag{30}$$

In order to determine these two matrices, singular value decomposition is applied to the Hankel matrix with $n = 0$:

$$\mathbf{H}(0) = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}} \tag{31}$$

where the matrices $\mathbf{U} \in \mathbf{R}^{s_1 N_{\mathrm{o}} \times s_1 N_{\mathrm{o}}}$ and $\mathbf{V} \in \mathbf{R}^{s_2 N_{\mathrm{F}} \times s_2 N_{\mathrm{F}}}$ are unitary. The singular value decomposition can be proceeded using the function 'svd' in MATLAB (MATLAB 2002). The matrix $\mathbf{S} \in \mathbf{R}^{s_1 N_{\mathrm{o}} \times s_2 N_{\mathrm{F}}}$ contains the singular values of $\mathbf{H}(0)$ on its diagonal entries and it can be partitioned as $\mathbf{S} = \begin{bmatrix} \mathbf{S}_{\mathrm{s}} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_0 \end{bmatrix}$ where $\mathbf{S}_{\mathrm{s}} \in \mathbf{R}^{2N_{\mathrm{d}} \times 2N_{\mathrm{d}}}$ and $\mathbf{S}_0 \in \mathbf{R}^{(s_1 N_{\mathrm{o}} - 2N_{\mathrm{d}}) \times (s_2 N_{\mathrm{F}} - 2N_{\mathrm{d}})}$. By using this partition, Eq. (31) can be rewritten as:

$$\mathbf{H}(0) = \mathbf{H}_{\mathrm{L}}\mathbf{H}_{\mathrm{R}} = \begin{bmatrix} \mathbf{U}_{\mathrm{s}} & \mathbf{U}_0 \end{bmatrix} \begin{bmatrix} \mathbf{S}_{\mathrm{s}} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{\mathrm{s}}^{\mathrm{T}} \\ \mathbf{V}_0^{\mathrm{T}} \end{bmatrix} \tag{32}$$

For noise-free cases, $\mathbf{S}_0 = \mathbf{0}$ and the rank of $\mathbf{S}$ is given by $\mathrm{rank}(\mathbf{S}) = 2N_{\mathrm{d}}$. Therefore, $\mathbf{H}(0) = \mathbf{U}_{\mathrm{s}}\mathbf{S}_{\mathrm{s}}\mathbf{V}_{\mathrm{s}}^{\mathrm{T}}$. For general noisy measurements, the values of the diagonal entries in $\mathbf{S}_0$ are closed to zero (Zeiger and McEwan 1974). Therefore, the Hankel matrix $\mathbf{H}(0)$

satisfies the following approximation:

$$\mathbf{H}(0) \approx \mathbf{U}_\mathrm{s}\mathbf{S}_\mathrm{s}\mathbf{V}_\mathrm{s}^\mathrm{T} \tag{33}$$

Then, the observability and controllability matrix can be determined as follows (Juang and Pappa 1985):

$$\begin{aligned} \mathbf{H}_\mathrm{L} &= \mathbf{U}_\mathrm{s}\mathbf{S}_\mathrm{s}^{1/2} \\ \mathbf{H}_\mathrm{R} &= \mathbf{S}_\mathrm{s}^{1/2}\mathbf{V}_\mathrm{s}^\mathrm{T} \end{aligned} \tag{34}$$

By using Eq. (30), one can extract the matrix $\mathbf{B}_\mathrm{d}$ from the first N_F columns of the controllability matrix $\mathbf{H}_\mathrm{R}$ and the matrix $\mathbf{C}$ from the first N_o rows of the observability matrix $\mathbf{H}_\mathrm{L}$.

Finally, the state-space system matrix $\mathbf{A}_\mathrm{d}$ can be determined by taking $n = 1$ in Eq. (29):

$$\mathbf{A}_\mathrm{d} = \mathbf{H}_\mathrm{L}^{+}\mathbf{H}(1)\mathbf{H}_\mathrm{R}^{+} = \mathbf{S}_\mathrm{s}^{-1/2}\mathbf{U}_\mathrm{s}^\mathrm{T}\mathbf{H}(1)\mathbf{V}_\mathrm{s}\mathbf{S}_\mathrm{s}^{-1/2} \tag{35}$$

where the superscript $^+$ denotes the generalized inverse of a matrix. Furthermore, the modal parameters of the system (i.e., the modal frequencies, damping ratios and mode shapes) can be obtained by solving the eigenvalue problem with the identified system matrix $\mathbf{A}_\mathrm{d}$.

The identification procedure of ERA can be summarized as follows:

(1) Construct the Hankel matrices $\mathbf{H}(0)$ and $\mathbf{H}(1)$ with measured pulse response using Eq. (28);
(2) Compute the matrix $\mathbf{D}$ by Eq. (26);
(3) Apply singular value decomposition to $\mathbf{H}(0)$ to obtain $\mathbf{U}_\mathrm{s}$, $\mathbf{S}_\mathrm{s}$ and $\mathbf{V}_\mathrm{s}$;
(4) Compute the observability matrix $\mathbf{H}_\mathrm{L}$ and controllability matrix $\mathbf{H}_\mathrm{R}$ using Eq. (34);
(5) Extract the matrices $\mathbf{B}_\mathrm{d}$ and $\mathbf{C}$ from $\mathbf{H}_\mathrm{L}$ and $\mathbf{H}_\mathrm{R}$ by Eq. (30);
(6) Compute $\mathbf{A}_\mathrm{d}$ using Eq. (35);
(7) Solve the eigenvalue problem of $\mathbf{A}_\mathrm{d}$ to obtain the modal parameters of the system.

In order to improve the accuracy of the ERA algorithm with noisy measurement, Juang et al. (1987) proposed an alternative approach, namely the eigensystem realization algorithm with data correlation (ERA/DC). Instead of using the measurements to form directly the Hankel matrix, the ERA/DC method uses the data correlation matrices (derived from the original Hankel matrix). This method was shown effective in reducing the bias due to measurement noise (Juang and Pappa 1986; Juang 1987). Significant research efforts have been devoted to improve this algorithm (Juang 1997; De Callafon et al. 2008; Chiang and Lin 2010).

On the other hand, ERA or ERA/DC were originally derived to handle pulse response data. However, the ERA or ERA/DC method can also handle response of broad band excitation which is usually encountered in *ambient vibration survey* (Doebling et al. 1998). In such case, preprocessing of the measured response is necessary. One popular approach is to use the random decrement technique to compute the pulse response from the broad band response measurement (Vandiver et al. 1982). Then, ERA or ERA/DC can be applied for parametric identification.

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- Member, SC-1 Subcommittee on Computational Stochastic Mechanics, International Association on Structural Safety and Reliability (since 2005)
- Member, SC-5 Subcommittee on Systems Identification and Structural Control, International Association on Structural Safety and Reliability (since 2002)
- Member, Dynamics Committee, American Society of Civil Engineers (ASCE) (2002-2006)
- Member, Probabilistic Methods Committee, American Society of Civil Engineers (ASCE) (2002-2006)
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