TRANSIENT STABILITY IN POWER SYSTEMS

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Contents

Summary

Power systems are constantly subject to disturbances. Such disturbances cause the power system to deviate from its steady state and experience transients. The ability of the power system to recover from transients is the subject of transient stability analysis, which is discussed in this chapter. Depending on the magnitude of the disturbance and its main effect, different types of stability are defined: rotor angle stability, voltage stability, and frequency stability, where the first two are further divided into small-signal stability and large-signal stability. This chapter discusses the Equal Area Criterion for assessing rotor stability for a single-machine connected to an infinite bus. This is then extended to the case of multiple machines. Transient stability of power systems with wind energy systems is also discussed in an in-depth example.

1. Introduction

Power systems never operate at steady state. The load on the system continuously changes and the generators continuously respond to the load change to maintain the system frequency within acceptable levels. The power system is also subject to disturbances due to faults. Faults are detected by protection systems, and faulted components are removed to prevent the disturbance from spreading into the rest of the network. These disturbances result in a mismatch of power generation and consumption, which in turn result in disturbing the system frequency, voltages, and the speed of generators. A stable power system is capable of returning to a new steady state operation with satisfactory voltage levels and system frequency. The stability of power systems can be classified based on the following considerations (IEEE/CIGRE Joint Task Force, 2004):

- The physical nature of the resulting mode of instability as indicated by the main variable in which instability can be observed.
- The size of the disturbance, which influences the method of calculation and prediction of stability.
- The devices, processes, and the time span that must be taken into consideration in order to assess stability.

This chapter is devoted to transient rotor angle stability. It is important to understand the different forms of power system instability before going into the details of Transient Rotor Angle Stability. A brief introduction to stability classification is given below. For a more complete discussion, refer to the work by the IEEE/CIGRE Joint Task Force (2004).

1.1. Rotor angle stability

In rotor angle instability, the instability is observed in the rotor angle of the machine, either as monotonically increasing rotor angle that leads to loss of synchronism, or as oscillatory swings of the rotor angle with increasing amplitude. This leads to the following classification of rotor angle instability:

Small-disturbance (*small-signal*) *rotor angle stability* is the ability of the power system to maintain synchronism under small disturbances. If the changes in system variables caused by the disturbance are sufficiently small that the behavior of the system can be studied using linear approximations to the system equations, then the disturbance is called a small disturbance. Instability occurs due to the insufficient damping torque.

Large-disturbance rotor angle stability (*transient stability*) is the ability of the power system to maintain synchronism under large disturbances. If the changes in system variables caused by the disturbance are large enough to make the linear approximation to the system equations unacceptable, then the disturbance is called a large disturbance. Instability occurs due to the insufficient synchronizing torque.

1.2. Voltage stability

Voltage stability is the ability of maintaining steady voltages at all busses after being subjected to a disturbance from a given initial operating condition. This depends on the ability to maintain/restore equilibrium between load demand and load supply from the power system. Voltage Collapse refers to the process by which the sequence of events accompanying voltage instability leads to a blackout or abnormally low voltages in a significant part of the power system.

A voltage drop can also be resulted from rotor angle instability. When the rotor angle separation between two groups of machines approaches 180º, the voltages at the intermediate points (electric centre) drop to a low value. Voltage stability is divided into two classes, large-disturbance and small-disturbance voltage stability.

Large-disturbance voltage stability refers to the systems ability to maintain steady voltages following large disturbances such as system faults, loss of generation, or circuit contingencies. Non-linear simulation is used to assess this.

Small-disturbance voltage stability refers to the systems ability to maintain steady voltages following small disturbances such as incremental changes in system load. Both linearized models and non-linear simulation are used to assess this.

The IEEE/CIGRE report clearly states that the distinction between rotor angle stability and voltage stability is not made based on weak coupling between variations in active power/angle and reactive power/voltage magnitudes. For highly stressed systems this coupling is strong. The distinction is based on the opposing force that experiences the sustained imbalance and the power system variable (rotor angle or voltage magnitude) in which the consequent instability is apparent.

1.3. Frequency stability

Frequency Stability refers to the ability of a power system to maintain steady frequency following a severe system upset resulting in a significant imbalance between generation and load (Christie & Bose, 1996). A common situation is when a large disturbance leads to a break up of the power system into smaller subsystems leaving each subsystem with a mismatch between the generation and load. In such situations the stability is maintained either by load shedding or by generator tripping.

2. Models for Transient Rotor Angle Stability

During steady state operation the rotor of the machine rotates at the synchronous speed. The balanced three phase currents in the stator winding produce a rotating magnetic field in the air gap. This rotating magnetic field also rotates at the synchronous speed. The flux linkages in the field winding (which is on the rotor) due to the rotating magnetic field are constant during the steady state operation as both the rotor and the rotating magnetic field rotate at the same speed.

During a disturbance, the rotor speed changes, and consequently, the field winding will no longer see a constant flux linkage in it due to the rotating magnetic field. When the disturbance occurs, the flux linkages in the field winding cannot change instantaneously. There will be induced currents in the field winding and the damper windings (including the rotor body) to counteract the change in flux. The currents induced in the damper windings will decay quickly. The induced currents in the field winding will take longer to decay. The initial period is known as the *subtransient* period, whereas the subsequent period in which the induced currents in the damper windings have decayed but the induced currents in the field winding are still present is called the *transient* period.

If we are interested only in studying the behavior of the system during the first swing of the rotor, the synchronous machine can be modeled as a constant internal voltage in series with the transient reactance, representing constant flux linkage in the field winding (Grainger & Stevenson, 1994). The phase angle of the constant internal voltage represents the position of the rotor with respect to a reference axis rotating at the synchronous speed.

For accurate studies, the transients in the damper windings and the field winding, as well as the saliency of the machine, must be taken into account (the flux linkage and hence the internal voltage are not constant) (IEEE Committee Report, 1981). In addition to these, the presence of the excitation system controller will cause the field voltage to change, and the presence of the speed governing system will cause the mechanical torque (power) on the shaft to change. These effects must also be modeled for accurate studies (IEEE Std 421.5, 1992).

The following example illustrates how to determine the variables of concern during steady state operation, prior to the disturbance.

The synchronous machine shown in Figure 1 generates 1.0 pu output power. The transient reactance x'_{d} of the generator is 0.2 pu as shown. The terminal voltage of the generator (V_t) and the infinite bus voltage (V_∞) are both 1.0 pu.

a) Determine the magnitude (E') and phase angle (δ) of the internal voltage.
b) One of the transmission lines is suddenly tripped off (see Figure 2). Determine the new phase angle of the internal voltage, assuming that E' does not change.

c) Can this angle change instantaneously?

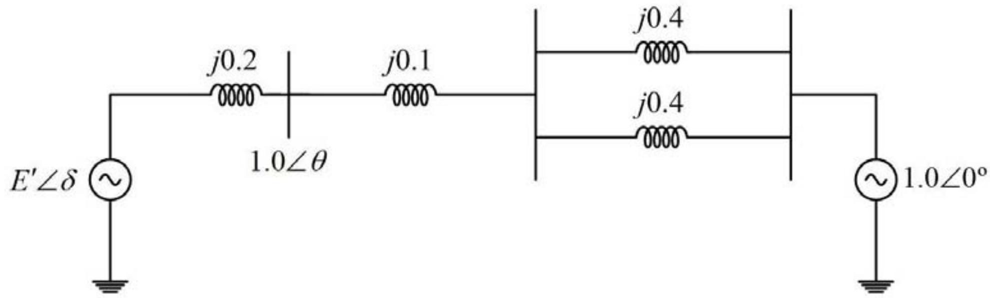

Figure 1. Simple power system.

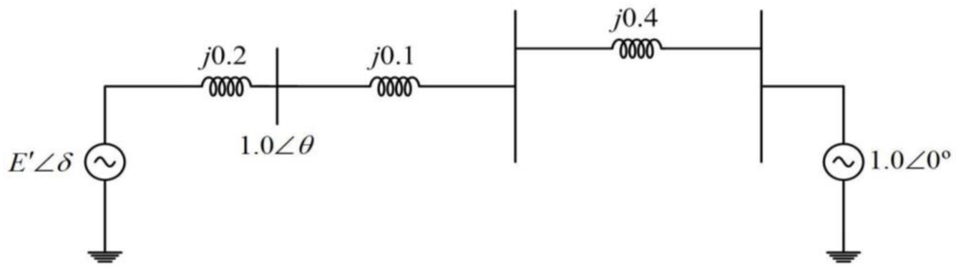

Figure 2. Simple power system after loss of a line.

a) Let $V_t = 1.0\angle\theta$. The net impedance between the generator terminal and the infinite bus is $j0.3$. The angle θ can be determined using the following expression:

$$P = \frac{V_t V_\infty}{X}\sin\delta$$

That is,

$$1.0 = \frac{(1.0)(1.0)}{0.3}\sin\theta \;\Rightarrow\; \theta = 0.3047$$

The generator current I is given by:

$$\frac{1.0\angle 0.3047 - 1.0}{j0.3} = 1.0 + j0.15$$

and,

$$E'\angle\delta = V_t + j0.2I = 1.05\angle 0.496$$

b) When one of the lines is tripped off, the net impedance between the generator internal voltage $E'\angle\delta$ and the infinite bus is 0.7 pu. Thus, the new value of δ can be calculated as follows:

$$1.0 = \frac{(1.05)(1.0)}{0.7}\sin\delta \quad \Rightarrow \quad \delta = 0.7297$$

c) The rotor angle δ does not change instantaneously from the initial value of 0.3047 to the new value of 0.7297. The rotor angle δ is the position of the rotor with respect to a reference axis rotating at the synchronous speed ω_s. Let the speed of the rotor be ω.

At steady state $\omega = \omega_\mathrm{s}$, δ is constant and $d\delta / dt = \omega - \omega_\mathrm{s} = 0$.

During the disturbance, the speed of the rotor ω increases/decreases when the rotor accelerates/ decelerates. The rotor position δ increases when the speed of the rotor is greater than the synchronous speed, and decreases when the speed of the rotor is smaller than the synchronous speed. Therefore, δ and ω do not change instantaneously. In the next section we model the rotor dynamics in order to investigate how δ and ω change with time subsequent to a disturbance in the power system.

3. The Swing Equation

The equation of motion of the rotor is given by (Grainger & Stevenson, 1994):

$$J\frac{d^2\theta_\mathrm{m}}{dt^2} + D\frac{d\theta_\mathrm{m}}{dt} = T_\mathrm{a} = T_\mathrm{m} - T_\mathrm{e} \qquad (1)$$

where J is the total moment of inertia of the rotor mass in kgm^2, D is the damping constant, θ_m is the angular displacement of the rotor with respect to a stationary axis in mechanical radians per second, t is time in seconds, T_m is the mechanical or shaft torque supplied by the prime mover less retarding torque due to rotational losses in Nm, T_e is the electromagnetic torque on the shaft (output electrical power plus losses) in Nm, and T_a is the net accelerating torque in Nm.

To simplify the equations, damping is neglected in the rest of this chapter. Since damping has a stabilizing effect, its omission leads to conservative results.

Under the steady state operation $T_\mathrm{m} = T_\mathrm{e}$, and there is no accelerating torque. The position of the rotating reference as a function of time is $\omega_\mathrm{sm}t$, where ω_sm is the synchronous speed in mechanical rad/s. The position of the rotor with respect to the reference is δ_m. Therefore, the absolute position of the rotor in mechanical radians is given by the following equation:

$$\theta_{\mathrm{m}} = \omega_{\mathrm{sm}} t + \delta_{\mathrm{m}} \tag{2}$$

Differentiating Eq. (2) with respect to time, we have:

$$\frac{d\theta_{\mathrm{m}}}{dt} = \omega_{\mathrm{sm}} + \frac{d\delta_{\mathrm{m}}}{dt} \tag{3}$$

Further, differentiating Eq. (2) one more time with respect to time, we have:

$$\frac{d^2\theta_{\mathrm{m}}}{dt^2} = \frac{d^2\delta_{\mathrm{m}}}{dt^2} \tag{4}$$

Therefore, Eq. (1) can be written as:

$$J\,\frac{d^2\delta_{\mathrm{m}}}{dt^2} = T_{\mathrm{a}} = T_{\mathrm{m}} - T_{\mathrm{e}} \tag{5}$$

Upon multiplying both sides of the equation by the angular velocity of the rotor, Eq. (5) can be expressed in terms of power as follows:

$$J\omega_{\mathrm{m}}\,\frac{d^2\delta_{\mathrm{m}}}{dt^2} = P_{\mathrm{a}} = P_{\mathrm{m}} - P_{\mathrm{e}} \tag{6}$$

where P_{m} is the shaft power, P_{e} is the electrical power crossing the air gap, P_{a} is the accelerating power, and ω_{m} is the angular velocity of the rotor in mechanical radians per second.

If the rotational losses and the ri^2 losses in the windings are neglected, P_{m} is the mechanical power input to the shaft and P_{e} is the electrical power output.

The product $J\omega_{\mathrm{sm}}$ is called the inertia constant of the machine and is denoted by M. Using M and assuming $\omega_{\mathrm{m}} \approx \omega_{\mathrm{sm}}$, Eq. (6) becomes:

$$M\,\frac{d^2\delta_{\mathrm{m}}}{dt^2} = P_{\mathrm{a}} = P_{\mathrm{m}} - P_{\mathrm{e}} \tag{7}$$

Since M depends on the machine size, its value has wide variations for different machines. Therefore, a normalized quantity H is used instead, which is defined as:

$$H = \frac{\text{Kinetic energy of the rotor at synchronous speed } \omega_{\text{sm}} \text{ in MJ}}{\text{Machine rating in MVA}}$$

$$= \frac{\frac{1}{2} J \omega_{\text{sm}}^2}{S_{\text{rated}}} \tag{8}$$

$$= \frac{\frac{1}{2} M \omega_{\text{sm}}}{S_{\text{rated}}} \quad \text{MJ/MVA}$$

Therefore, M can be expressed in terms of H as:

$$M = \frac{2H}{\omega_{\text{sm}}} S_{\text{rated}} \quad \text{MJ/mechanical radians} \tag{9}$$

Substituting for M in Eq. (7) and dividing the resulting equation by S_{rated} gives:

$$\frac{2H}{\omega_{\text{sm}}} \frac{d^2 \delta_{\text{m}}}{dt^2} = \frac{P_{\text{a}}}{S_{\text{rated}}} = \frac{P_{\text{m}} - P_{\text{e}}}{S_{\text{rated}}} \tag{10}$$

Note that the right hand side of Eq. (10) is the per unit acceleration power.

If we use electrical degrees for δ, electrical radians per second for ω, and per unit for acceleration power, Eq. (10) can be written as:

$$\frac{2H}{\omega_{\text{s}}} \frac{d^2 \delta}{dt^2} = P_{\text{m}} - P_{\text{e}} \tag{11}$$

which is called the swing equation.

Alternative forms of the swing equation can be obtained by substituting $\omega = 2\pi f$, where f is the power frequency. If δ_{m} is expressed in electrical radians, then:

$$\frac{H}{\pi f} \frac{d^2 \delta}{dt^2} = P_{\text{m}} - P_{\text{e}} \tag{12}$$

When δ is expressed in electrical degrees,

$$\frac{H}{180 f} \frac{d^2 \delta}{dt^2} = P_{\text{m}} - P_{\text{e}} \tag{13}$$

The second-order swing equation given in (12) can be split into two simultaneous first-order equations as follows:

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_\mathrm{m} - P_\mathrm{e} \tag{14a}$$

$$\frac{d\delta}{dt^2} = \omega - \omega_\mathrm{s} \tag{14b}$$

Consider the small power system in Figure 1 where there is a generator connected to a large power system (which can be considered as an infinite bus) through parallel transmission lines and a transformer. The impedances shown are in per unit on a common system base. The machine is delivering 1.0 per unit power and both the terminal voltage and the infinite bus voltage are 1.0 per unit.

The following MATLAB m-file can be used to solve the swing equation. The method used to solve the swing equation is RK-2 (Runge-Kutta). Figure 3 shows the variation of δ with time when there is no damping and the damping is present ($D = 10$). The value of $D = 10$ is not typical, but chosen to demonstrate that δ reaches a different steady state value. Also note that, although the simulation has been carried out for 5 seconds, our assumption that E' is constant is not good for that long. However, the two figures give a qualitative idea on what happens to the rotor angle δ during the disturbance.

```
clear all
%Enter the mechanical power and Pmax for pre-fault,
%during the fault and
%post-fault power angle equations below.
pm= ;
pmax_pref=;
pmax_f=;
pmax_postf=;
%Enter the H constant of the machine
H=;
%Enter the fault clearing time
tc=;
delta(1)=asin(pm/pmax_pref); del_wr(1)=0.0; dt=0.01; w0=377.0;
D=10.0; tfinal=5.0; M=2*H;
%-------------------------------------------------
% d/dt(del_wr)=(1/2H)(pm-pe)
% d/dt(delta)=w0*del_wr
% Pe = Pmax sin (delta) + D del_wr
%-------------------------------------------------
max_index=round(tfinal/dt);
   for index=1:max_index
   time(index)=index*dt;
   if (index*dt<tc)
   k11=dt*(pm-pmax_f*sin(delta(index))-D*del_wr(index))/M;
   k21=dt*w0*del_wr(index);
   k12=dt*(pm-pmax_f*sin(delta(index)+k21)-D*(del_wr(index)+k11))/M;
   k22=dt*w0*(del_wr(index)+k11);
   del_wr(index+1)=del_wr(index)+(k11+k12)/2;
   delta(index+1)=delta(index)+(k21+k22)/2;
   else
   k11=dt*(pm-pmax_postf*sin(delta(index))-D*del_wr(index))/M;
   k21=dt*w0*del_wr(index);
```

```
    k12=dt*(pm-pmax_postf*sin(delta(index)+k21)-
    D*(del_wr(index)+k11))/M;
    k22=dt*w0*(del_wr(index)+k11);
    del_wr(index+1)=del_wr(index)+(k11+k12)/2;
    delta(index+1)=delta(index)+(k21+k22)/2;
    end
end
len=max(size(time));
figure(1)
plot(time,delta(1:len))
figure(2)
plot(time,del_wr(1:len))
```

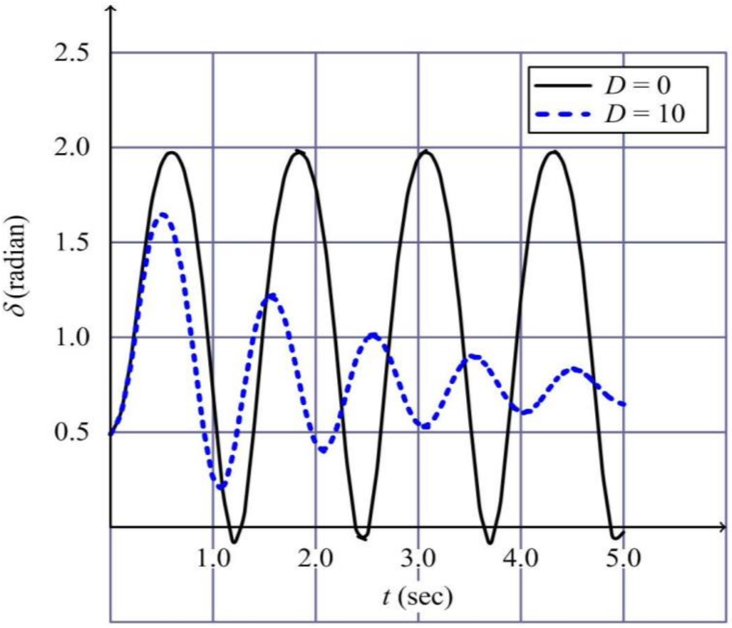

Figure 3. Rotor angle δ vs time - $D=0$ and $D=10$.

The response of the rotor of the generator to a small disturbance can be studied by approximating the nonlinear swing Eq. (11) with a linear second order differential equation. Consider a small change in δ_{Δ} around its steady state value of δ_0. The electrical power P_e is given by:

$$P_e = P_{max}\sin(\delta_0 + \delta_{\Delta})$$

The swing Eq. (11) can be written as:

$$\frac{2H}{\omega_s}\frac{d^2(\delta_0 + \delta_{\Delta})}{dt^2} = P_m - P_{max}\sin(\delta_0 + \delta_{\Delta}) \tag{15}$$

The nonlinear term on the right hand side can be expanded as a Taylor series as follows:

$$P_{\max}\sin(\delta_0+\delta_\Delta)=P_{\max}\sin(\delta_0)+P_{\max}\cos(\delta_0)\delta_\Delta-\frac{1}{2}P_{\max}\sin(\delta_0)\delta_\Delta^2+\ldots$$

For small δ_Δ, the term δ_Δ^2 and higher order terms can be ignored. Then, Eq. (15) can be written as:

$$\frac{2H}{\omega_\mathrm{s}}\frac{d^2(\delta_\Delta)}{dt^2}=P_\mathrm{m}-P_{\max}\sin(\delta_0)-P_{\max}\cos(\delta_0)\delta_\Delta \quad (16)$$

But, $P_\mathrm{m}=P_{\max}\sin(\delta_0)$. Therefore, Eq. (15) simplifies to:

$$\frac{2H}{\omega_\mathrm{s}}\frac{d^2(\delta_\Delta)}{dt^2}=-P_{\max}\cos(\delta_0)\delta_\Delta \quad (17)$$

Definition: The sensitivity of P_e to δ at $\delta=\delta_0$ is defined as the *synchronizing power coefficient* S_p. Note that $S_\mathrm{p}=P_{\max}\cos(\delta_0)$. That is:

$$\frac{d^2(\delta_\Delta)}{dt^2}=-\frac{\omega_\mathrm{s}S_\mathrm{p}}{2H}\delta_\Delta \quad (18)$$

Equation (18) describes a simple harmonic motion with the natural frequency of oscillation f_n given by:

$$\omega_\mathrm{n}=\sqrt{\frac{\omega_\mathrm{s}S_\mathrm{p}}{2H}}$$

4. Equal Area Criterion

The stability of one synchronous machine connected to a large system or two synchronous machines connected through a transmission system can be analyzed using the equal area criterion. The equal area criterion is explained using the example power system of Figure 4: a synchronous generator connected to an infinite bus through a double-circuit line and a step-up transformer. When using this method, the synchronous machine is modeled as a constant voltage *E*' behind the transient reactance of the machine x'_d (Anderson & Fouad, 2002; Xue, van Cutsem, & Ribbens-Pavella, 1989).

At time zero, a three-phase to ground fault occurs at point P on line 2. The fault changes the power flow in line 2, but line 1 continues to deliver power to the infinite bus. However, the reduction in the available ampacity of the lines may cause instability depending on (i) the parameters of the machine and network before and after the fault (e.g., line impedances, transient reactance, H constant) and level of loading, and (ii) the fault location and clearing time. The equal area criterion provides a simple means to determine if the power system remains stable following this disturbance.

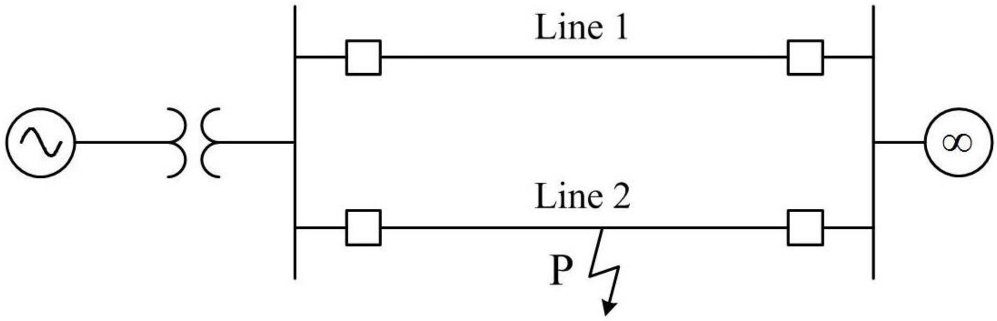

Figure 4. One-line diagram of a synchronous generator connected to an infinite bus.

Figure 5(a) shows the mechanical input power to the shaft of the machine P_{m} and three power-angle curves for three conditions: the curve P_{pre} characterizes the prefault system, P_{fault} characterizes the faulted system, and P_{post} characterizes the system after the operation of the breakers (causing line 2 to go out of operation). These curves are obtained from the following expression, assuming the transmission lines are purely reactive:

$$P = \frac{E' V_\infty}{X} \sin \delta \qquad (19)$$

where E' is the internal voltage of the generator, V_∞ is the voltage of the infinite bus, δ is the phase angle between E' and V_∞, and X is the total impedance between E' and V_∞, which includes the internal transient impedance of the synchronous generator x'_{d}, the impedance of the step-up transformer, and the equivalent impedance of transmission lines. The equivalent impedance X has three distinct values in pre-, mid-, and postfault intervals. At the initial steady-state angle δ_0, P_{pre} equals the mechanical input power P_{m}.

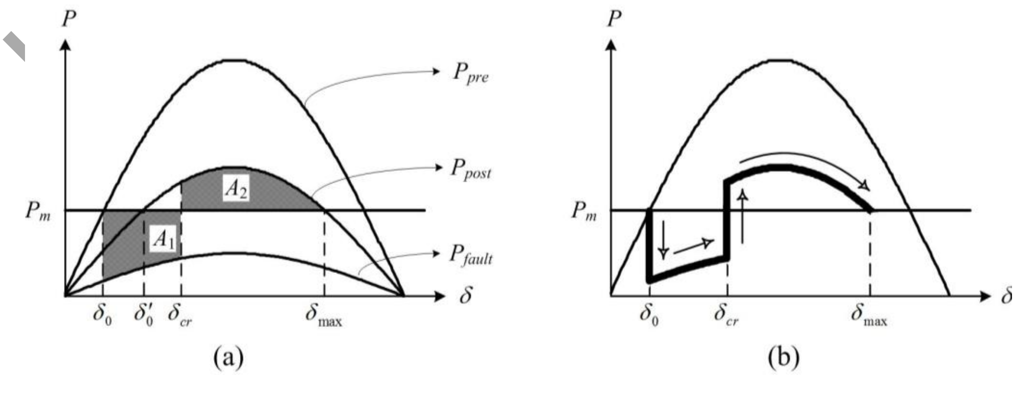

Figure 5. Equal area criterion. (a) areas A_1 and A_2, (b) evolution of the machine angle.

Upon occurrence of the fault, δ cannot change instantaneously; however, the power-angle characteristic of the system instantaneously shifts to the P_{fault} curve, Figure 5(a), causing the electric power P_{e} delivered to the infinite bus to decrease. From the swing equation, the mismatch between the mechanical input power and electrical output power will cause the accelerating power to be positive. Therefore, the machine speed increases beyond the synchronous speed, and as shown in Figure 5(b), δ increases. The increase in the machine rotor speed (defined as $\omega_{\mathrm{r}} = \omega - \omega_{\mathrm{s}}$) increases the kinetic energy of the machine. This energy is proportional to the area A_1, Figure 5(a), between P_{m} and P_{fault} and between δ_0 and δ_{cr}. At δ_{cr}, the fault is cleared by tripping out line 2, and the power-angle characteristic of the system suddenly shifts to the P_{post} curve, Figure 5(b). In this case, the new value of electrical output power P_{e} is greater than the mechanical input power P_{m}. As a result, the machine decelerates and its kinetic energy decreases. However, as long as the rotor speed is higher than the synchronous speed ($\omega_r > 0$), δ continues to increase. The maximum allowable increase of δ is $\delta_{\max}$; if δ increases beyond $\delta_{\max}$, the accelerating power $P_{\mathrm{m}} - P_{\mathrm{post}}$ becomes positive, causing the rotor to accelerate and the angle to increase further. This eventually leads to too high a deviation in the machine speed, and the machine will finally trip out.

The equal area criterion states that for the power system to remain stable, the area A_1 must be less than the area A_2 (the area between P_{m} and P_{post}, and between δ_{cr} and $\delta_{\max}$). That is, the increase in the rotor energy before clearing the fault must be equal to the decrease in the rotor energy after clearing the fault. In this case, the rotor swings between $\delta_{\max}$ and δ_0. Due to mechanical losses, the oscillations gradually damp until the rotor angle settles to δ_0', see Figure 5(a).

Consider the swing equation, reproduced below.

$$\frac{2H}{\omega_{\mathrm{s}}}\frac{d^2\delta}{dt^2} = P_{\mathrm{m}} - P_{\mathrm{e}} \tag{20}$$

Since

$$\frac{d\delta}{dt} = \omega_{\mathrm{r}} = \omega - \omega_{\mathrm{s}} \tag{21}$$

we have

$$\frac{2H}{\omega_{\mathrm{s}}}\frac{d\omega_{\mathrm{r}}}{dt} = P_{\mathrm{m}} - P_{\mathrm{e}} \tag{22}$$

Multiplying both sides of Eq. (22) by ω_{r},

$$\frac{H}{\omega_{\mathrm{s}}} 2\omega_{\mathrm{r}} \frac{d\omega_{\mathrm{r}}}{dt} = \left(P_{\mathrm{m}} - P_{\mathrm{e}}\right)\frac{d\delta}{dt} \tag{23}$$

and

$$\frac{H}{\omega_{\mathrm{s}}} \frac{d\omega_{\mathrm{r}}^{2}}{dt} = \left(P_{\mathrm{m}} - P_{\mathrm{e}}\right)\frac{d\delta}{dt} \tag{24}$$

Integrating Eq. (24) with respect to time from (ω_{r0} ; δ_0) to ($\omega_{\mathrm{r}x}$; δ_x),

$$\frac{2H}{\omega_{\mathrm{s}}}\left(\omega_{\mathrm{r}x}^{2} - \omega_{\mathrm{r0}}^{2}\right) = \int_{\delta_0}^{\delta_x}\left(P_{\mathrm{m}} - P_{\mathrm{e}}\right)d\delta \tag{25}$$

But $\omega_{\mathrm{r0}} = 0$ and $\omega_{\mathrm{r}x} = 0$. Therefore,

$$\int_{\delta_0}^{\delta_x}\left(P_{\mathrm{m}} - P_{\mathrm{e}}\right)d\delta = 0 \tag{26}$$

which can be expressed as:

$$\int_{\delta_0}^{\delta_c}\left(P_{\mathrm{m}} - P_{\mathrm{e}}\right)d\delta = \int_{\delta_c}^{\delta_x}\left(P_{\mathrm{e}} - P_{\mathrm{m}}\right)d\delta \tag{27}$$

Note that the term

$$\int_{\delta_0}^{\delta_c}\left(P_{\mathrm{m}} - P_{\mathrm{e}}\right)d\delta$$

is proportional to the kinetic energy absorbed by the rotating mass during the fault and

$$\int_{\delta_c}^{\delta_x}\left(P_e - P_m\right)d\delta$$

is proportional to the kinetic energy released by the rotating mass after clearing the fault. If the absorbed kinetic energy can be released to the network, then the machine is stable.

The critical clearing time (angle) is the maximum time (angle) of clearing a fault upon which the system can maintain its stability. This angle is found using the equal area criterion $A_1 = A_2$. The areas A_1 and A_2 in Figure 5(a) are obtained as follows:

$$A_1 = \int_{\delta_0}^{\delta_{\mathrm{cr}}}\left(P_{\mathrm{m}} - P_{\mathrm{fault}}\sin\delta\right)d\delta = P_{\mathrm{m}}\left(\delta_{\mathrm{cr}} - \delta_0\right) + P_{\mathrm{fault}}\left(\cos\delta_{\mathrm{cr}} - \cos\delta_0\right) \tag{28a}$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} \left(P_{post}\sin\delta - P_m\right) d\delta = P_{post}\left(\cos\delta_{cr} - \cos\delta_{max}\right) - P_m\left(\delta_{max} - \delta_{cr}\right) \qquad (28b)$$

Equate A_1 and A_2 to calculate the critical clearing angle as:

$$\cos\delta_{cr} = \frac{P_m\left(\delta_{max} - \delta_0\right) - P_{fault}\cos\delta_0 + P_{post}\cos\delta_{max}}{P_{post} - P_{fault}} \qquad (29)$$

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