

# POWER AND ENERGY MEASUREMENTS

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## Summary

This work presents the theoretical and more practical aspects related to electric power and energy measurements, by introducing the methodology to perform such measurements, then the instruments to implement it. Section 2 deals mainly with the theorem necessary to understand power and energy relationships in the time and phasor domain.

The most important results for analyzing any single-phase or three-phase electrical systems are shown. Section 3 describes the basic structure of wattmeters and energy meters, and presents practical considerations.

## 1. Introduction

Energy is a word that comes from the ancient Greek  $\epsilon\nu\epsilon\rho\gamma\epsilon\iota\alpha$  ( $\epsilon\nu\epsilon\rho\gamma\epsilon\iota\alpha$ ), which means both action and energy. These two meanings exemplify the dynamic character of energy. For example, the derivative of energy,  $E$ , versus any spatial variable,  $x$ , distance or angle:  $\frac{dE}{dx}$ , represents a mechanical action, such as strength or torque; while the

derivative of energy versus the time,  $t$ :  $\frac{dE}{dt}$ , defines the important physical concept of power.

In this article power and energy measurement is explained in both its theoretical and practical implementations. Section 2 is allocated to the theorems necessary to understand power and energy relationships. Section 3 provides an overview of wattmeters and energy meters, and their practical implementations.

## 2. Electric Power Measurements

### 2.1. DC Electric Circuits

In order to identify an electrical system in DC, a measurement of the electrical current,  $I$ , flowing through the bipoles and their electric voltage,  $V$ , across the terminals has to be performed (see Figure 1.). These measurements can be arranged by means of a DC ammeter and voltmeter. The electric power,  $P$ , related to such a bipole, can be defined as the product of the voltage and current applied to:  $P = V \times I$ . The power generated by an active bipole can be defined in the same way

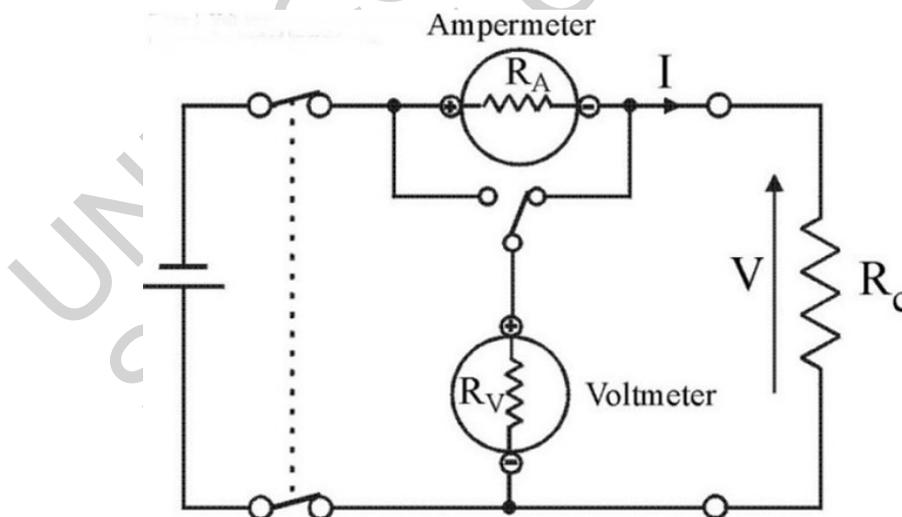


Figure 1. Volt-ampmetric measurement of power,  $P$ , adsorbed by resistor  $R_C$

### 2.2. Single-Phase AC Electric Circuits

The electric power in a single-phase AC circuit can be defined as:

$$p(t) = v(t)i(t) \quad (1)$$

where  $p$ ,  $v$ ,  $i$  are instantaneous power, voltage, and current, respectively.

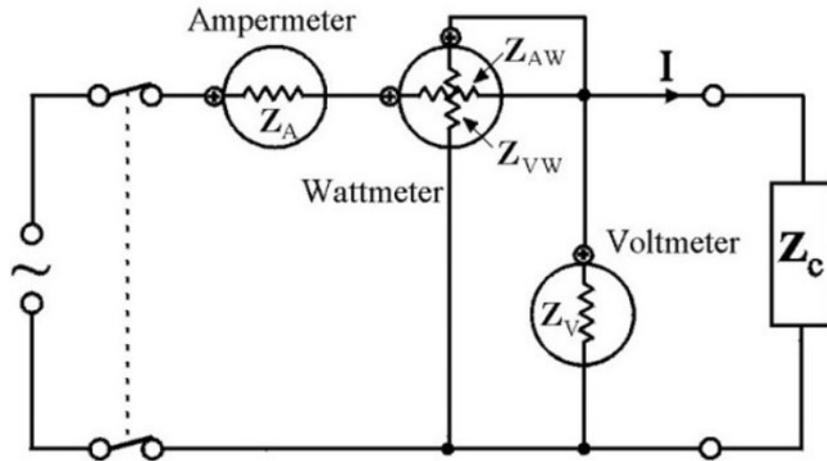


Figure 2a. Measurement of active power,  $P$ , by means of the wattmeter, and apparent power, by means of the voltmeter and ammeter, to identify the power exchange related to the impedance,  $Z_C$

Voltage and current are considered to be sinusoidal, and these electric quantities can be represented by means of phasors and vector operators. It is possible to define the mean power,  $P$ , active power, as the scalar product of the voltage and current phasors. By defining the voltage and current in the following way:

$$v(t) = \sqrt{2}V \sin(\omega t) \text{ and } i(t) = \sqrt{2}I \sin(\omega t + \varphi) \quad (2)$$

the mean power in the period,  $T$ , is equal to:

$$P = \frac{1}{T} \int_0^T v(t)i(t)dt = VI \cos \varphi \quad (3)$$

or, by using the phasor notation:

$$P = \mathbf{V} \bullet \mathbf{I} = VI \cos \varphi \quad (4)$$

where  $\varphi$  is the phase between the current and voltage signal—or between  $\mathbf{V}$  and  $\mathbf{I}$ .

It is important to note that by means of the phasor notation, and remembering the impedance definition for a generic bipole, it is possible to conclude that the mean adsorbed power is:

$$P = \mathbf{V} \bullet \mathbf{I} = (\mathbf{Z}\mathbf{I}) \bullet \mathbf{I} = Z \cos \varphi I^2 = RI^2 \quad (5)$$

When dealing with the sinusoidal regime it is possible to define the apparent power,  $A$ , as a direct product of the voltage and current rms values:

$$A = VI \quad (6)$$

The apparent power,  $A$ , is dimensionally equal to electric power, without representing the effective power responsible for the energy transfer in a section connecting two networks. This type of functionality is demanded by the mean value of the instantaneous power—defined for such reason as active power.

There is a third type of power in AC system: reactive power,  $Q$ , defined as:

$$Q = \pm \sqrt{(A^2 - P^2)} \quad (7)$$

Reactive power can also be defined using the notation of inner product module by multiplying the voltage and current phasors:

$$Q = \pm |\mathbf{V} \times \mathbf{I}| = V \sin \varphi \quad (8)$$

In the same way as arranged in Eq. (5), reactive power can be defined as:

$$Q = XI^2 \quad (9)$$

where  $X$  is the reactance of the equivalent bipole in a series arrangement.

In a single-phase AC system the measurement of active and reactive power can be performed by means of measurement instruments called wattmeters and varmeters, respectively. The measurement of active power requires just one wattmeter—single phase. This instrument is characterized by two pairs of terminals, one for the current circuit, one for the voltage (see Figure 2.a). The first pair of connections have to be inserted to the current circuit; the second to the proper section of the single-phase circuit to which the power flow is related.

The measurement of the complete power flow in single-phase systems—and also polyphase—can be performed by means of computer-based processing systems (see Figure 2b). These devices are able to compute power from voltage and current signals sampled from the network.

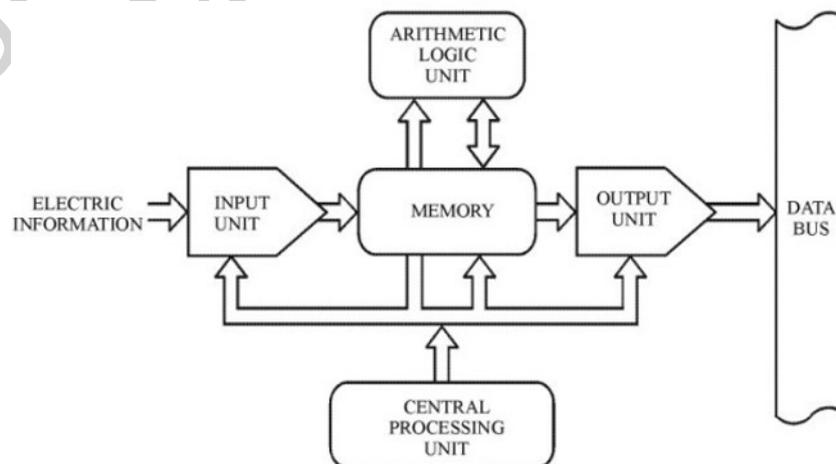


Figure 2b. Block structure of a computer-based processing system able to measure the complete power flow—under the Von Neumann scheme

## 2.3. Power Measurement in Three-Phase Systems

### 2.3.1. Triple Single-Phase Circuits and Three-Phase Circuits

Three-phase circuits—as with single-phase—represent a particular case of polyphase circuits. From the theoretical point of view three-phase circuits can be defined by means of three separate single-phase circuits. For this reason they can allow a power flow three time greater than the corresponding single-phase circuit.

A triple single-phase circuit can be represented in two ways (see Figures 3 and 4).

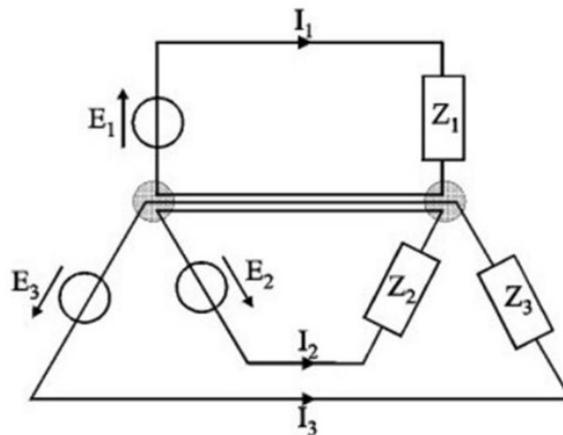


Figure 3. Triple single-phase system built using three separate single-phase systems—six conductors

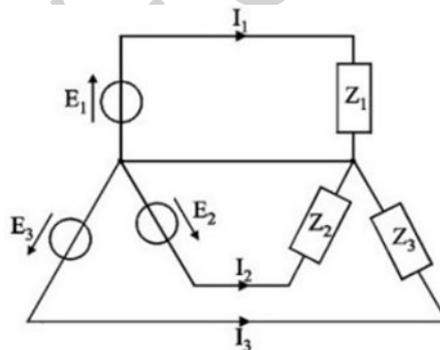


Figure 4. Triple single-phase system having three separate single-phase systems, but with one common return-path—four conductors

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## Biographical Sketches

**Arnaldo Brandolini** was born in 1930 in Calolziocorte, Italy. He received his Master's degree in Electrical Engineering at the Politecnico di Milano, Italy, in 1955. In 1958 he joined the same university as Assistant Professor of Electrical Measurements. He became Associate Professor in 1968. Since 1980 he has been a Full Professor and holds the Chair of Electrical Measurements. In 1996 he was appointed Head of the Department of Electrical Engineering, Politecnico di Milano. His research activities and scientific interests deal with the following: electrical and electronic measurement topics; the design and development of digital computer-based measuring systems; power network digital-protections; new A/D testing methodologies; advanced sensors and sensing systems for robotics; and remote sensing in geophysics. He has published about 100 papers on these subjects and five textbooks in the field of electrical and electronic measurements.

**Alessandro Gandelli** was born in 1960 in Milan, Italy. He received his Bachelor's, Master's, and Ph.D. degrees in Electrical Engineering from the Politecnico di Milano, Italy. In 1990 he joined the faculty of Engineering at the same university, where he is now teaching electrical engineering and applied electronics. Since 1998 he has been a member of the Academic Senate. His research activities and scientific interests deal with: the basic theory of electrical circuit and systems; microelectronics and packaging; power electronics and switching networks; advanced sensors and transducers; and electronic instrumentation. He has published about 85 papers on these subjects and three textbooks in the field of electrical and electronics engineering.