TURBULENT FLOW MODELING

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Contents

Summary

After a historical introduction, this article describes the state of the art and the trails for new developments in the field of turbulent flow modeling. O. Reynolds showed that when a nondimensional parameter called the Reynolds number becomes high, the flow becomes turbulent. This means that perturbations can be amplified and can generate unsteady eddies. Consequently, a turbulent flow is usually unsteady, three-dimensional, and contains eddies of very different sizes. Due to their large Reynolds numbers, most real life flows are turbulent. A first approach for simulating turbulent flows is to compute only the averaged motion. Incompressible turbulent flows obey the Navier-Stokes equations. When the averaged motion is concerned, these equations are averaged using the Reynolds averaging. Then the Reynolds averaged Navier-Stokes equations are obtained. Unfortunately, due to the nonlinear advective term of the Navier-Stokes equations, new unknowns are obtained—the exact Reynolds stresses equations can be derived for the Reynolds stresses, but again, they involve higher order statistical correlations that are unknown. Consequently, the Reynolds averaged Navier-Stokes equations have to be modeled; this is called the "turbulence closure problem." Before introducing the different closures used in practice, this article introduces the Kolmogorov theory, which explains how turbulence is possible, i.e., how turbulent kinetic energy is produced, and how it is dissipated, explaining why various sizes of eddies exist in a turbulent flow. Then the different closures are presented, first the one based on the Boussinesq eddy viscosity concept (the mixing length model and the k-ε model) and then Reynolds stress transport equations models. In parallel, recent approaches based on the direct resolution of the unsteady Navier-Stokes equations are

introduced—direct numerical simulation if all the sizes of eddies are computed, or large eddy simulation if only the largest eddies are computed, the effect of the smallest eddies being introduced through modeling. As a conclusion, the trails for developments in the first decade of the twenty-first century are presented in the fields of both the Reynolds averaged Navier-Stokes equations, and direct numerical simulation.

1. Introduction

"Real flows are beautiful. They are always changing, with eddies randomly forming…" At the end of the fifteenth century, just imagine Leonardo da Vinci (1452–1519), bent over a bridge and gazing at the river, fascinated by these eddies, and trying to catch their beauty by drawing on his notebook. Real flows are beautiful because they are turbulent. A turbulent flow is characterized by its unsteadiness and the presence in the flow of eddies of various sizes. Thus, a turbulent flow is usually three-dimensional. Most flows in nature (flow in atmosphere, flow in rivers) and in industrial devices (piping systems, power plants) are turbulent. After Leonardo's pictures, there was a wait until the nineteenth century for a description of the turbulent motion. The first qualitative description of turbulent flows was published by Hagen in 1854. He obtained flow visualizations by using glass pipes and particle laden flows. He exhibited two regimes of flow: a regular regime with smooth particle paths (laminar flow), and a regime where these paths become chaotic with fluctuations (turbulent flow).

Barré de Saint Venant (1797–1886) was the first to link the stresses in a turbulent flow to the intensity of the eddies created by the flow. Then, Boussinesq (1842–1929), in his "essai sur la theorie des eaux courantes" (theory of fresh waters) presented at the Academy of Sciences of Paris in 1877, demonstrated the necessity of working on averaged quantities for the study of turbulent flows. He also introduced the eddy viscosity concept, which is used in a large number of turbulence models. He linked the eddy viscosity to the intensity of the mean fluctuations produced in a turbulent flow.

Osborne Reynolds (1842–1912) exhibited a nondimensional parameter which is related to the transition between the laminar and the turbulent flow regime. This parameter is called the Reynolds number, $\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$, where V is the velocity, D a length scale, ρ the density, μ the dynamic viscosity of the fluid, and $\nu = \frac{\mu}{\rho}$ the kinematic viscosity of the fluid). However, it was Lord Kelvin (1824–1907) who introduced the denomination of "turbulence."

At the beginning of the twentieth century, it was the German school of Göttingen University, created by Ludwig Prandtl (1875–1953), which generated advances in turbulent flow modeling, and especially in the field of turbulent near-wall flows. Prandtl introduced the first turbulence model—the mixing length model, which is well suited for the computation of turbulent near-wall flows. Blasius (1883–1970) produced experimental results of turbulent channel flows and Von Karman (1881–1963) worked on turbulent wakes behind obstacles and on turbulent boundary layers.

Kolmogorov (1903–1987) introduced in 1942 the concept of the turbulent energy cascade from large (size L_t) to small (size λ_0) eddies. He demonstrated that theses eddies of different length scale also have very different velocity scales, and that the ratio of the largest to the smallest scale is $\frac{L_t}{\lambda_0} = \mathrm{Re}^{3/4}$, explaining the coexistence in a turbulent flow of eddies of very different length scales.

The development of powerful tools for scientific computing (including both computers and accurate numerical methods) enabled advances in turbulence modeling in the 1970s. Launder and Spalding introduced in 1972 the so-called k-ε model, based on the Boussinesq eddy viscosity concept, which is still commonly used for simulating turbulent flows. In 1975, Launder, Reece and Rodi proposed the first Reynolds stress transport model, removing the eddy viscosity assumption, and opening the way for more accurate simulations of turbulent flows.

The mathematical presentation of turbulence modeling is summarized in the first section of this article, focusing on incompressible flows, i.e., flows with constant density. In such a flow, motion of fluid particles is described by the so-called Navier-Stokes equations. These equations cannot be solved exactly for a turbulent flow, because time scale and length scale of the smallest eddies in a turbulent flow are very tiny, and their exact simulation would require too much memory and time, even on the most advanced computers. Following the Boussinesq idea, it is necessary to introduce equations on averaged velocity and pressure. This averaging was introduced by Reynolds and led to Reynolds averaged Navier-Stokes equations. Unfortunately, these equations contain new unknowns coming from the averaging of the nonlinear advective terms—the Reynolds stresses related to the fluctuating velocity components. Exact equations can be obtained for the Reynolds stresses, but they involve again higher order unknown statistical correlations. A turbulence model using these Reynolds stress transport equations is called a second order turbulence model.

Section 2 of the article focuses on the energy transfer in turbulent flows, based on the Kolmogorov model for homogeneous turbulence. First the kinetic turbulent energy equation is established. It demonstrates that a turbulent flow is possible because the mean flow produces turbulent energy through the interaction of large eddies with mean flow strain. The Kolmogorov model explains that this energy is cascading from large eddies towards small eddies (Kolmogorov length scale) where it is dissipated by viscosity.

Section 3 presents various turbulence models. The simplest one is the mixing length model. A more advanced and predictive model is the k-ε model. This commonly used for real life flow simulation. More accurate models based on the Reynolds stress transports equations are also presented. In parallel, the way of directly solving the Navier-Stokes equations is introduced. It is very demanding in term of computer power, and is thus limited to simple flows. Nevertheless, this approach is very useful for producing rich numerical experiments.

Section 4 gives examples of turbulent flow simulations in various configurations, and using different turbulence models. As a conclusion, trails for future developments are presented.

2. The Reynolds Averaged Navier-Stokes Equations

2.1 Introduction

The modeling of turbulent flows is introduced here for incompressible flows, which are representative of a large number of flows encountered in practice: air flows in the atmosphere, river flows, water flows in pipes, etc.

Usual incompressible viscous flows are represented by the Navier-Stokes equations. If $\mathbf{V} = (u_1, u_2, u_3)$ denotes the velocity vector, p the pressure, ρ the density and $\boldsymbol{g} = (g_1, g_2, g_3)$ the gravity acceleration, these equations can be written as:

$$\begin{cases} \dfrac{\partial u_i}{\partial t} + u_j \dfrac{\partial u_i}{\partial x_j} = \dfrac{1}{\rho} \dfrac{\partial \sigma_{ij}}{\partial x_j} + g_i \\ \dfrac{\partial u_i}{\partial x_i} = 0 \end{cases} \tag{1}$$

In the previous equations and in all of this article, the Einstein notation is used, which means summing on repeated indices, i.e., $u_j \dfrac{\partial u_i}{\partial x_j} = \sum_{j=1}^{3} u_j \dfrac{\partial u_i}{\partial x_j}$

$\boldsymbol{\sigma} = (\sigma_{ij})$ stands for the stress tensor. Usual flows follow the Newtonian constitutive law:

$$\sigma_{ij} = -p\,\delta_{ij} + 2\mu S_{ij} = -p\,\delta_{ij} + 2\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

Where μ is the dynamic viscosity of the fluid. Then the Navier-Stokes equations become:

$$\begin{cases} \dfrac{\partial u_i}{\partial t} + u_j \dfrac{\partial u_i}{\partial x_j} = \dfrac{1}{\rho} \dfrac{\partial \rho}{\partial x_i} \dfrac{\partial^2 u_i}{\partial x_k \partial x_k} + g_i \\ \dfrac{\partial u_i}{\partial x_i} = 0 \end{cases} \tag{2}$$

where $\nu = \dfrac{\mu}{\rho}$ is the kinematic viscosity of the fluid (unit: m^2/s).

The Navier-Stokes equations remain applicable when a flow becomes turbulent. Starting from a steady laminar solution and increasing the velocity leads to amplification of small perturbations through the nonlinear advective term. These perturbations ultimately occupy the whole domain and are almost of comparable magnitude to that of the mean flow. Turbulent eddies with a wide variety of scales appear. Their evolution is still given by the Navier-Stokes equations, but an increasing number of degrees of freedom is required to represent the smaller details of the turbulent flow properly.

The difficulty of modeling turbulent flows arises from the fact that the fluctuations associated with these perturbations have a size which is often too small to be captured by measuring devices or computational meshes.

The Navier-Stokes equations can be seen as the averaged motion of a large ensemble (called "fluid particle") of individual molecules. In the same way, the equations of the mean velocity can be derived by averaging over a large ensemble of eddies or flow realizations. The resulting equations, however, will require hypotheses, and result in model equations that are far from universal. The main reason is that while the scale of "a fluid particle" is several orders of magnitude larger than that of the individual molecules, the scales of the mean and turbulent motions, on the other hand, are often of comparable magnitude and interact strongly.

2.2 Reynolds Experiment (1986)

Osborne Reynolds (1842–1912) was the first to quantify the transition between a regular laminar flow and a turbulent flow. The Reynolds experiment was very simple—a water tank discharging through a glass pipe of diameter D. The flow is visualized by using dye which is introduced through a small tube into the large pipe. The bulk velocity (flow rate divided by section area) in the large pipe is V. For small values of V the dye draws a straight line because the molecular diffusion is slow compared with the advection by the mean flow. The flow is said to be laminar. By gradually increasing V, the dye streak starts to oscillate; this first appearance of unsteadiness is called transition. Further increase in V results in a chaotic pattern, and dye is rapidly dispersed in the entire cross section of the tube. The flow is said to be turbulent.

The same observations can be made if the viscosity (or the diameter of the pipe) is changed, while V is kept constant. That is, the transition from laminar to turbulent flow regimes is only characterized by the Reynolds number defined as:

$$\mathrm{Re} = \frac{VD}{\nu} \qquad (3)$$

In a circular pipe, transition occurs for a critical Reynolds number of $\mathrm{Re}_c = 2500$. If $D =$ 10 cm, the flow becomes turbulent above a velocity of only 2.5 cm/s. Flows in nature or industrial systems are thus very turbulent.

2.3 Mean and Fluctuating Components of the Flow

Assume that some parameter F is measured by some adequate device; repeat N times the same experiment ; N measured values of F are obtained: F_i, (i = 1,...,N). If N is large enough, then the mean value of F is approximated by:

$$\overline{F} = \frac{F_1 + \ldots + F_N}{N}$$

The rigorous mean value would be obtained with N tending toward infinity. The mean value obtained by averaging over N experiments is called an ensemble average. It is more convenient to average over time in a single experiment but this time should be sufficient so that a large number of different eddies have gone past the sensor.

2.3.1 Ergodicity Hypothesis

The ergodicity hypothesis assumes that the average defined by the following time averaging is identical to the ensemble average:

$$\overline{F} = \lim_{T\to\infty} \frac{1}{T}\int_0^T F(x,t)\,dt$$

In practice, T should be large compared with the integral time-scale defined further. This procedure only makes sense if the mean flow is considered to be at steady state (constant flow-rate for instance).

When the mean flow can be considered homogeneous (i.e., statistical values are constant in space), the mean defined by time averaging is the same as the space averaged value, on a space volume over which the flow is homogeneous:

$$\overline{F} = \lim_{\Omega\to\infty} \frac{1}{\Omega}\int_\Omega F(x,y,z)\,dx\,dy\,dz$$

The size of this domain should be much larger than typical eddy scales, or more precisely the integral length-scale defined further.

2.3.2 Probability Density Functions

In the mathematical sense, probabilities allow the definition of the averaging operator independently from the previous hypotheses. The probability that a random parameter F should happen to fall inside a certain interval is:

$$\text{Probability}\left(F \in [F_0, F_0 + dF]\right) = P(F_0)dF$$

This defines the probability density function *P(F)*. Naturally:

$$P(F) \geq 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} P(F)dF = 1$$

The knowledge of *P(F)* allows the construction of any statistical quantity related to F:
F' is the fluctuation, defined by $F' = F - \overline{F}$, and has a mean of zero.

The probability density function allows a rigorous mathematical approach. It is useful for demonstrating that the averaging operator commutes with space or time, and for obtaining the following properties for the averaging operator:

$$
\begin{aligned}
&\left(\overline{aF+bG}\right)=a\overline{F}+b\overline{G} \quad (a \text{ and } b \text{ are constants})\\
&\left(\overline{\frac{\partial F}{\partial x_i}}\right)=\frac{\partial \overline{F}}{\partial x_i} \quad \text{and} \quad \left(\overline{\frac{\partial F}{\partial t}}\right)=\frac{\partial \overline{F}}{\partial t}\\
&\left(\overline{\overline{F}}\right)=\overline{F} \quad \text{and} \quad \left(\overline{\overline{F}.G}\right)=\overline{F}.\overline{G}\\
&F'=F-\overline{F} \quad \text{and } \overline{F'}=0\\
&\text{but } \left(\overline{F.G}\right)\neq \overline{F}.\overline{G}
\end{aligned}
\tag{4}
$$

2.3.3 Mean Velocity Equations

Starting from the Navier-Stokes equations, Reynolds introduced for a turbulent flow the decomposition of the velocity and pressure into mean and fluctuating components:

$$
\begin{cases}
u_i=\overline{u_i}+u'_i\\
p=\overline{p}+p'
\end{cases}
$$

and then applied the averaging operator. Given relations in Equation (4), the linear terms in the Navier-Stokes equations give rise to identical terms in the mean velocity equations. Only the nonlinear advection term requires special attention. Using the incompressibility condition this term becomes:

$$
\begin{aligned}
(\overline{u_j}+u'_j)\frac{\partial(\overline{u_i}+u'_i)}{\partial x_j}&=\frac{\partial\left[(\overline{u_j}+u'_j)(\overline{u_i}+u'_i)\right]}{\partial x_j}\\
&=\frac{\partial\left[\overline{u_j}\,\overline{u_i}+u'_j\,\overline{u_i}+\overline{u_j}u'_i+u'_j\,u'_i\right]}{\partial x_j}
\end{aligned}
$$

Applying the averaging operator, it is noted that:

$\overline{u_i}\,\overline{u_j}$ is already a mean quantity, so averaging it again has no effect. Moreover:

$\left(\overline{u'_j\,\overline{u_i}}\right)=\overline{u'_j}\,\overline{u_i}=0\times\overline{u_i}=0$, but on the other hand $\overline{u'_i}\,\overline{u'_j}\neq 0$

This term is carried over to the right hand side, leaving on the left hand side only the advection by the mean velocity. The averaged Navier-Stokes equations, called Reynolds averaged Navier-Stokes equations (or Reynolds equations), are thus:

$$
\begin{cases}
\dfrac{\partial\overline{u_i}}{\partial t}+\overline{u_j}\dfrac{\partial\overline{u_i}}{\partial x_j}=\dfrac{1}{\rho}\dfrac{\partial\overline{p}}{\partial x_i}+\nu\dfrac{\partial^2\overline{u_i}}{\partial x_k\partial x_k}-\left(\dfrac{\partial\overline{u'_i\,u'_j}}{\partial x_j}\right)+g_i\\
\dfrac{\partial u_i}{\partial x_i}=0
\end{cases}
\tag{5}
$$

The Reynolds stress tensor is defined by $R_{ij}=\overline{u'_i\,u'_j}$, i.e.:

$$\mathbf{R} = \begin{bmatrix} \overline{u'_1 u'_1} & \overline{u'_1 u'_2} & \overline{u'_1 u'_3} \\ \overline{u'_1 u'_2} & \overline{u'_2 u'_2} & \overline{u'_2 u'_3} \\ \overline{u'_1 u'_3} & \overline{u'_2 u'_3} & \overline{u'_3 u'_3} \end{bmatrix} \qquad (6)$$

Now the Navier-Stokes equations can be written in a vector form, with $\boldsymbol{\sigma}$ as the stress tensor, then:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\mathbf{grad})\mathbf{V} = \frac{1}{\rho}\mathrm{div}(\boldsymbol{\sigma}) + \mathbf{g}$$

$$\mathrm{div}(\overline{\mathbf{V}}) = 0$$

thus the Reynolds equations are written as follows:

$$\frac{\partial \overline{\mathbf{V}}}{\partial t} + (\overline{\mathbf{V}}.\mathbf{grad})\overline{\mathbf{V}} = \frac{1}{\rho}\mathrm{div}(\overline{\boldsymbol{\sigma}} - \rho\mathbf{R}) + \mathbf{g} \qquad (7)$$

$$\mathrm{div}(\overline{\mathbf{V}}) = 0$$

This clearly demonstrates that **R**, which is the mean of the exterior product of the velocity fluctuation vector: $\mathbf{R} = \overline{\mathbf{V}' \otimes \mathbf{V}'}$, appears as a new stress tensor in the equations.

2.3.4 Necessity of a Closure Hypothesis

To obtain the mean velocity by solving Equation (7), an evaluation of **R** and subsequently of the fluctuating motion, is needed.

The equation of the fluctuating velocity is obtained by subtracting the Reynolds equation from the Navier-Stokes equation, then multiplying by the fluctuation and averaging:

$$\frac{\partial u'_i}{\partial t} = \frac{\partial u_i}{\partial t} - \frac{\partial \overline{u_i}}{\partial t} = \ldots \qquad \frac{\partial \overline{u'_i u'_j}}{\partial t} = \overline{\frac{\partial u'_i}{\partial t}u'_j + \frac{\partial u'_j}{\partial t}u'_i} = \ldots$$

Leading to:

$$\frac{\partial u'_i}{\partial t} + \overline{u}_j \frac{\partial u'_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u'_i}{\partial x_k \partial x_k} - \left(u'_j \frac{\partial \overline{u_i}}{\partial x_j} + u'_j \frac{\partial u'_i}{\partial x_j} \right) + \left(\frac{\partial \overline{u'_i u'_j}}{\partial x_j} \right) \qquad (8)$$

$$\frac{\partial u'_i}{\partial x_i} = 0 \quad \text{or} \quad \mathrm{div}(\mathbf{V}') = 0 \qquad (9)$$

In a compact notation, from the momentum equation, this gives:

$$\frac{\partial \overline{\mathbf{V}}}{\partial t} = f(\overline{\mathbf{V}}, \overline{p}, \overline{\mathbf{V}' \otimes \mathbf{V}'})$$

A transport equation can be obtained for the Reynolds stresses:

$$\frac{\partial \overline{\mathbf{V}' \otimes \mathbf{V}'}}{\partial t} = f(\overline{\mathbf{V}}, \overline{\mathbf{V}' \otimes \mathbf{V}'}, \overline{\mathbf{V}' p'}, \overline{\mathbf{V}' \otimes \mathbf{V}' \otimes \mathbf{V}'}) \qquad (10)$$

This new set of equations depends on new unknowns, for instance $\overline{\mathbf{V}' \otimes \mathbf{V}' \otimes \mathbf{V}'}$. Again, an equation can be obtained for this third order tensor:

$$\frac{\partial \overline{\mathbf{V}' \otimes \mathbf{V}' \otimes \mathbf{V}'}}{\partial t} = f(\overline{\mathbf{V}}, \overline{\mathbf{V}' \otimes \mathbf{V}'}, \overline{\mathbf{V}' p'}, \ldots, \overline{\mathbf{V}' \otimes \mathbf{V}' \otimes \mathbf{V}' \otimes \mathbf{V}'}) \qquad (11)$$

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Biographical Sketches

Jean-Paul Chabard graduated Master of Fluid Mechanics, Ecole Central des Arts et Manufactures. Since October 1997 he has been Director of the Thermal Transfer and Aerodynamics Branch of the EDF (Electricité de France), Research and Development Division. Formerly he was Director of the Laboratoire National d'Hydraulique (LNH) (1994–1997), and in the Research Division of EDF was responsible for experimental and numerical studies in fluid mechanics and hydraulics as well as for the development of general purpose Computational Fluid Dynamics computer software (LNH devotes resources to studies in fluid mechanics and hydraulics on behalf of French and foreign organizations—Maritime Sections of Public Works Contractors, River Basin Agencies Private Industries). Chabard lectures at the French High Engineering Schools (Ecole Nationale des Ponts et Chaussées and the Ecole Centrale des Arts et Manufactures), teaching Applied Fluid Mechanics and Numerical Turbulence Computation Methods. He is a member of the IAHR and Chairman of its Division II: Applied Hydraulics.

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