MATHEMATICAL MODELS OF MIGRATION

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Summary

Currently, the increasing role of migration in the social, economic and demographic development of countries, regions and the whole of the world is becoming more and more apparent, thus stimulating interest in mathematical modeling of migration. The objective of this chapter is to provide a general framework for analyzing migration: to highlight the goals and objectives of the analysis, to classify migration models, and analyze main migration models at macro and micro levels. The chapter also provides an overview of the practical application for the models and summarizes the opportunities for the development of migration models and establishes a basis for policy-analysis.

1. Introduction

Human migration is one of the few truly interdisciplinary fields of research of social processes which has widespread consequences for both, individuals and the society, and which can be considered as demographic, economic, ethnologic, geographic, political, psychological or social process. So, it is hardly surprising that the number of social sciences, involved in its study, is continually increasing: demography, economics, geography, political science, sociology and others. However, each of the disciplines listed above has its particular orientation. For instance, demography treats migration as one of the main demographic processes affecting the population dynamics and structure of population, while economics focuses on economic aspects of migration, its impact on a country's, region's or individual's welfare.

One of the first analysts, who attempted to formalize the study of migration processes, was the English geographer Ernest Ravenstein. Over a hundred years ago he formulated seven "laws of migration", or generalizations associated with migration, which were presumed to explain and predict migration patterns both within and between nations. These "laws" were inductively derived from the data of population censuses in European countries and in the USA. The original seven as Ravenstein originally set forth are as follows:

1. Most migrants rarely move to long distance; the majority is absorbed by rapidly developing commercial and industrial centers.
2. As migrants move towards absorption centers, they leave free "gaps" that are filled by migrants, moving from more remote districts, creating migration flows that reach, step by step, the most remote corners of the country. The number of migrants within a certain centre of absorption will increase the slower, the farer is the starting point and in proportion to the number of native population which furnishes them.
3. The process of dispersion is the inverse of absorption, and exhibits similar features.
4. Each basic migration flow produces a compensating counter-flow. (Hence the net migration from region *A* to region *B* will always be less than gross migration between regions *A* and *B*.)
5. Migrants proceeding the long distances generally target large commercial and industrial centers.
6. The native dwellers of towns are less migratory than those of the rural parts of the country. (Hence migration flows increase with development of industry, commerce and, especially, transport.)
7. Females are more migratory than males.

At the core of Ravenstein's contribution was the idea that the principal factors of migration, though not the only cause of migration, are economic. As pointed by him, bad or oppressive laws, heavy taxation, an unattractive climate, uncongenial social surroundings, and even compulsion (slave trade, transportation) produce flows of migrants, but none of these flows can be compared in volume with that which arises from the desire inherent in the most men to "better" themselves in material respects.

Most of this "natural regularities" have not only kept, but even increased their importance by now. Ravenstein's basic laws amended by additional laws subsequently, which were derived from his work, serve as the starting point for virtually all mathematical models of migration processes.

A breakthrough in mathematical modeling of migration dates back to the 1930s, marked with the appearance of the models by W. Reilly (1931), later – S. Stouffer (1940), G. Zipf (1946), S. Dood (1950), T. Hägerstrand (1967), I. Lowry (1966), M. Todaro

(1966), A. Rogers (1966), M. Greenwood (1969), L. Sjasstad (1962), J. Wolpert (1964), L. Rybakovskiy (1978), O. Staroverov (1979), W. Alonso (1978), S. Soboleva (1982), G. Borjas (1987), M. Cadwallader (1989) and other micro and macro models. A significant contribution to the development of migration modeling in the 20th century was made by the improvement of the mathematical methods used in demography, first of all – econometrics. Development and expansion of personal computers as well as information technologies substantially enhanced the migration modeling, providing necessary technical facilities and capacity for complicated calculations, required for processing and generalization of large amounts of statistical data.

At the same time, currently out of three demographic processes directly influencing the change in population size and structure – fertility, mortality and migration – only the third has the least developed mathematical tooling. One of the reasons for this is the absence of a unified theory of migration, which could become a basis for modeling, as well as the absence of a common approach in statistics, registering and terminology of migration. Therefore, despite the fact that a considerable amount of different models of migration movement has been developed, most of them have lower prognostic potential than models of natural movement of the population and its sex and age structure do .

2. Classification of Migration Models

A mathematical model of migration may be defined as a simplified description of real migration processes, where all the crucial links between the real “participants” of spatial movements – migrants and factors of migration – are expressed mathematically (see *Fundamentals of Mathematical Modeling for Complex Systems*).

As a social-economic process, migration modeling can be applied to both, macro and micro levels. Given the same object of research – population – these two approaches differ in the subject and the goals of research. The selection of modeling level (macro or micro) does not depend on the size of the area unit, as well as migration links under consideration.

The macro approach studies the patterns of migration of the whole population or certain social groups (for example, seniors, population in working-age period, ethic Germans, etc.) within a framework of a given territory and is based on either census data or current statistics. Characteristics of the origin and destination regions (such as climate, income, unemployment, etc.) are used as input variables for macro models of migration, migration processes indicators (such as migration increase, etc.) – as output variables.

The core ultimate goals of application of migration modeling at macro level are:

(a) migration processes analysis, seeking for the key determinants of migration (i.e. influence of interregional differences in wages and unemployment rates on the intention of migration flows);
(b) migration indicators forecast (i.e. forecast of the scale of migration flows from one region into another);
(c) simulation of migration process development using the observed interconnections (i.e. analysis of possible changes in emigration under different scenarios of

economic growth in the emigration countries).

The micro approach focuses on the migration behavior of individuals (families, households) and intends to explain the decision-making process by potential migrants to remain in a current residence or to migrate to another one. Micro models of migration usually are based on disaggregated data (i.e. the characteristics of an individual) delivered from censuses or sociologic surveys. As input variables micro models use both the characteristics of the origin and destination regions and the characteristics of individuals involved in migration processes. Output variables can describe migration behavior of a representative individual (i.e. who follows the patterns of migration behavior common for the observed territory) or, as in the case of macro models, aggregate indicators of migration processes.

The core ultimate goals of application of migration modeling at micro level are:

(a) analysis of the decision-making process by potential migrant ("to move or to stay?"), seeking for the key determinants of the decision identification of the most "dynamic" (apt to migration) social groups;
(b) analysis of the individuals' selection process between alternatives ("where to move?"), seeking for crucial factors affecting the "choice" of destination if a movement is started.

Despite the fact that macro- and micro-approaches to modeling have traditionally evolved independently from each other, actual migration flows are always the result of a combination of decisions made by individuals. The necessity to combine two approaches to migration modeling was emphasized in 1970s by Peter Morrison and a conceptual framework for so-called meso-approach has been proposed in 1980s–1990s by Martin Cadwallader and Oleg Staroverov.

According to the underlying basics and the mathematical methods used, micro and macro models of migration can be classified into several categories.

According to the modeling approach migration models can be characterized as *deterministic* and *stochastic models*. A model is classified as *deterministic*, if the result of any system change can be predicted unambiguously. Models of this type do not reflect any possible variance of the indicators (predicators) used in the model, instead – it is assumed that their values unambiguously determine the resulting value at a fixed point of time. In the migration modeling the deterministic approach had prevailed for a long period of time since it is evidently simpler than the stochastic approach. Nevertheless some degree of uncertainty concerning any social processes is always inevitable: due to the effects of coincidental factors the values of the determinants cannot predict the resulting value precisely enough. *Stochastic models* were developed to reduce this uncertainty; they provide the researchers with several output values given the same input value, whereas some likelihood values correspond to all the attainable outcomes. At the moment the stochastic approach of migration modeling is becoming more popular.

According to the time period migration models can be classified as *static* or *dynamic*.

Static models refer to the state of migration processes at a particular point in time, whereas *dynamic* models consider the interaction between the variables in time. In dynamic models time can be regarded as either discrete or continuous. In the first case, during one period of time (usually a year) only one change in the system is possible (i.e. only one movement of an individual), while in the second case the number of such changes is unlimited. The "real" time is continuous, but for modeling purposes it is sometimes convenient to assume, that the changes can only be observed at some fixed points in time. Furthermore, discrete models provide the researchers with equations more tractable for solving. Dynamic models are mainly used for the purposes of migration processes forecasting.

According to the type of interaction between input and output variables migration models can be classified as *single equation models* and *simultaneous equations models*. Single equation models are the ones considering a one-way dependence of migration on some determinants (activities) and not taking into account it's counter-influence on the predicating activities and any other elements of the social system. Usually these models are used solely for the purposes of theoretical explanation of the observed processes, as well as for discovering of quantitative and qualitative interdependence between particular activities. In simultaneous equations models migration is regarded as one of the interconnected elements of a certain system. These models enable us to consider both, direct and counter-flow connections of migration and indicators of the development of a particular territory unit (region, country etc.). Simultaneous equations models are typical mainly for the macro level of migration modeling.

Listed below are the main types of migration models at macro and micro levels, for simplicity grouped by modeling assumptions of the same kind.

3. Macro models of Migration

3.1. Interaction Models of Migration

Chronologically the *interaction models*, or the *models of spatial interaction* were the first. They are based on the idea of analogy of the processes undergoing within physic and social systems and, correspondingly, the idea of the methodological unity. This idea was first pronounced by a Belgian scientist Jacque Adolphe Quetelet in the beginning of the 19^{th} century and developed within the science of "social physics" concept.

In the middle of the 19^{th} century the American economist K. Kerry stated that the population can be compared to a simple system of elementary particles, whose existence and motion follow the rules similar to physical, and the society is a vast mechanism, where migration and concentration of people-molecules proceed as the result of the implementation of the "gravity forces" between aggregate groups of people, i.e. the distance of motion rises as the size of the group goes up, and falls in case the distance between two groups increases. The latter was empirically testified by Ernest Ravenstein.

Most commonly the models of this type can be represented as follows:

$$M_{ij} = k\frac{f\left(R_i, A_j\right)}{\varphi\left(d_{ij}\right)}, \qquad (1)$$

where M_{ij} is the amount of interaction of two "bodies" (here the number of migrants from region i in region j); $f\left(R_i, A_j\right)$ is a function of repulsive forces (R_i) at region of origin i and attractive forces (A_j) at region of destination *j*, R_i is a parameter representing repulsive factors which are associated with "leaving" region *i*, A_j is a parameter representing attractive factors related to going to region *j*, $\varphi\left(d_{ij}\right)$ is a function of the distance (d_{ij}) between regions *i* and *j*, k is a proportionality coefficient.

The followers of the "social physics" concept consider that $f\left(R_i, A_j\right) = W_i W_j$, where W_i and W_j are the masses of "bodies" *i* and *j*, correspondingly. Therefore the interaction model can be represented as follows:

$$M_{ij} = k\frac{W_i W_j}{\varphi\left(d_{ij}\right)}, \qquad (2)$$

If a territory system of *n* regions is concerned, then the total migration outflow from region *i* equals $M_i = \sum_{j=1}^{n} M_{ij}$, and total migration inflow in region *j* equals, accordingly, $M_j = \sum_{i=1}^{n} M_{ij}$.

One of the main problems in implementation of the interaction migration modeling is the necessity of a highly significant level of population aggregation. That is, for example, a given model can provide a good approximation for migration processes of a region as a whole, but at the same time an adequate interaction model for its smaller parts can not be derived. An important condition for the use of the interaction model is the comparability of the interacting territory units' sizes.

With respect to peculiarities of the migration modeling, interaction models can be classified into *gravity models*, *intervening opportunities models* and *models based on the Alonso's general theory of movement*.

3.1.1. Gravity Models of Migration

The majority of the interaction models is attributed to *gravity models of migration*. These models consider spatial mobility of population in analogy with Isaac Newton's Law of Gravitation (1687) as interaction between two territorial units (i.e. countries, regions, etc.) (see Figure 1).

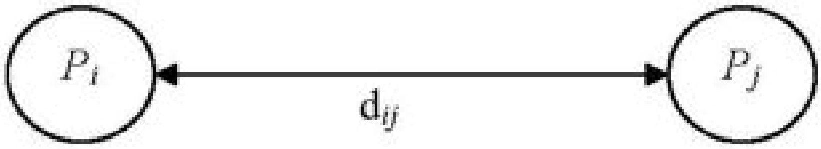

Figure 1. Scheme of spatial interaction of two territorial units within the gravity model of migration

English scientist George Stuart was the first to notice this peculiarity, having stated the so-called *gravity law of spatial interaction* in 1941:

$$F = kP_i P_j d_{ij}^{-2}, \tag{3}$$

where F is gravity or demographic force, k is a proportion coefficient, P_i is a population of the region of origin (i); P_j is population of region of destination (j), d_{ij} is a distance between interacting regions i and j.

Thus this "law" says that migration between two regions *i* and *j* is directly proportional to the product of population sizes of these regions and inversely proportional to the square of the distance between them.

More generally classical gravity model of migration can be represented as follows:

$$M_{ij} = kP_i P_j d_{ij}^{-\alpha}, \tag{4}$$

where coefficient α ($\alpha > 0$) reflecting the friction associated with distance as *k* should be determined in the model.

Hence classical gravity model is a special case of migration interaction model where population of regions of origin and destination of migrants are used as the masses of interacting "bodies (see *Macrosystem Modeling in System Analysis*).

Empirical verification of gravity model was conducted by different researchers during the last sixty years of the 20th Century. In 1979 R. Flowerdew and J. Salt used the gravity model to explore the migration flows between a Standard Metropolitan Labour Areas in Great Britain in 1970–1971. Fitting gravity model they derived a multiple coefficient of determination of 0.527, thus implying that over 50 percent of the variation in migration flows could be statistically accounted for by the gravity model. All coefficients were significantly different from zero and, as expected in theory, the coefficients associated with both population variables were positive, while the variable characterizing distance between two regions was negative.

Advantages of models of this type include, first of all, relative simplicity of developing and availability of statistical data for all level of analysis (inter-state, interregional migration, etc.).

Major disadvantages of the gravity models comprise the migration flow symmetry assumption ($M_{ij} = M_{ji}$), which basically never happens in reality, as well as low goodness of fit and predictive power of the models, which is mainly attributed to the consideration of only three migration factors. To eliminate these shortcomings different attempts to modify the gravity model have been made. Thus, in 1950 Stewart Dodd suggested using of compensation coefficients adjusting the indicators of the number of population. Such indicators as the proportion of female and male population, the level of income per capita, the share of population in working age, etc. can be used as the set of weights. Later the weight degrees were used.

Generalized gravity model can be represented as follows:

$$M_{ij} = kP_i^{\beta} P_j^{\gamma} d_{ij}^{-\alpha}, \quad (5)$$

where β, γ, α are constant that reflect the relative weightings of the constituent variables.

The "distance" indicator requires an additional explanation. Geographical distance, measured in time and length units, used in the gravity models can be treated differently by individuals. Hence modified indicators of distance between regions seem to be more appropriate. In 1940 an American sociologist Samuel Stouffer suggested the way of modification of distance indicator.

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Bibliography

Cadwallader M.T. (1992). *Migration and Residential Mobility: Macro and Micro Approaches*. Madison, Wis.: University of Wisconsin Press. [This book presents major theoretical and empirical issues in the field of migration processes modeling, while describing and explaining various kinds of mathematical models for migration both at macro and micro levels.]

Greenwood M.J. (1969). An Analysis of the Determinants of Geographic Labor Mobility in the United States. *Review of Economics and Statistics* **51**(2), 189–194. [Here the dynamic regression model of internal migration is presented.]

Mark Rosenzweig and Oded Stark (Eds.) (1997) *Handbook of Population and Family Economics.* Volume 1B. Amsterdam: Elsevier Science B.V.. [This handbook presents major theoretical and empirical issues of population economics, i.e. in the field of migration processes modeling.]

Lee E.S. (1966). A Theory of Migration, *Demography*, **3**(1), 47–57. [This paper presents the fundamentals of pull-push migration theory.]

Lindsay I., Brenton B. (1972). Two Stochastic Approaches to Migration: Comparison of Monte Carlo Simulation and Markov Chain Model, *Geografiska Annaler. Series B, Human Geography* **54**(1), 56–67.

[Here stochastic models of migration processes are presented.]

John Stillwell and Peter Congdon (eds) (1992).*Migration Models: Macro and Micro Approaches* / Belhaven Press. [Here is presented a summary of modern approaches to migration modeling, a wide range of migration models is illustrated.]

Ravenstein E.G. (1885). The Laws of Migration, *Journal of the Statistical Society of London* **48**(2), 167–235. [This is one of the pioneering presentations in the field of statistical analysis of migration flows. Here the famous Ravenstein's "laws of migration" are presented.]

Rogers A. (1967). A Regression Analysis of Interregional Migration in California, *The Review of Economics and Statistics* **49**(2), 262–267. [This paper presents a famous Lowry and Lowry-Rogers model of internal migration in the United States which based on pull-push theory.]

Sandell S.H. (1977). Women and the Economics of Family Migration, *The Review of Economics and Statistics* **59**(4), 406–411. [This paper describes the model of migration decision-making process at the family level.]

Sagradov A. (2004). Economic Demography. Moscow: Infra-M. [In Russian.] [Here numerous models of demographic processes, i.e. migration, at micro and macro scale are presented.]

Sjaastad L.A. (1962). The Costs and Returns of Human Migration, *The Journal of Political Economy* **70**(5), 80–93. [This paper outlines the concept of human capital and shows why migration can be depicted as investment in the human capital.]

Staroverov O.V. (1997). Fundamentals of Mathematical Demography. Moscow: Nauka. [in Russian] [Here the fundamental mathematical models used in demography are presented.]

Vries J., Nijkamp P., Rietveld P. (2000). Alonso's General Theory of Movement, *Tinbergen Institute Discussion Paper* (TI 2000–062/3). [This paper includes a general presentation of the Alonso's general theory of movement.]

Wadycki W.J. (1975). Stouffer's Model of Migration: A Comparison of Interstate and Metropolitan Flows, *Demography* **12**(1), 121–128. [This paper presents main theoretical empirical issues of the intervening opportunities models of migration.]

Biographical Sketches

VLADIMIR IONTSEV graduated in 1977 from the Faculty of Economics at the Moscow State 'Lomonosov' University. From the same year he has been working at the Population Department, Faculty of Economics, Moscow University. In 1980 he received Ph.D. in Economics (*Directions of international migration policy in economic developed countries*) and in 2000 he received D.Sc. in Economics. In 2002 he became a Professor. Presently Vladimir Iontsev is a Head of the Department of Population. He specializes in migration issues, in particular international migration, studying also different aspects of demography and economics of population. He authored over 130 publications, including *The Role of Demographic Factor in the Economic Development of the Region (Perm' Region Example)*. Moscow, 2004 (editor and co-author); *Introduction to Demography*: Textbook. Moscow: TEIS, 2002, 2003 (co-editor and co-author); *International Migration*. Scientific Series "Migration of Population". Volume 3. Moscow, 2001; *International Migration of Population: Theory and History of Scientific Study*. Moscow, 1999; *Contemporary Demography*: Textbook. Moscow: Publishing House of the Moscow University, 1995 (co-editor and co-author); *Migration of Population in the Contemporary World*. Moscow, 1992. He is a member of International Union of Scientific Studies of Population (IUSSP) and European Association of Population Studies (EAPS). He is an Editor-in-Chief of the scientific series *International Migration of Population: Russia and the Contemporary World founded in 1998* (15 volumes published).

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