FUZZY LOGIC

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Contents

Summary

We describe the basics of fuzzy sets and fuzzy logic. Based upon the concept of linguistic values, which describe imprecise concepts using words, the basics of fuzzy rules and fuzzy inference are introduced. In the second part we briefly explain applications of fuzzy rules for function approximation using fuzzy graphs, clustering using fuzzy algorithms, and classification under uncertainty using fuzzy decision trees.

1. Introduction

In the past years fuzzy logic has raised increasing attention in real world scenarios. This is due to the fact that most approaches from classical statistics assume that we deal with exact measurements. But in most, if not all real-world scenarios, we will never have a precise measurement. There is always going to be a degree of uncertainty. Even if we are able to measure a temperature of 32.42 degrees with two significant numbers, we will never know the exact temperature. The only thing we can really say is that a measurement is somewhere in a certain range, in this case (32.41,32.43) degree. In effect, all recorded data are really intervals, with a width depending on the accuracy of the measurement. It is important to stress that this is different from probability, where we deal with the likelihood that a certain crisp measurement is being obtained. In the context of uncertainty we are interested in the range into which our measurement falls. Several approaches to handle information about uncertainty have already been proposed, for example interval arithmetic allows us to deal and compute with intervals rather than crisp numbers, and also numerical analysis offers ways to propagate errors along with the normal computation. The following will concentrate on presenting an approach to deal with imprecise concepts based on fuzzy logic.

This type of logic enables us to handle uncertainty in a very intuitive and natural manner. In addition to making it possible to formalize imprecise numbers, it also enables us to do arithmetic using such fuzzy numbers. Classical set theory can be extended to handle partial memberships, thus making it possible to express vague human concepts using fuzzy sets and also describe the corresponding inference systems based on fuzzy rules.

Another intriguing feature of using fuzzy systems is the ability to granulate information. Using fuzzy clusters of similarity we can hide unwanted or useless information, ultimately leading to systems where the granulation can be used to focus the analysis on aspects of interest to the user.

This chapter will start out by explaining the basic ideas behind fuzzy logic and fuzzy sets, followed by a brief discussion of fuzzy numbers. We will then concentrate on fuzzy rules and how we can generate sets of fuzzy rules from data. We will close with a discussion of Fuzzy Information Theory by showing how Fuzzy Decision Trees can be constructed.

2. Basics of Fuzzy Sets and Fuzzy Logic

Before introducing the concept of fuzzy sets it is beneficial to recall classical sets using a slightly different point of view. Consider for example the set of "young people", assuming that our perception of a young person is someone with an age of not more than 20 years:

$$\text{young} = \{x \in P \mid \text{age}(x) \leq 20\}$$

over some domain *P* of all people and using a function "age" that returns the age of some person *x* in years. We can also define a characteristic function:

$$m_{\text{young}}(x) = \begin{cases} 1 & \text{age}(x) \leq 20 \\ 0 & \text{age}(x) > 20 \end{cases}$$

which assigns to elements of *P* a value of 1 whenever this element belongs to the set of young people, and 0 otherwise. This characteristic function can be seen as a membership function for our set "young", defining this set on P.

Someone could now argue with us that he, being just barely over 20 years old, still considers himself young to a very high degree. Defining our set young using such a sharp boundary seems therefore not very appropriate. The fundamental idea behind fuzzy set theory is now a variable notion of membership; that is, elements can belong to sets to a certain degree. For our example we could now specify that a person with an age of, let's say, 21 years, still belongs to the set of young people, but only to a degree of less than one, maybe 0.9. The corresponding membership function would look slightly different:

$$\mu_{\text{young}}(x) = \begin{cases} 1 & \text{age}(x) \le 20 \\ 1 - \dfrac{\text{age}(x) - 20}{10} & 20 < \text{age}(x) \le 30 \\ 0 & \text{age}(x) > 30 \end{cases}$$

Now our set "young" contains people with ages between 20 and 30 with a linearly decreasing degree of membership, that is, the closer someone's age approaches 30, the closer his degree of membership to the set of young people approaches zero (see Figure 1).

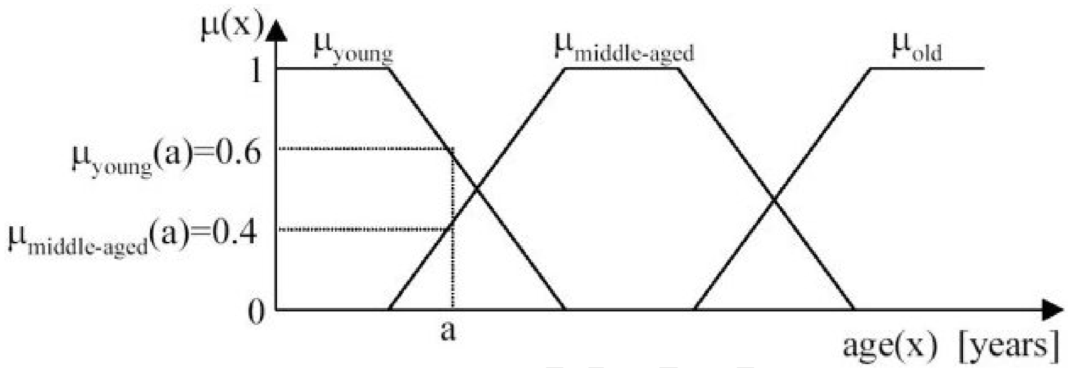

Figure 1: A linguistic variable age with three fuzzy sets and degrees of memberships for a certain age *a*

The above is a very commonly used example for a fuzzy set. In contrast to classical sets, where an element can either belong to a set or lies completely outside of this set, fuzzy sets allow also partial memberships. A fuzzy set *A* is thus defined through specification of a membership function μ_A that assigns each element *x* a degree of membership to *A*: $\mu_A \in [0,1]$. Classical sets only allow values 1 (entirely contained) or 0 (not contained), whereas fuzzy set theory also deals with values in between 0 and 1. This idea was introduced in 1965 by Lotfi A Zadeh.

3. Linguistic Variables and Fuzzy Sets

Covering the domain of a variable with several such fuzzy sets together with a corresponding semantic results in linguistic variables, allowing the computation with words. For our example this could mean that we define two more membership functions for middle-aged and old people, covering the entire domain of the variable age. This type of representation is especially appropriate for many real-world applications, where certain concepts are inherently vague in nature, either due to imprecise measurements or subjectivity. The above example for a linguistic variable is shown in Figure 1. People are distinguished using their age (a function defined for all people) in groups of young, middle-aged and old people. Using fuzzy sets allows us to incorporate the fact that no sharp boundary between these groups exists. Figure 1 also illustrates how the corresponding fuzzy sets overlap in these areas, forming non-crisp or fuzzy boundaries. Elements at the border between two sets belong to both. For example some person *p* with an age of age(*p*)=24 years belongs to both groups "young" and "middle-aged" to a degree of 0.6 and 0.4 resp.; that is, $\mu_{\text{young}}(p) = 0.6$ and $\mu_{\text{middle-aged}}(p) = 0.4$. With an increase in age, the degree of membership to the group of young people will decrease whereas $\mu_{\text{middle-aged}}$ increases. The linguistic variable age is therefore described through three linguistic values, namely "young", "middle-aged", and "old". The overlap between the membership functions reflects the imprecise nature of the underlying concept. We should keep in mind, however, that most concepts depend on the respective context. An old student can easily be a young professor.

This way of defining fuzzy sets over the domain of a variable is often referred to as granulation, in contrast to the division into crisp sets (quantization) which is used by classical sets. Granulation results in a grouping of objects into imprecise clusters or fuzzy granules, with the objects forming a granule drawn together by similarity. Thus fuzzy quantization or granulation could also be seen as a form of fuzzy data compression. Often the granulation for some or all variables is obtained manually through expert interviews. If such expert knowledge is not available or the usage of a predefined granulation seems harmful, it is also possible to find a suitable granulation automatically, for example from available data.

If no real semantic about the variable is known, a commonly used approach to label fuzzy sets is illustrated in Figure 2.

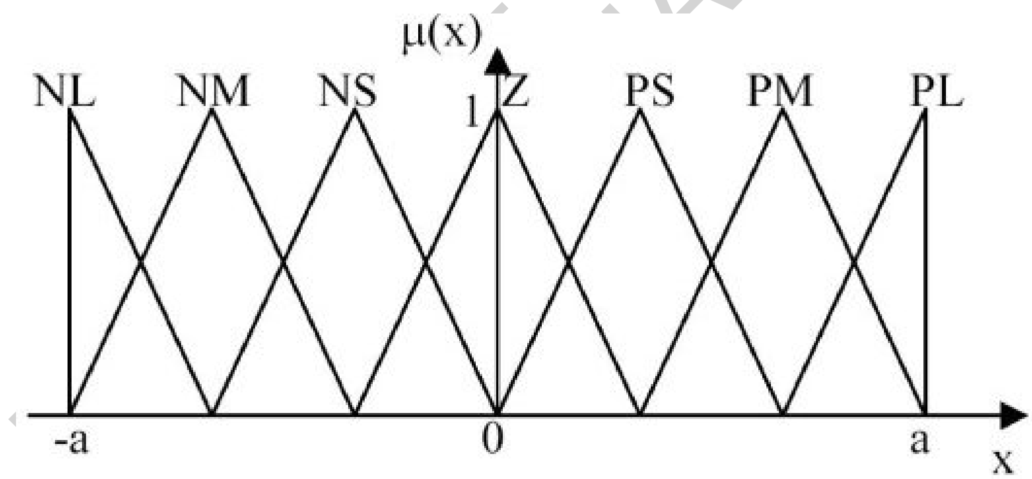

Figure 2: The standard granulation using an odd number (here seven) of membership functions

For a symmetrical domain *[-a,a]* of the variable often an odd number of membership functions is used, usually five or seven. The membership functions are then labeled NL (for "negative large"), NM ("negative medium"), NS ("negative small"), Z ("zero"), and the corresponding labels PS, PM, PL for the positive side.

In real applications the shape of membership functions is usually restricted to a certain class of functions that can be specified with only few parameters. Figure 3 shows the most commonly used shapes for membership functions.

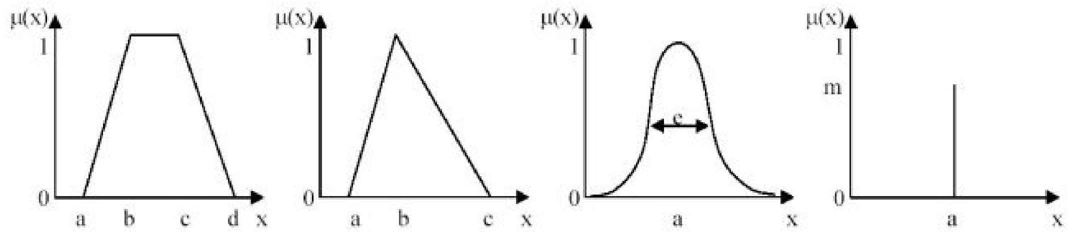

Figure 3: Most commonly used shapes for membership functions (trapezoidal, triangular, Gaussian, singleton)

On the left a trapezoidal function is depicted which can be specified through the four corner-points <*a,b,c,d*>. The triangular membership function can be seen as a special case of this trapezoidal function. Often used is also a Gaussian membership function which can be simply specified through two parameters *a* and *e* and offers nice mathematical properties such as continuity and differentiability. This is an often required property when membership functions are to be fine-tuned automatically during a training stage. Finally the singleton <*a*|*m*> on the right can be used to define a fuzzy set containing only one element to a certain degree $m \le 1$. The choice of membership function is mostly driven by the application. The Gaussian membership functions are usually used when the resulting system has to be adapted through gradient-descent methods. Knowledge retrieved from expert interviews will usually be modeled through triangular or trapezoidal membership functions, since the three resp. four parameters used to define these functions are intuitively easier to understand. An expert will prefer to define his notion of a fuzzy set by specifying the area where the degree of membership should be 1 (*[b,c]* for the trapezoid) and where it should be zero (outside of *(a,d)*), rather than giving mean *a* and standard deviation *e* of a Gaussian. The resulting fuzzy system will not be affected drastically. Changing from one form of the membership function to another will affect the system only within the boundaries of its granulation.

The following parameters can be defined and are often used to characterize any fuzzy membership function:

- support: $s_A = \{x : \mu_A(x) > 0\}$, the area where the membership function is greater than 0.
- core: $c_A = \{x : \mu_A(x) = 1\}$, the area for which elements have maximum degree of membership to the fuzzy set *A*. Note that the core can be empty when the membership function stays below 1 over the entire domain.
- alpha-cut: $A_\alpha = \{x : \mu_A(x) \ge \alpha\}$, the cut through the membership function of *A* at height α.
- height: $h_A = \max_x \{\mu_A(x)\}$, the maximum value of the membership function of *A*.

Before discussing operators on fuzzy sets, the following section will discuss how fuzzy numbers can be treated.

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Biographical Sketch

Michael R. Berthold received his Diploma and Doctorate in Computer Science from the University of Karlsruhe. From fall 1997 until early 2000 he was a Research Fellow at the Berkeley Initiative in Soft Computing (BISC) and a Lecturer at the University of California, Berkeley. He was a Visiting Researcher at Carnegie Mellon University in 1991/92 and at Sydney University in 1994 and worked as a Research Engineer at Intel Corp., Santa Clara in 1993. He is currently Director of Data Analysis at Tripos' Research Center in the Bay Area.

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