INTERNAL MODEL CONTROL

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Keywords: Internal Model Control, Q-parameterization, PID control, feedback control, first-order systems, second-order systems, delayed systems, plants with integrator.

Contents

Summary

The Internal Model Control (IMC) design procedure, with emphasis on its implications for proportional-integral-derivative (PID) controller tuning, is presented in this chapter. The basis for IMC is the so called Q-parameterization structure. The IMC design procedure is a two step design process that aims to provide a suitable tradeoff between performance and robustness. In Step 1 a stable and causal controller is obtained that is optimal with respect to either the integral of squared error (ISE) or integral of absolute error (IAE) criteria for step changes to the control system; the second step augments the controller from Step 1 with a filter to insure that the IMC controller is proper. For many simple processes of interest the IMC controller, when implemented in classical feedback form, leads to a PID-type controller. Various illustrative examples are developed in this chapter and evaluated under a common setting. The performance of IMC-based PID

controller tuning for systems with delay is examined and contrasted with popular classical Ziegler-Nichols and Cohen-Coon PID controller tuning rules.

1. Introduction

Internal Model Control (IMC) refers to a systematic procedure for control system design based on the Q-parameterization concept that is the basis for many modern control techniques. What makes IMC particularly appealing is that it presents a methodology for designing Q-parameterized controllers that has both fundamental and practical appeal. As a consequence, IMC has been a popular design procedure in the process industries, particularly as a means for tuning single loop, PID-type controllers.

The IMC design procedure is quite extensive and diverse. It has been developed in many forms; these include single-input, single-output (SISO) and multi-input, multi-output (MIMO) formulations, continuous-time and discrete-time design procedures, design procedures for unstable open-loop systems, combined feedback-feedforward IMC design, and so forth. The focus of this chapter is on the feedback-only SISO design procedure for open-loop stable systems, with particular emphasis on its relationship to PID controller tuning.

Aside from controller design, IMC is helpful in assessing the fundamental requirements associated with feedback control, such as determining the effect of non-minimum phase elements (delays and Right-Half Plane (RHP) zeros) on achievable control performance. Since the sophistication of the IMC controller depends on the order of the model and control performance requirements, the IMC design procedure is also helpful in determining when simple feedback control structures (such as PID controllers) are adequate.

2. The Internal Model Control Structure

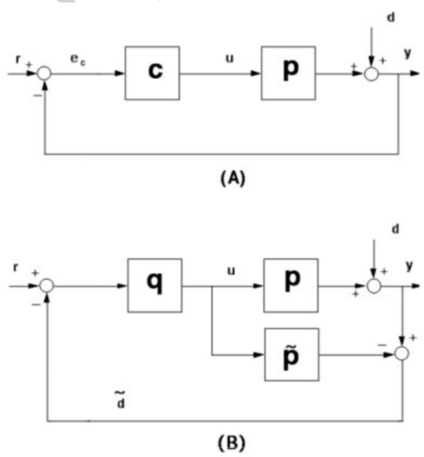

Figure 1: Classical (A) and Internal Model Control (B) Feedback Structures. p is the plant model, c is the classical feedback controller, $\tilde{p}$ is the internal model, and q is the IMC controller.

The first issue one needs to understand regarding IMC is the IMC *structure* (to be distinguished from the IMC design procedure). Figure 1B is the "Internal Model Control" or "Q-parameterization" structure. It consists of an internal model $\tilde{p}(s)$ and an IMC controller $q(s)$. The IMC structure and the classical feedback structure (Figure 1A) are equivalent representations; Figure 2 demonstrates the evolution of the IMC structure. A significant benefit of the IMC structure is that a design procedure for $q(s)$ can be developed that is more straightforward and intuitive than the direct design of a classical feedback controller $c(s)$. Having designed $q(s)$, its equivalent classical feedback controller $c(s)$ can be readily obtained via algebraic transformations, and vice-versa

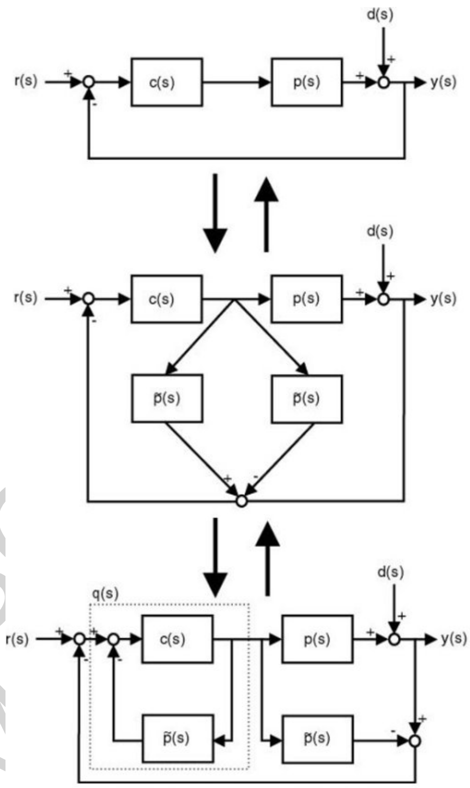

Figure 2: Evolution of the Internal Model Control Feedback Structure.

$$c = \frac{q}{1-\tilde{p}q}, \tag{1}$$

$$q = \frac{c}{1+\tilde{p}c}. \tag{2}$$

For linear, stable plants in the absence of constraints on u, it makes no difference to implement the controller either through c or q. However, in the presence of actuator constraints, one can use the IMC structure to avoid stability problems arising from input saturation without the need for special anti-windup measures.

2.3. Closed-loop Transfer Functions for IMC

The sensitivity ε and complementary sensitivity η operators define the closed-loop behavior of a classical feedback linear control system.

$$y = \eta r + \varepsilon d \tag{3}$$

$$u = p^{-1}\eta\left(r-d\right) \tag{4}$$

$$e_c = \varepsilon\left(r-d\right). \tag{5}$$

Recall that $\eta = pc(1+pc)^{-1}$ and $\varepsilon = (1+pc)^{-1}$ for classical feedback control system. A statement of the sensitivity and complementary sensitivity operators in terms of the internal model $\tilde{p}$ and the IMC controller $q(s)$ corresponds to:

$$\eta(s) = \frac{pq}{1+q\left(p-\tilde{p}\right)} \tag{6}$$

$$\varepsilon(s) = \frac{1-\tilde{p}q}{1+q\left(p-\tilde{p}\right)}. \tag{7}$$

In the absence of plant/model mismatch $(p=\tilde{p})$, these functions simplify to

$$\tilde{\eta}(s) = \tilde{p}q \qquad \tilde{\varepsilon}(s) = 1-\tilde{\eta}(s) = 1-\tilde{p}q \qquad \tilde{p}^{-1}\tilde{\eta} = q \tag{8}$$

which lead to the following expressions for the input/output relationships between y, u, e_c, r and d:

$$y = \tilde{p}qr \_(1-\tilde{p}q)d \tag{9}$$

$$u = q(r-d) \tag{10}$$

$$e_c = (1-\tilde{p}q)(r-d). \tag{11}$$

From examining Eqs.(9)-(11), one is able to recognize the benefits of the IMC parameterization. The closed-loop response between set-point r and output y is readily determined from the properties of the simple product $\tilde{p}q$. Furthermore, the manipulated variable response is determined through the design of q. As a consequence, both analysis and synthesis tasks in the control system are simplified.

2.4. Internal Stability

Internal Stability (IS) is a critical theoretical requirement for any control system. In an internally stable control system, bounded input signals introduced anywhere in the control system result in bounded output signals everywhere in the control system. For the IMC structure we have the following important internal stability results:

1. Assume a perfect internal model $(p = \tilde{p})$. The IMC control system (Figure 1B) is internally stable if and only if both p and q are stable.
2. Assume that p id stable and $p = \tilde{p}$. then the classical feedback system (Figure 1A) with controller according to Eq.(1) is IS if and only if q is stable.

These results apply for the IMC structure even if $\tilde{p}$ and q are nonlinear operators. For the case of an open-loop, linear stable system under no plant model mismatch, the IMC structure thus offers the following benefits with respect to classical feedback:

- It eliminates the need to solve for the roots of the characteristic polynomial $1+pc$; stability can be determined "by inspection" by examining only the poles of q.
- It is possible to search for q instead of c without any loss of generality.

In the case of open-loop unstable plants (which includes systems with integrators), the requirement that both p and q be stable eliminates the use of the IMC structure for the purposes of implementation; however, IMC still serves a useful role for the design of the compensator q, which can be implemented in classical feedback form according to Eq.(1).

2.3. Asymptotic closed-loop behavior (System Type)

Another important requirement for a feedback control system is that it leads to no offset for set-point and disturbance changes. Meeting this requirement for so-called Type 1 and Type 2 inputs is described in the following:

Type 1 (step Inputs): No offset to asymptotically step set-point/disturbance changes is obtained if

$$\lim_{s\to 0} \tilde{p}q = \tilde{\eta}(0) = 1 \tag{12}$$

Type 2 (Ramp Inputs): For no offset to ramp inputs, it is required that

$$\lim_{s\to 0} \tilde{p}q = \tilde{\eta}(0) = 1 \tag{13}$$

$$\lim_{s\to 0} \frac{d}{ds}(\tilde{p}q) = \left.\frac{d\tilde{\eta}}{ds}\right|_{s=0} = 0 \tag{14}$$

These requirements will form part of the IMC design procedure, as noted in Section 3.

2.5. Performance Measures

Performance measures in process control are varied but norm criteria are commonly used. Among the most popular choices are the Integral of Square Error (ISE)

$$J_{ISE} = \int_0^\infty (y-r)^2 dt = \int_0^\infty e_c^2 dt \qquad (15)$$

and the Integral of Absolute Error (IAE)

$$J_{IAE} = \int_0^\infty |y-r| dt = \int_0^\infty |e_c| dt \quad . \qquad (16)$$

Time-weighted modifications of the ISE and IAE such as the Integral of Time-Weighted Square Error (ITSE)

$$J_{ITSE} = \int_0^\infty t^2 (y-r)^2 dt \qquad (17)$$

and the Integral of Time-Weighted Absolute Error (ITAE)

$$J_{ITAE} = \int_0^\infty t|y-r| dt \qquad (18)$$

tend to emphasize a decrease in the control error at long times. For these performance measures to be well-posed they require that the external signal acting upon the control system be defined; usually a step set-point change is assumed.

3. Internal Model Control Design Procedure

The IMC design procedure is a two-step approach that, although sub-optimal in a norm-oriented sense, provides a simple means for achieving a reasonable tradeoff between performance and robustness. A significant benefit of the IMC approach is the ability to directly specify the complementary sensitivity and sensitivity functions η and ε, which as noted in Section 2.1, directly specify the nature of the closed-loop response. The presentation in this section is as follows: after developing the basic theoretical requirements for the design of q (Section 3.1), an IMC design procedure leading to PID-type controllers for simple process systems will be developed. The more formal design procedure relying on H_2-optimality is briefly described in Section 6.

3.1. Requirements for Physical Realizability on q, the IMC Controller

In order for $u(t)$, the manipulated variable response, to be physically realizable, the IMC controller q must be *stable*, *proper*, and *causal*. These criteria are described below:

1. S*tability*. The controller must generate bounded responses to bounded inputs; therefore all poles of q must lie in the open Left-Half Plane.
2. "*Properness*". Differentiation of step inputs by a feedback controller leads to impulse changes in u, which are not physically realizable. In order to avoid pure

differentiation of signals, $q(s)$ must be **proper**, which implies that the quantity

$$\lim_{|s|\to\infty} q(s) \tag{19}$$

is finite. $q(s)$ is referred to as **strictly proper** if

$$\lim_{|s|\to\infty} |q(s)| = 0\,. \tag{20}$$

A strictly proper transfer function has a denominator order greater than the numerator order. $q(s)$ is **semi-proper**, that is,

$$\lim_{|s|\to\infty} |q(s)| > 0 \tag{21}$$

if the denominator order is equal to the numerator order. A system that is not strictly proper or semi-proper is called **improper**.

1. *Causality*. $q(s)$ must be causal, which implies that the controller must rely on *current* and *previous* plant measurements in order to determine the current or future values of the manipulated variable. A simple example of a noncausal transfer function is the *inverse* of a time delay

$$c(s) = \frac{u(s)}{e_c(s)} = \frac{K_c}{e^{-\theta s}} = K_c e^{+\theta s}\,. \tag{22}$$

The inverse transform of (22) relies on *future* inputs to generate a *current* output; it is clearly not physically realizable:

$$u(t) = K_c e_c(t+\theta) \qquad . \tag{23}$$

3.2. Limitations to Perfect Control: the Need for an IMC Design Procedure

From examining Eqs.(9)-(11) for no plant/model mismatch $(p = \tilde{p})$, "perfect" control (meaning $y = r$ for all time) is achieved when $\tilde{\eta} = 1$ and $\tilde{\varepsilon} = 0,$ which implies that

$$q = \tilde{p}^{-1}. \tag{24}$$

In general, q arising from "perfect" control rarely results in physically realizable responses in the $u(t)$. Non-minimum phase elements such as dead-time and RHP zeros will cause $q = \tilde{p}^{-1}$ to be noncausal and unstable, respectively; if $\tilde{p}$ is strictly proper, then q according to (24) will be improper as well. One can better understand the situation with a simple example. Consider the plant model

$$\tilde{p}(s) = \frac{K(-\beta s + 1)e^{-\theta}}{\tau^2 s^2 + 2\xi\tau s + 1}, \tag{25}$$

where $\beta > 0$, which implies the presence of a Right-Half Plane zero. Non-minimum phase elements for this plant are $e^{-\theta s}$ and $(-\beta s + 1)$. The “perfect” IMC controller for this system corresponds to

$$q = \tilde{p}^{-1} = \frac{\tau^2 s^2 + 2\xi\tau s + 1}{K(-\beta s + 1)} e^{+\theta s}.$$

Despite perfect control on y, the manipulated variable response is physically unrealizable for three reasons. First, q is unstable as a result of a Right-Half Plane pole arising from $(-\beta s + 1)$. Secondly, q is noncausal because of the presence of the time *lead* term $e^{+\theta s}$. Finally, q is improper because the numerator order is greater than the denominator order, leading to the differentiation of step changes in the control system.

In the ensuing sections we will show that defining the IMC controller based on a suitable factorization of the plant model will result in stable, causal control action; augmenting the controller with a properly chosen filter will insure properness and a physically realizable response. One must keep in mind, however, that the non-minimum phase elements $e^{-\theta s}(-\beta s + 1)$ will always form part of the closed-loop response; these cannot be removed by feedback action alone.

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Biographical Sketches

Daniel E. Rivera is an Associate Professor in the Department of Chemical and Materials Engineering at Arizona State University, and Program Director for the ASU Control Systems Engineering Laboratory. Prior to joining Arizona State in 1990 he worked for four years as an Associate Research Engineer in the Control Systems Section of Shell Development Company. He received his Ph.D. in chemical engineering from the California Institute of Technology in 1987, and holds B.S. and M.S. degrees from the University of Rochester and the University of Wisconsin-Madison, respectively. His research interests include a wide range of issues in control systems engineering, which include the topics of dynamic modeling via system identification, robust process control, and extensions to problems in semiconductor manufacturing and the social sciences. Dr. Rivera is the recipient of the 1994-1995 Outstanding Undergraduate Educator Award by the ASU student chapter of AIChE and the 1997-1998 Teaching Excellence Award from the ASU College of Engineering and Applied Sciences (now the Ira A. Fulton School of Engineering).

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