DESIGN OF STATE SPACE CONTROLLERS (POLE PLACEMENT) FOR SISO SYSTEMS

Lohmann, Boris
Institut für Automatisierungstechnik, Universität Bremen, Germany

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Contents

Summary

The state variable representation of dynamic systems is the basis of different and very direct approaches to the analysis and design of control systems. One of these approaches is introduced in this topic contribution with its four articles, serving in particular the treatment of linear systems with one control input and one control output variable, so-called single-input-single-output (SISO) systems. The design idea is to place the poles, characterizing stability and dynamic properties, at desired locations specified by the control-system designer.

Before presenting the corresponding analysis and design procedures in the articles *Description and Analysis of Dynamic Systems in State Space, Controller Design, Observer Design* and *Extended Control Structure*, the design objectives and the general approaches to its solutions shall be outlined in the subsequent sections, together with some general remarks on state-space design and on the system class under consideration. For illustration, the state variable representation of a balanced pendulum system is given. It also serves as an accompanying example in the subsequent four articles.

1. Design Objective

Consider a closed-loop control system consisting of a plant and a controller as implied by figure 1 where either the *state variables* $x_1,\ldots,x_n$ or the control output variable *y* are fed back, referred to as state-feedback or output-feedback respectively. The control output variable is the main variable to be influenced and formed by the control design. The state variables as introduced in article *Description of Continuous Linear Time-Invariant Systems in Time-Domain* describe the internal behavior of the control plant. Typically, they represent physical quantities resulting from a mathematical modeling of the plant based on physical laws. For abbreviation, the state variables are combined into the so-called state vector $\boldsymbol{x}$. As this vector is a function of time, $\boldsymbol{x}(t)$, moving in an n-dimensional vector space, the methods based on the corresponding system description are referred to as state-space methods. The vector $\boldsymbol{x}(t)$ at some fixed time t is called the *state* of the system at time t.

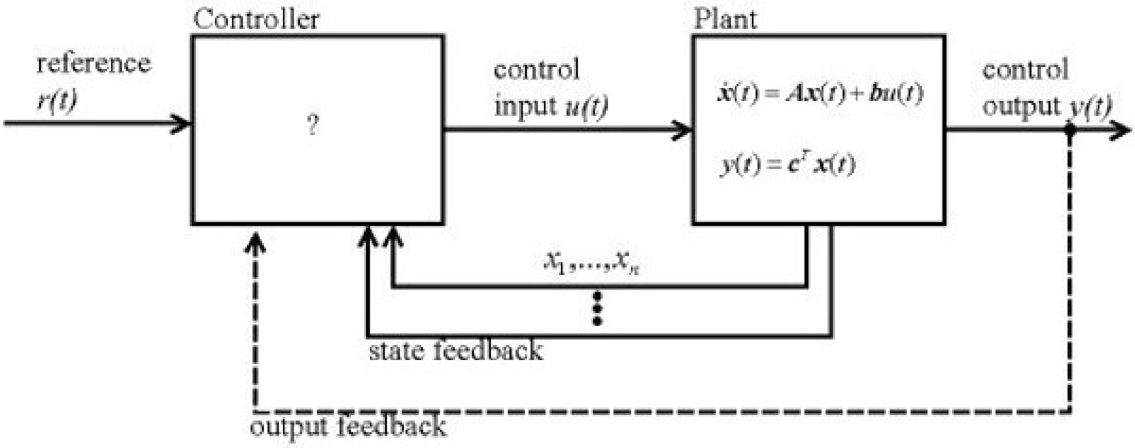

Figure 1. General structure of a closed-loop control system

In case of state feedback, the objective is to design the block "controller" such that it generates a control input signal $u(t)$ (from a reference signal $r(t)$ and the state variables $x_1,\ldots,x_n$) with the following properties:

- The effects of initial values of the state variables "decay" as $t \to \infty$.
- The control output $y(t)$ tracks the reference signal $r(t)$ "as well as possible".
- Both of these processes run with "desired dynamics" (which will be achieved by specifying poles).

The design of such state-feedback controllers is presented in *Controller Design*. It will emerge that state-feedback is a powerful tool allowing the specific influence of the internal structure and the dynamic properties of the plant.

As introduced so far, the technical realization of pure state-feedback requires that all of the state variables are accessible, i.e., that they are continuously measured. This is not always the case, for instance if measurement is expensive, difficult, or impossible for principal reasons. In such cases, it is desirable to reduce the number of variables fed back. As an extreme, only one single variable, typically the control output variable, is measured and fed back while the three design objectives just listed remain unchanged. The resulting controller is called output-feedback controller. In order to preserve the advantages of state feedback it is a good idea to estimate the inaccessible state variables from the accessible ones and to run the state-feedback controller with the estimated variables. Such a state estimator is also called a state observer. Its design is presented in the article *Observer Design*. By introducing the estimated state variables $\hat{x}_1,\ldots,\hat{x}_n$ into the state-feedback controlled system, the structure as implied by the results shown in Figure 2: From the control input u and the control output y the observer continuously generates the estimated state variables $\hat{x}_1(t),...,\hat{x}_n(t)$ which supply the state-feedback controller. By combining the observer and the state-feedback controller into one single block (dashed lines) it is easy to observe that in fact an output-feedback control structure results.

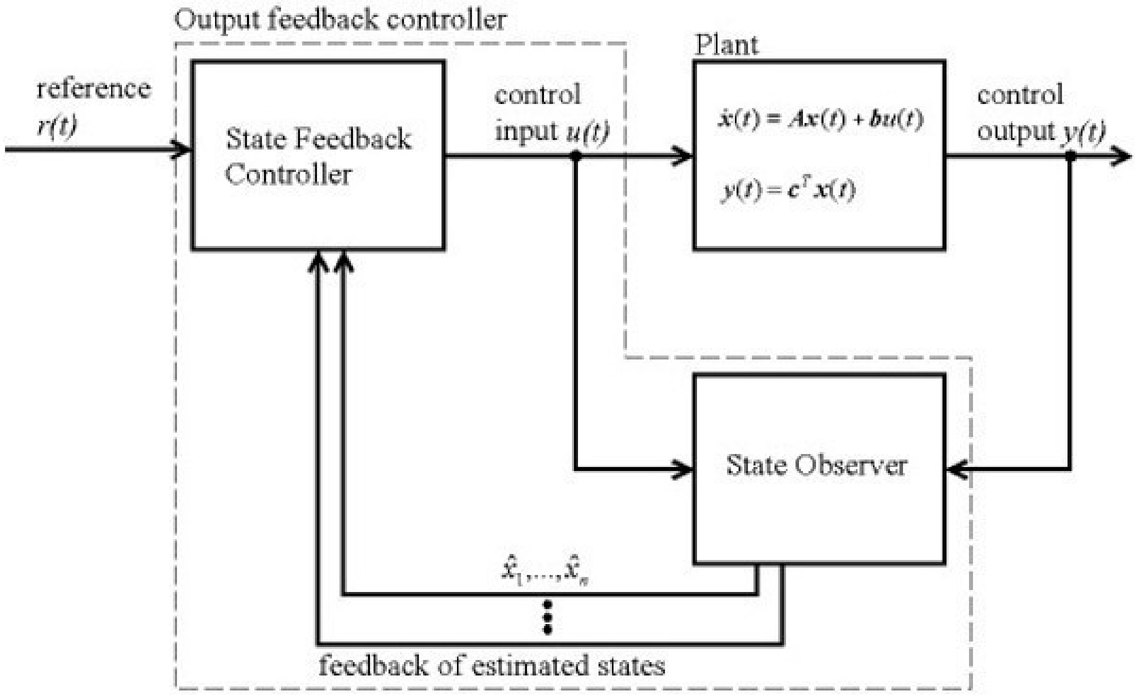

Figure 2. Output feedback by combining state observer and state-feedback controller

State observation is not possible in all situations, i.e., not for any plant and any choice of control output variable. For instance, if some state variable does neither directly nor indirectly take effect on the control output, then it is impossible to estimate this state variable from measuring the output; the system is not completely observable. Corresponding restrictions apply in the design of state feedback: If some part system is neither directly nor indirectly affected by the control input, then it is impossible to specifically influence its dynamics; the system is not completely controllable. The notions of controllability and observability are key concepts in the analysis and design of state-space control systems. They are discussed and defined more precisely in article *Description and Analysis of Dynamic Systems in State Space*.

2. General Remarks on State Space Design

By inserting a state observer, the powerful tools of state-feedback design become applicable to a wide range of practical control problems. Starting from aerospace, state-space methods have spread within the past 30 years into the fields of chemical, bio-technological, and other production process automation, automotive control, shipping and navigation, and mechatronics. Also in non-technical fields like meteorology, economy, and biological processes the modeling, analysis, and control are frequently done by state-space methods. A little heard opinion is that state-space methods were abstract and not applicable in practice. This is untenable for the following reasons:

- State space design avoids the "detour" Laplace transformation – design – back transformation, which is inherent in classical frequency domain approaches. Analysis and design are almost fully carried out in time domain, i.e., in the real-world physical quantities.
- State space design gives a deeper insight into the system and its properties than classical frequency domain approaches can. The terms of controllability, observability, zeros, and others are related to this, and will be introduced in the subsequent articles. Roughly speaking, frequency-domain approaches aim at shaping the input-output behavior of a system, while the state-space approach also takes internal system variables into consideration.
- By state space control, desired system dynamics can be realized in a straight-forward manner by pole placement. Advantages of state-space design are especially apparent, when the system to be controlled has more than one control input and more than one control output: The results presented here can easily be generalized in this case. The multi-input-multi-output (MIMO) case is presented in the topic contribution *Control of Linear Multivariable Systems*.
- State space methods can be applied to time varying and to nonlinear systems. Many results of linear state space theory have been transferred to and extended to the nonlinear case.

However, state space design is not a universal remedy. It is true that the system dynamics and speed of the transient behavior can in principal be arbitrarily specified. However, in any real system, the control input variable and others are limited for physical reasons which, consequently, limits the dynamics of the controlled system. If a linear system description is assumed, as will be done in this contribution, then all results obtained will only be valid as long as the system in fact behaves "almost" linear. Otherwise, methods of nonlinear control theory may be helpful. Also, significant uncertainties and variations of the system parameters can make the system behavior worse than expected. The specific methods of robust control can take remedial action and can guarantee certain system performance even in the presence of uncertainties and parameter variations.

In all, it is to be noticed that linear state space methods are quite advanced today and have brought a variety of methods for different applications and design objectives. So, as an addition or extension to pole placement, other objectives are pursued, like optimizing certain performance indices, specifically reducing the influence of disturbances, reducing the interactions between system variables, or reducing the sensitivity to model uncertainty.

An important initiator of state space methods was, starting in the 1960s, Rudolf E. Kalman with his results on system analysis, including controllability and observability and on optimal estimation and filtering. Important criteria of controllability and observability are due to Gilbert and Hautus. A universal and straight-forward design procedure for state-feedback controllers by pole placement was presented by J. Ackermann, known as Ackermann's formula. The state observers presented here are due to Luenberger and are sometimes referred to as Luenberger-observers.

Nowadays, the results presented in the subsequent four articles are typical subjects of lectures on automatic control for advanced students of engineering and applied mathematics. Hence, corresponding chapters can be found in many textbooks on modern control design methods. For reference, an incomplete list of such textbooks is given in the Bibliography.

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Bibliography

Ackermann J. (1972). Der Entwurf linearer Regelungssysteme im Zustandsraum. *Regelungstechnik* **20**, 297-300. [Presents Ackermann's formula]

DeCarlo R.A. (1989). *Linear Systems*. Prentice Hall. [Includes an introduction to the description and analysis by state space methods].

Franklin G.F., Powell J.D. and Emami-Naeini A. (1994). *Feedback Control of Dynamic Systems*, 3rd edition. Addison-Wesley. [Textbook with good introduction to the key objectives of automatic control. Mathematical appendix].

Gilbert E.G., (1963). Controllability and Observability in Multivariable Control Systems. *J. Control, Ser. A*, **1**,. 128-151. [Includes the controllability/observability criteria presented].

Hautus M.L.J. (1969). Controllability and Observability Conditions of Linear Autonomous Systems. *Indagationes Mathematicae* **31**, 443-448. . [Includes the controllability/observability criteria presented].

Kaczorek T. (1992). *Linear Control Systems. Volume 1 and 2*. Research Studies Press. [Extensive theoretical work; includes linear differential algebraic systems and many details].

Kailath, T. (1980). *Linear Systems*. Prentice Hall. [A standard textbook on linear theory].

Kalman, R.E. (1960). A new Approach to Linear Filtering and Prediction Problems. *J. Basic Engineering* **85**, 394-400. [An initial work on state space methods].

Kalman R.E. (1960). *On the General Theory of Control Systems*. Proc. 1st Int. Congr. Autom. Control, Moscow, pp. 481-492. [An initial work on state space methods].

Kuo B.C. (1995). *Automatic Control Systems*, 7th edition. Prentice Hall. [Extensive textbook. Some details missing in favor of good readability].

Luenberger D.G. (1964). Observing the State of a Linear System. *IEEE Transactions on Military Electronics* **8**, 74-80. [Presents Luenberger's approach to state observers].

Luenberger D.G. (1966). Observers for Multivariable Systems. *IEEE Transactions on Automatic Control* **11**, 190-197. [Presents Luenberger's approach to state observers].

Luenberger D.G. (1971). An Introduction to Observers. *IEEE Transactions on Automatic Control* **16**, 596-602. [Presents Luenberger's approach to state observers].

MacFarlane A.G.J. and Karcanias N. (1976). Poles and Zeroes of linear multivariable systems: a survey of the algebraic, geometric and complex-variable theory. *Int. J. Control* **24**, 33-74. [Introduces the concept of invariant zeros from a state variable approach].

Shinners S.M. (1998). *Advanced Modern Control System Theory and Design*. John Wiley & Sons. [A recent textbook].

MATLAB applications: The following two web-addresses provide introductory examples on how to use the software package MATLAB for control system design purposes: http://tech.buffalostate.edu/ctm/ and http://www.ee.usyd.edu.au/tutorials_online/matlab/index.html

Biographical Sketch

Boris Lohmann received the Dipl.-Ing. and Dr.-Ing. degrees in electrical engineering from the Technical University of Karlsruhe, Germany, in 1987 and 1991 respectively. From 1987 to 1991, he was with the Fraunhofer Institut (IITB) and with the Institute of Control Systems, Karlsruhe, working in the fields of autonomous vehicles control and multi-variable state space design. From 1991 to 1997, he was with SIEMENS Electrocom Automation Systems in the development department for postal sorting machines, latterly as the head of mechanical development. In 1994 he received the 'Habilitation' degree in the field of system dynamics and control from the Universität der Bundeswehr, Hamburg, for his results on model order reduction of nonlinear dynamic systems. Since 1997, he has been full professor at the University of Bremen, Germany, and head of the Institute of Automation Systems. His fields of research include nonlinear multivariable control theory; system modeling, simplification, and simulation; and image-based control systems, with industrial applications in the fields of autonomous vehicle navigation, active noise reduction, and others.