IDENTIFIABILITY OF LINEAR CLOSED-LOOP SYSTEMS

G. Bretthauer
Institute for Applied Computer Science, Karlsruhe Research Center, Germany

M. Kaufmann
Institute for Applied Computer Science/Automatic Control, University of Karlsruhe (TH), Germany

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Summary

This paper deals with the identifiability of linear closed-loop systems. The term "identifiability" means that there is a unique solution for the identification problem, i.e., that for the experimental conditions given, the best model can be determined by the identification method used. For that purpose, some identifiability concepts exist. They are described briefly in Section 0. Depending on the concept used, different identifiability conditions are known.

They are summarized for Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) systems in Section **Error! Reference source not found.**. In considering the input-output behavior of a system, the concept of I/O-identifiability is of great

relevance. This concept is used for the test of identifiability for closed-loop systems. As in closed-loop systems generally, where more than two signals can be measured, several signal combinations exist that can be used for the identification.

Hence, one problem is to determine the signal combinations for which I/O-identifiability is guaranteed. Another problem is to determine the identifiability of all unknown quantities (system and signal identification). These two problems are the subject of Section **Error! Reference source not found.**, where necessary conditions for I/O-identifiability are given and some results for complete and partial I/O-identifiability are summarized.

1. Introduction

A central problem of each identification is to prove that there is a unique solution for the identification problem under consideration. In the literature this is denoted by the term "identifiability". It means that for the class of models used, for the identification method applied and for the experimental conditions given, a successful identification of the system is guaranteed. Some related basic concepts for the identifiability of deterministic and stochastic systems have been developed. These concepts are:

- Deterministic identifiability
- Stochastic identifiability
- Structural identifiability.

They were first applied to SISO linear, stable, constant, discrete and continuous-time dynamic systems. It was shown that for input-output models (difference equations, transfer functions, impulse responses) as well as for state space models, a system is identifiable if it is completely controllable and if the input signal $u(t)$ is persistently exciting. In this case, the input signal $u(t)$ and the noise signal $v_S(t)$ (see Figure 1) are uncorrelated.

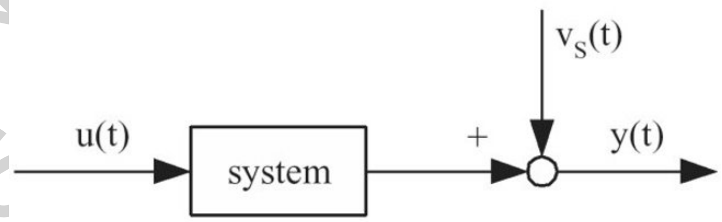

Figure 1: Open-loop system

However, if such a system operates in a closed loop (see Figure 2), then the above assumptions about $u(t)$ and $v_S(t)$ are violated and some difficulties regarding the identifiability of the plant may occur. For $w_S(t) = 0$ and $v_C(t) = 0$, it can be shown that the plant in the closed-loop system of Figure 2 cannot be identified from measurements of $u(t)$ and $y(t)$ by noncausal methods such as correlation or spectral analysis. On the other hand, the identifiability of the plant can be shown under other experimental conditions, e.g., an extra input signal $w_S(t)$. (*see Closed-loop Behavior*)

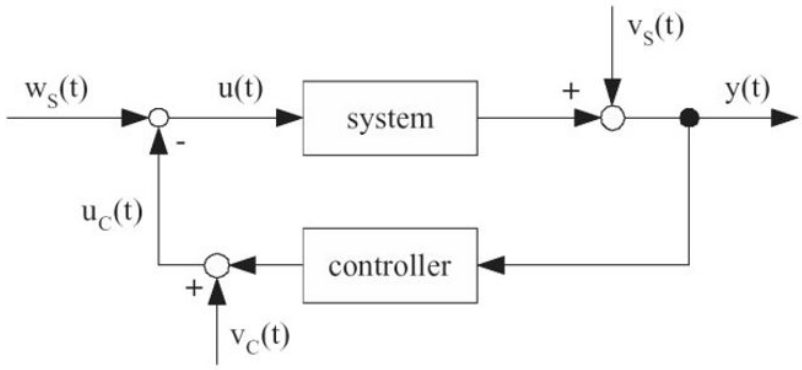

Figure 1: Closed-loop system

In contrast to open-loop systems, additional conditions, e.g., on the system and the controller, must be fulfilled for identifiability of closed-loop systems. (*see Identification for Control*).

As identification experiments must frequently be performed on closed-loop systems, the identifiability of this class of system has been the subject of much research. For example, the concepts of system and parameter identifiability were created and applied to SISO and MIMO closed-loop systems that were described by difference equations and vector difference equations, respectively. As a first goal of this paper, the five basic concepts of identifiability are summarized briefly in Section 0.

The result of an identification experiment depends on

- the system to be identified,
- the class of models used,
- the identification method applied, and
- the experimental conditions given.

Each of these items affects identifiability. Hence, several identifiability conditions are known in the literature. In recent years, some new identifiability conditions have been derived. Therefore, a second goal of this paper is a short overview of some important identifiability conditions for closed-loop systems, given in Section 0.

In most cases, the identifiability conditions known for closed-loop systems are based on the assumption that only the input and output signals $u(t)$ and $y(t)$, or an additional test signal $w_S(t)$, are measured. However, in a closed-loop system (see Figure 2) some other signals, for instance $u_C(t)$, can often also be measured. In this case, novel identifiability problems occur. Some of these are as follows:

- How many (two or three) and what signals (loop or outer) should be measured?

- What a priori information on the closed-loop system is necessary for the identifiability of the plant?
- What other characteristics (signal and/or system) can be determined simultaneously?

The solutions to these problems are of practical relevance for the optimal design of an identification experiment in multivariable closed-loop systems, and in such large-scale systems as interconnected systems or hierarchically structured systems.

These identifiability problems will be investigated for linear MIMO closed-loop systems in Section **Error! Reference source not found.**.

2. Identifiability Concepts

In this section, the five basic concepts of identifiability are summarized. To illustrate the different concepts, we consider a SISO, linear, stable, constant, discrete, or continuous-time dynamic system. The system is denoted by $\mathcal{S}$ and described by

- a difference equation or
- a differential equation.

To identify the system $\mathcal{S}$, we require a class of models denoted by $\mathcal{M}$. This class of models is parameterized by a parameter vector Θ containing the unknown parameters a_i and b_i of the system. (*see General Models of Dynamic Systems*) To evaluate the equivalence between $\mathcal{E}$ and $\mathcal{M}$, a criterion is necessary. A quadratic loss function V is frequently used. To find the best model within the class of models, an identification method denoted by $\mathcal{I}$ is required.

Furthermore, each identification experiment is carried out under defined experimental conditions, such as measurements of $u(t)$ and $y(t)$, use of special input signals, or the presence of feedback. They are denoted by $\mathcal{E}$, $\mathcal{S}$, $\mathcal{M}$, $\mathcal{I}$ and $\mathcal{E}$ influence the identifiability.

Now let us specify the meanings of the different identifiability concepts.

2.1 Deterministic Identifiability

Definition 1

A linear, stable, constant dynamic system is said to be identifiable in the deterministic sense if the parameter vector Θ *and the initial state* $\mathbf{x}_0$ *are uniquely determined from a finite number N of undisturbed observations of* $u(t)$ *and* $y(t)$.

For deterministic identifiability, necessary and sufficient conditions regarding the number of observations and the number of unknown parameters, as well as the properties of the system and the input signal $u(t)$ have been derived. Because in this case the system is undisturbed, i.e., $v_S(t) = 0$ (see Figure 1), the determination of these identifiability conditions is a problem of algebraic nature. This concept is of theoretical interest only.

2.2 Stochastic Identifiability

Definition 2

A linear, stable, constant dynamic system is said to be identifiable if the sequence of estimates $\hat{\Theta}(N)$ converges to Θ in a stochastic sense.

For the identification method $\mathcal{I}$, the Maximum Likelihood method has been used. Here, the identifiability is defined in a probabilistic framework, and the identification problem is reduced to an estimation problem. For stochastic identifiability, other definitions have been given and many necessary and sufficient identifiability conditions have been derived This concept is of great practical relevance because a real system is usually affected by noise, and parameter estimation methods are commonly used.

2.3 Structural Identifiability

Definition 3

A linear, stable, constant dynamic system is said to be structurally identifiable if the loss function V has a minimum. If the minimum is local, the system is said to be locally identifiable. If the minimum is global, the system is said to be globally identifiable.

In contrast to stochastic identifiability, the identification problem is here connected with an optimization problem. Accordingly, the system is structurally identifiable if the optimization problem has a unique solution. A sufficient condition has been derived and applied to some special classes of linear systems. This concept is of practical importance for the identifiability of compartmental systems, and some papers related to this problem are known.

These basic concepts are characterized by the following properties:

- they are parameter-oriented, and
- the different experimental conditions $\mathcal{E}$ for closed-loop systems are not explicitly stated.

To consider these properties, the concepts of system and parameter identifiability were developed. (*see System Description in Time-Domain*)

2.4 System Identifiability

Definition 4

A linear, stable, constant dynamic system is said to be system identifiable under given

$\mathcal{M}$, $\mathcal{I}$, *and* $\mathcal{E}$ *if*

$$\hat{\Theta}(N) \to D_T(\mathcal{S}+\mathcal{M}) \;\; w.p.1 \;\; as \;\; N \to \infty \,. \qquad (1)$$

$D_T\left(\mathcal{S},\mathcal{M}\right)$ is the set consisting of all parameters that give models that describe the system without error in the mean square sense. System identifiability is denoted by $SI\left(\mathcal{M},\mathcal{I},\mathcal{E}\right)$.

2.5 Strong System Identifiability

Definition 5

A linear, stable, constant dynamic system is said to be strongly system identifiable, denoted by $SII\left(\mathcal{I},\mathcal{E}\right)$*, under given* $\mathcal{I}$ *and* $\mathcal{E}$*, if it is system identifiable for all* $\mathcal{M}$ *such that the set* $D_T\left(\mathcal{S},\mathcal{M}\right)$ *is nonempty.*

If the objective of the identification is to obtain a model for control purposes, then the concept of system identifiability is quite adequate.

2.6 Parameter Identifiability

Definition 6

A linear, stable, constant dynamic system is said to be parameter identifiable under given $\mathcal{M}$, $\mathcal{I}$ and $\mathcal{E}$ *if it is system identifiable and the set* $D_T\left(\mathcal{S},\mathcal{M}\right)$) *consists of only one element. It is denoted by* $PI\left(\mathcal{M},\mathcal{I},\mathcal{E}\right)$.

This concept is a natural one if the objective of the identification is to determine some parameters that have physical significance. It is more general than the concept of stochastic identifiability.

The concepts of system and parameter identifiability contain the experimental conditions explicitly. Therefore, they are favorable for the examination of the identifiability of closed-loop systems.

All the concepts described in this section are based on the assumption that only the input and output signals $u\left(t\right)$ and $y\left(t\right)$ are measured. However, in a closed-loop system, usually more than these two signals can be measured. This point of view has not yet been taken into consideration.

To solve this problem, the concepts of complete and partial I/O-identifiability, which depend on the measured signals and on the information known a priori, have been developed. These concepts will be explained in Section **Error! Reference source not found.**. Before doing so for closed-loop systems, some essential identifiability conditions based on the above concepts will be summarized.

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Biographical Sketches

Georg Bretthauer obtained the Dipl.-Ing., Dr.-Ing., and Dr.-Ing. habil. degrees in Automatic Control at the University of Technology, Dresden in 1970, 1977, and 1983, respectively. He is now Professor of Applied Computer Science and Automatic Control at the University of Karlsruhe and the Head of the Institute of Applied Computer Science at the Karlsruhe Research Center. His research interests are in the fields of computational intelligence, knowledge-based systems, and automatic control.

He is elected Chairman of the German Society of Measurement and Automatic Control. He works as a technical expert of German research in the field of automatic control and is member of the administrative council of the European Union of Control associations.

Michael Kaufmann received his Dipl.-Ing. degree at the Freiberg University of Mining and Technology in 1990 and his Dr.-Ing. degree in Mechanical Engineering at the University of Karlsruhe (TH) in 2002. He worked in industrial companies as a development engineer and manager in the field of measuring and process control. He is now a researcher at the Institute for Applied Computer Science / Automatic Control at the University of Karlsruhe (TH). His research interests include applications of computer science in mechanical and chemical engineering and mechatronics.