DESIGN TECHNIQUES IN THE FREQUENCY DOMAIN

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Summary

Several multivariable frequency response design methods have been developed, based on extensions to Nyquist's stability theory, and Bode's single loop design approach. Several of these approaches have involved reducing the multivariable design to a set of single loop designs. The different methods can be combined to give multivariable designs aiming at independent control of the individual outputs. These methods are illustrated using a design for an unstable chemical reactor.

1. Frequency responses and stability

A linear system will produce an output that is in proportion to its input, and so the response to a sine wave input, $\sin(\omega\theta)$, is a sine wave with the same frequency but different magnitude and phase, $k \sin(\omega t+\theta)$, and so can be thought of as a complex gain $g(j\omega)$. The measurable frequency response of a system is this complex gain as a function of frequency. A frequency 's' on the frequency plane represents an input signal equal to the real part of e^{st}. So, $s = j\omega$ corresponds to a signal $\sin(\omega\theta)$.

A frequency $s = j\omega+r$ corresponds to a signal $e^{r} \sin(\omega\theta)$, which is an exponentially growing oscillation if r is positive, and shrinking if r is negative. So, the frequency response g(s) for values of s off the imaginary axis represents the response of the system to growing and shrinking input oscillations.

1.1. Single loop stability

The poles of a system are those frequencies at which the system gain is infinite. A system will be unstable if any of its poles, s_i, are in the right half plane, since the infinite gain will magnify any noise to ensure that the corresponding exponentially growing oscillation occurs. The Nyquist stability criterion says that the number of closed loop right half plane poles for a negative feedback gain k is equal to the number of open loop right half plane poles minus the number of anticlockwise encirclements of the critical point –1/k by the open loop frequency response.

So, the number of anticlockwise encirclements of the critical point is equal to the net number of poles that have migrated out of the right half plane as the feedback has been increased from zero to –k. This can be proved by contour integration, or by considering the continuity of the frequency response function. So, the Nyquist diagram of the open loop frequency response shows the stability for any closed loop gain.

The closed loop gain is g(s)k/(1+g(s)k). So, the closed loop gain at a particular frequency is given by the ratio of g(s)k to (1+g(s)k). For a particular sine wave sin(ωθ), the closed loop gain is the ratio of {the distance of the frequency response from the origin g(jω)k)} to {the distance from the critical point (1+g(jω)k)}. As the negative feedback gain increases from zero, the closed loop poles move along the lines for which g(s)k is real and negative, and are at the points where (1+g(s)k) = 0.

1.2. Multivariable stability using Characteristic loci

The characteristic loci also form a natural extension of frequency responses to the multi-loop situation. Consider the case of equal negative feedback gains k on each loop. The number of closed loop right half plane poles for a negative feedback gain kI is equal to the number of open loop right half plane poles minus the number of anticlockwise encirclements of the critical point –1/k by plots of the eigenvalues of the open loop frequency response.

These plots are called the characteristic loci. Once again this can be proved by contour integration, or by considering the continuity of the frequency response function. The closed loop gain is $G(s)k(I+G(s)k)^{-1}$. As the negative feedback gain increases from zero, the closed loop poles move along the lines for which an eigenvalue, λ (s), of G(s)k is real and negative, and are at the points where |I+G(s)k| = 0, and so (1+λ (s)k) = 0 for at least one of the eigenvalues. When the eigenvectors are nearly orthogonal, the characteristic loci also give a good indication of the closed loop performance.

1.3. Multivariable stability using Gershgorin bands on Nyquist arrays

However, the characteristic loci do not give stability information for different gains on different loops. Also, in triangular systems the characteristic loci do not indicate the effect of off-diagonal elements on performance. The Nyquist Array and Inverse Nyquist

Array can give approximate stability information for different gains on the loops, and also indicate the effects of the off-diagonal elements.

The key theorem to obtain an approximate stability result is Gershgorin's theorem, which states that the eigenvalues of a matrix are contained within the union of a set of disks centered on the diagonal elements of the matrix and with radius equal to the sum of the moduli of the off-diagonal elements in the corresponding row or column. For example, the radii of the column disks are

$$d_i^c(j\omega) = \sum_{\substack{j=1 \\ j \neq i}}^{m} \left| g_{ji}(j\omega) \right| \tag{1}$$

When these Gershgorin circles are superimposed on the diagonal elements of the Nyquist array, they form the Gershgorin bands. For diagonally dominant systems, one of the characteristic loci will be in each of the Gershgorin bands, and so these bands indicate the number of encirclements of different points on the negative real axis by the characteristic loci.

The number of closed loop right half plane poles for negative feedback gains $k_1,k_2,..,k_n$ is equal to the number of open loop right half plane poles minus the number of anticlockwise encirclements of the critical points $-1/k_i$ by the Gershgorin bands on the diagonal elements $G_{i,i}(s)$.

So, the stability of all the combinations of gains $k_1,k_2,..,k_n$ can be seen simultaneously from a single diagram, for all the gains such that the critical points are not in the corresponding Gershgorin band. For example, Figure 8 is the Nyquist array of a stabilized chemical reactor, the Gershgorin bands extend to 1.16 and 1.0024 so it shows that extra negative feedback loops with gains $k_1 > -1/1.16$ and $k_2 > -1/1.0024$ will leave the system stable.

1.4. Diagonal Dominance

A rational m x m matrix G(s) is column diagonal dominant if the following inequality is satisfied

$$\left| g_{ii}(j\omega) \right| > \sum_{\substack{j=1 \\ j \neq i}}^{m} \left| g_{ji}(j\omega) \right| \tag{2}$$

for i = 1,...,m and all s on the Nyquist D-contour; i.e., each of the bands produced by the Gershgorin disks excludes the origin, for i = 1,...,m. Non-interacting control at high frequencies requires that the system is diagonal dominant at high frequencies.

However, diagonal dominance is affected by the scaling of the rows and columns, so it is often possible to improve row dominance at the expense of making column dominance worse. Since scaling is just equivalent to changing the measurement units, measuring the first output in Fahrenheit instead of Celsius would increase the first row by a factor of 1.8, leaving the row dominance unchanged but improving the column dominance of the first column and worsening the dominance of the second column. A system is said to be “generalized diagonally dominant” if there are row and column scales that make it dominant.

2. Basic Design

Nyquist’s stability proofs were complemented by the design methods introduced by Bode. His frequency response designs were based on plots of the gain as a function of frequency using logarithmic scales for the axes, enabling high, medium and low frequency gains to be seen simultaneously. One of his main aims was to minimize the variations of the controlled response when the system response changed. This was achieved by high loop gains within the required bandwidth.

His second aim was to keep the bandwidth as small as possible since high bandwidth actuators are expensive. Bode’s single degree of freedom approach was extended to a two degree of freedom approach by Horowitz, allowing the noise rejection and sensitivity to be designed independently of the input output response. He also developed an ideal frequency response, which had greater phase margins than those proposed by Bode. The increased phase margins give a more robust design.

2.1. Multivariable Design Methods

One natural extension of single loop control to multivariable systems is the sequential loop difference approach of Mayne. Here the system’s inputs and outputs are reordered to make the transfer function as near as possible to diagonal or lower triangular. A single loop design is performed on the first loop and the transfer functions re-evaluated with this control included.

Next, the second loop is designed and included, and the process is repeated until all the loops have been controlled. This approach works best when applied to diagonal dominant systems, which often occur. However, in some systems there is significant interaction between loops requiring a multi-loop design.

Rosenbrock introduced a method based on the Nyquist and Inverse Nyquist arrays where the diagonal elements of the arrays have Gershgorin circles superimposed on them. These circles indicate the effects of the loops on each other. Pre-compensation is used to make the system more diagonal dominant, and then single loop dynamic controllers are designed on the basis that the frequency response of each loop lies within its Gershgorin band. One way of calculating a suitable pre-compensator is to choose the compensator to minimize the sum of the squares of the off-diagonal elements in the compensated transfer function G(s)K subject to having the sum of the squares of the elements of each column of the controller equal to unity.

The characteristic locus stability test is also used as the basis for a design method by MacFarlane. In this method, high frequency interaction is decreased by using a constant matrix K_h to make the system more diagonally dominant at high frequencies. The compensated characteristic loci are then calculated and single loop compensators $\lambda_k(s)$ are designed for each of the loci. These could not be directly implemented since the individual loci do not correspond with individual inputs. To implement these, a constant approximation A is made to the eigenvector matrix U(s) of the system $G(s)K_h$ and a constant approximation B is made to the inverse eigenvector matrix V(s). The approximate commutative controller is then implemented as $A\lambda_k(s)B$, where the individual controllers are put together as a diagonal matrix $\lambda_k(s)$. The eigenvector structure of the controller then approximates that of the $G(s)K_h$, and so the diagonal elements are approximately applied to the individual characteristic gains. Integral action is then added to remove low frequency interaction. The initial high frequency compensator is designed using the ALIGN algorithm, which minimizes the squares of the errors E between the compensated frequency response $G(jw)K_h$ and a diagonal matrix with unit modulus elements J, where

$$J = G(jw)Kh + E \tag{3}$$

This same algorithm is used to calculate the A and B matrices for the approximate commutative controller, with A chosen to make V(s)A approximately diagonal, and B chosen to make BU(s) approximately diagonal.

2.2. Integrating the multivariable design methods

These three stability tests lead to frequency response design and analysis methods. The main design steps are: -

1. Scale and reorder the inputs and outputs, to decouple into subsystems.
2. Sequentially design and add controllers to the subsystems, starting with the fastest.
3. Test the complete set of controllers.

The steps for the controller design for each subsystem are: -

1. Use a constant pre-compensator to reduce the interaction at medium and high frequencies.
2. Design dynamic compensators either for each loop, or for each characteristic locus, concentrating on medium and high frequencies, subject to the extra compensator being close to a unit matrix at high frequencies.
3. Design and add low frequency compensators to increase the low frequency gain, subject to not changing the high and medium frequency loop gains too much.

The aims of the design usually include, stability, low sensitivity to noise and parameter variations, and robustness.

3. A design example for an unstable chemical reactor

Using the frequency-domain tools described above, a control system design is often carried out by using both Nyquist Arrays and Characteristic Loci at different stages in the design process. This can best be illustrated by considering an actual design study, as follows.

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Biographical Sketches

Dr John M Edmunds obtained a B.A. in Mathematics at Cambridge University in 1971, and an M.Sc. (1973) and Ph.D. (1976) in automatic control at UMIST. My research started at UMIST in 1973 working on Self Tuning Control schemes for controlling unknown systems. In 1976, I moved to Cambridge and worked with Professor A.G.J. MacFarlane on multivariable control theory and computer aided design in the form of the Cambridge Linear Analysis and Design Program. I also extended various multivariable frequency response design and analysis methods, in particular I found stability theorems for linear and non-linear systems, and formulated a closed loop design method going by the name of KQ approach or the Edmunds approach. In 1981, I returned to UMIST where I continued working of frequency response optimization including two degree of freedom H∞ control. In particular, I have suggested systematic methods of choosing the loss functions to be optimized. The main theoretical areas I have worked on have been the use of uncertainty in design and analysis, estimation and real time control. The uncertainty approach leads to good statistical methods for model order reduction, model validation, and condition monitoring. It also can be used to widen the scope of control from robust control to reliable control, where the probabilities of poor behavior can be estimated. More recently, I have moved onto control applications in the area of building control and real time control.

Professor Neil Munro obtained his B.Sc. in Electrical Engineering from Strathclyde University in 1962, and later his M.Sc. in 1967 and Ph.D. in 1969 from UMIST, and D.Sc. in 1982 in Control Engineering from the University of Manchester. He was appointed Professor of Applied Control Engineering at UMIST in 1979. From 1988 to 1991, he served as Vice-Principal for External Affairs and currently is Head of the Control Systems Centre, having held this post previously for some 15 years. He was Chairman of the UK SERC's Control & Instrumentation Sub-Committee (1983-85), was the Director of Control and Instrumentation within the SERC/DTI Joint Framework for Information Technology (1991/92), and was the National Consultant on Control & Instrumentation for the EPSRC (1992/97). He was founding co-editor of the IEE Proceedings on Control Theory and Applications (1980/89), and was awarded the Sir Harold Hartley Medal (1982), and the Honeywell International Medal (1997), of the Institute of Measurement & Control. He has spent some 7 years in industry, working on submarine radar systems, radar guidance systems for ground-to-air missiles, industrial liquid metering controllers, and the design of alpha-numeric displays working for Barr & Stroud Ltd, Ferranti, Parkinson Cowan Measurements, and ICL, respectively. He is co-author of 1 book and editor/co-editor of 3 books. He has published over 200 papers in journals and conference proceedings, and has supervised 70 M.Sc. and 30 Ph.D. students who graduated successfully. His major research interests are in the development, implementation and application of CAD techniques for multivariable systems, with particular interest now in using symbolic algebra languages for the design of robust controllers for uncertain multivariable systems. Application areas include aerospace and process control.