CONTROL OF LINEAR MULTIVARIABLE SYSTEMS

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Keywords: Multivariable system, Impulse response, Internal model principle, Separation theorem, Regulator, Discrete-time control, Continuous-time control, Canonical form

Contents

Summary

This chapter concerns the analysis and control system design of linear multivariable systems. The systems are represented in several different forms in state space.

1. Linear Multivariable Systems

1.1. Emergence of State Space Approach

A linear system with multiple-inputs and/or -outputs is called a linear multivariable system (or linear a MIMO system). The history of the emergence of multivariable linear control systems theory is written nicely in Pearson (1991) describing how Kalman's state space approach appeared after Freeman and Kavanagh's multivariable control approaches based on transfer function models. The state space approach has introduced an effective and systematic method for the design of a controller for a multivariable system. Description of a multivariable system by a set of first order simultaneous differential equations is called a state space representation which had been used by Lyapunov for stability analysis. Parks (1992) wrote the story of stability of Lyapunov in his article: " A.M.Lyapunov's stability theory —100 years on". Control system design in today's sense had been initiated by a series of seminal papers of Kalman, which not only proposed optimal control for Linear Quadratic (LQ) criterion (Kalman 1960) and the so-called Kalman-Bucy Filter (Kalman and Bucy 1961) but also introduced the structural properties of linear multivariable systems like controllability and observability with the idea of duality (Kalman 1960). Controllability is the existence of a control input to transfer the initial state to the origin, which gives a sufficient condition for the existence of an optimal control. Observability which assures the existence of a filter is related to the unobservable subspace, which consists of states yielding zero outputs for zero inputs. The most impressive result in LQ optimal control is that the control law is a state feedback law given by the solution of a Riccati equation and the state is estimated by a filter constructed using the solution of Riccati equation. The duality of control and state estimation is clarified by Kalman in 1960 in his paper at IFAC Moscow Congress. Wonham later showed that the control law and the filter can be designed independently, which is known as the *Separation Principle* (Wonham 1968). For his contributions to the development of system theory, *IEEE Trans. on Automatic Control* dedicated the December 1971 *Special Issue on Linear-Quadratic-Gaussian Problem* to R.E.Kalman which included bibliography spanning the literature on the subject until 1971 (Mendel and Gieseking 1971). Kalman was awarded IEEE Medal of Honor in 1974 and the Kyoto Prize in 1985. A list of Kalman's publications is available in the book edited by Antoulas (1991).

The state space representation or the internal description of an n-dimensional linear multivariable system with m-inputs, p-output, is expressed as

$$\frac{d}{dt}x = Ax + Bu \tag{1}$$

$$y = Cx + Du \tag{2}$$

where x is an n-state vector, u is an m-input vector, y is a p-output vector, and A, B, C, D are $n\times n$, $n\times m$, $p\times n$ and $p\times m$ real coefficients matrices. The transfer function of the system is

$$H(s) = C(sI - A)^{-1}B + D \tag{3}$$

Thus the transfer function is uniquely determined from the state space representation but the state space representation can not be given uniquely from a transfer function. The state space representation constructed from the input-output relation like transfer function or Markov Parameters is called a realization and the minimal dimension of the realization called the *minimal realization* is related to the structure of the system from the controllability and observability viewpoint (Gilbert 1963), where the Markov parameters are $D, CA^iB, i = 0,1,2,\cdots$ for the above given system. The state space representation depends on the choice of the coordinates, and the system with the state represented by different coordinates is said to be an equivalent system which has the same transfer function. Since a class of such systems with the same dimension has the same transfer function, the standard form representing the class is called the *canonical form*, and the controllable and observable canonical forms have been used as the standard forms of systems (Luenberger 1967). The structure of systems from either controllability or observability point of view has been studied extensively (Kalman

1963, Popov 1972). The control input to stabilize the system described in state space is achieved by the state feedback

$$u = Fx \tag{4}$$

if the system is stabilizable. In this case, the control law can be determined so that the closed loop system

$$\frac{d}{dt}x = (A + BF)x \tag{5}$$

is stable. Equivalently, all eigenvalues of $(A + BF)$ can be assigned in the left half of the complex plane by properly choosing the control matrix F. If (A, B) is controllable, these eigenvalues can be assigned arbitrarily (see *Pole Placement Control*). Thus controllable system can be stabilized. The controllability of (A, B) is that the controllable subspace ($\mathrm{Im}\, age[B, AB, A^2B, \cdots, A^{n-1}B]$) is equal to the whole state space. The invariant class of controllable systems for state feedbacks is represented by the Brunovsky canonical form (1970) (see *Canonical Forms for State Space Descriptions*). The realization of the system from a practical viewpoint has been studied by many people in the field of identification and Moore (1981) gave the idea of balanced realization, which is not only used in model reduction (Pernebo and Silvermann 1982) but also in robust control synthesis later. Robust control is a topic outside the scope of this chapter and will not be discussed. The realization from the Markov parameters was presented by Ho and Kalman (1966) (see *Frequency Domain Representation and Singular Value Decomposition*). Gopinath (1969) presented a method to determine a state space model from input-output data. The relation of the input output behavior to the state space was discussed in Willems (1986).

The structure of the system has also been interpreted by the geometric point of view, which is called "Geometric Approach". The approach has been effectively used for disturbance decoupling and decoupling control by finding the input matrix generating the controllability subspace in the unobservable subspace. The details of the approach are found in the textbook by Wonham (1974).

Since all state variables are not directly measurable, the state should be reconstructed from the input and output through the linear system

$$\frac{d}{dt}z = \hat{A}z + \hat{B}y + \hat{J}u \tag{6}$$

$$\hat{x} = \hat{C}z + \hat{D}y \tag{7}$$

which is called the *observer*, where $\hat{x}$ is the estimate of the state x, and $\hat{A}$ should be stable. $\hat{x}$ satisfies

$$\lim_{t\to\infty} \| x(t) - \hat{x}(t) \| = 0 \tag{8}$$

The existence of the observer estimating the state is assured by the *detectability* of the system (C, A). Detectability means that all states in the unobservable subspace are stable, so the observable system is detectable. If the dimension of the observer is equal to that of the system n, it is called the full order state observer (see *Full-Order State Observers*) and if it is less than n, it is called the reduced order state observer (see *Reduced-Order State Observers*).

Multivariable control based on the state space description has been applied in the design of real systems like industrial processes, transportation systems, robotics, manufacturing systems and others. Bryson and Ho (1969) described how to use optimal control in the practical problems. The applications to mechanical systems are also found in *ASME J. of Dynamic Systems, Measurement and Control 50th Anniversary Issue* (June, 1993). The approach based on the transfer function has been studied by Rosenbrock (1970) and Wolovich (1974), and the relation of the state space to the rational transfer function has become clearer.

Many text books on linear multivariable control have been published and also coming to be published. Some of the early books are by Zadeh and Desoer (1963), Ogata (1967), Brockett (1969), Chen (1970), Desoer (1970) and others.

1.2. Discrete-time Control

The state space representation was first used for the analysis and design of discrete-time systems. Kalman started to use the state space approach for the analysis and design of discrete-time systems in order to evaluate the inter-sampling behavior. Tou (1964) wrote a book entitled "Modern Control Theory", since the state space approach was said to be *modern control theory* in those days. Text books on discrete-time control based on the state space have been written by Tou (1964), Kuo (1980) and Ackermann (1985), and those based on the transfer function were written by Wolovich (1974), Kucera (1979) and Åström and Wittenmark (1984). Åström developed control system design combined with identification, which is called *self-tuning control*. He has received the IEEE Medal of Honor in 1993. A list of Åström’s publications is available in (Wittenmark and Rantzer 1999).

In designing a discrete-time controller for a continuous-time system, we have to consider a) hold devices and b) redesigned problems.

A typical discrete-time system is one in which measurement of the output and the control action of the input take place at $k\Delta, k = 0, 1, 2, \cdots$, where Δ is the sampling interval. If the control input $u(t)$ is keeping its value during the sampling interval,

$$u(t) = u(k\Delta), k\Delta \le t < (k+1)\Delta \tag{9}$$

the input is considered as the output of a zero order hold. Using the above relation in the continuous-time system, the state space representation is rewritten as

$$x_{k+1} = \Phi x_k + \Gamma u_k \tag{10}$$

$$y_k = Cx_k + Du_k \tag{11}$$

where

$$x_k = x(k\Delta), u_k = u(k\Delta), y_k = y(k\Delta)$$

and

$$\Phi = e^{A\Delta}, \Gamma = \int_0^{\Delta} e^{At} B dt$$

They are computed by

$$e^{\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}} = \begin{bmatrix} \Phi & \Gamma \\ 0 & I \end{bmatrix} \tag{12}$$

since

$$\frac{d}{dt}\begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ u \end{bmatrix}, k\Delta \le t < (k+1)\Delta \tag{13}$$

In the transformation of a continuous-time system to a discrete-time system, the poles (λ) of the continuous-time system are transformed into $e^{\lambda\Delta}$. The left half of the complex plane is transformed into the unit disk centered at the origin. All poles of a stable discrete-time system are placed inside the unit disk. This shows that a stable continuous-time system is transformed into a stable discrete-time system by discretization. For stabilization of a discrete-time system, the control input given by the state feedback

$$u_k = Fx_k \tag{14}$$

should locate all eigenvalues of $(\Phi + \Gamma F)$ inside the unit disk.

Concerning zeros, Åström, Hagander and Sternby (1984) had shown that a continuous-time system is transformed into a discrete-time system with more zeros some of which may lie outside the unit disk (loosely termed as unstable zeros). The discretization of a continuous system may make the system uncontrollable or unobservable depending on the choice of the sampling interval (Kalman, Ho, Narendra 1963) due to the zeros outside the unit circle (systems having such zeros are also known as nonminimum phase systems).

This may be illustrated by the following example: A system with the transfer function

$$H(s) = \frac{s + (2\pi)^2 + 1}{s^2 + 2s + (2\pi)^2 + 1}$$

corresponds to the continuous-time system

$$\dot{x} = \begin{bmatrix} -1-2\pi j & 0 \\ 0 & -1+2\pi j \end{bmatrix} x + \begin{bmatrix} -1+2\pi j \\ -1-2\pi j \end{bmatrix} u$$

$$y = [0.5,\, 0.5]x$$

If this system is sampled at every unit time, i.e., the sampling interval Δ=1, the discrete-time system with zero order hold for the input is given by

$$x_{k+1} = \begin{bmatrix} e^{-1} & 0 \\ 0 & e^{-1} \end{bmatrix} x_k + \begin{bmatrix} 1-e^{-1} \\ 1-e^{-1} \end{bmatrix} u_k$$

$$y_k = [0.5,\, 0.5]x_k$$

Since the system is neither controllable nor observable, the input-output relation of this system is equivalent to the following first order system:

$$\bar{x}_{k+1} = e^{-1}\bar{x}_k + (1-e^{-1})u_k$$

$$y_k = \bar{x}_k$$

This is the discrete-time system obtained by sampling and zero order holding the continuous-time system with the transfer function

$$H(s) = \frac{1}{s+1}$$

A different feature of a discrete-time system is the use of nonzero order hold for the control input and the multirate sampling of the output. This kind of mechanism makes it possible to enhance the characteristics (Kabamba 1987, Hamby, Juan, Kabambab 1996).

Multirate sampling gives not only improvement of the performance but also some interesting characteristics of control (Araki 1993). Werner (1995) used multirate sampling for simultaneously stabilizing control systems.

1.3. Riccati Equation and Stabilization for Continuous-time Systems

Riccati differential equation was shown to be related to the adjoint equation of the system's equation for the LQ optimal control (Kalman 1960) and filtering problems with white Gaussian measurement noise (Kalman and Bucy 1961). The optimal control minimizing the following quadratic criterion function:

$$J = \int_0^{t_f} (x^T Q x + u^T R u)dt + x^T(t_f)P_f x(t_f) \tag{15}$$

for the linear system represented by the state space is given by

$$u(t) = -R^{-1}B^T P(t)x(t) \tag{16}$$

where R is positive definite and $P(t)$ is the solution of the following Riccati Differential Equation with terminal constraint:

$$-\frac{d}{dt}P = A^T P + PA + Q - PBR^{-1}B^T P,\ P(t_f) = P_f \tag{17}$$

The solution of the Riccati Differential Equation $P(t)$ is given by (Bucy and Joseph 1968)

$$P(t) = [\Phi_{21}(t-t_f) + \Phi_{22}(t-t_f)P(t_f)][\Phi_{11}(t-t_f) + \Phi_{12}(t-t_f)P(t_f)]^{-1} \tag{18}$$

where

$$\Phi(t) = \begin{bmatrix} \Phi_{11}(t) & \Phi_{12}(t) \\ \Phi_{21}(t) & \Phi_{22}(t) \end{bmatrix} = e^{Ht}$$

where H is the Hamiltonian matrix

$$H = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \tag{19}$$

When the criterion considered is defined over a infinite time horizon, i.e., t_f is infinite, the LQ optimal control is given by constant state feedback which makes the closed loop system stable when the system is controllable and the pair of the criterion and the system is observable.

The steady state solution of the Riccati differential equation is given by the Algebraic Riccati Equation (ARE). The Algebraic Riccati Equation corresponding to above LQ control is

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{20}$$

The positive definite solutions of Algebraic Riccati Equations determine the control law for the LQ optimal control (see *Optimal Linear Quadratic Control* for LQ). The Kalman Filter for the system subject to the disturbance and measurement noise

$$\frac{d}{dt}x = Ax + Bu + Dw \tag{21}$$

$$y = Cx + n \tag{22}$$

is given by

$$\frac{d}{dt}\hat{x} = A\hat{x} + Bu + K(y - C\hat{x}) \tag{23}$$

where w and n are zero mean independent white Gaussian noise terms with variance Q and R, and the filter gain K is

$$K = PC^T R^{-1} \tag{24}$$

where P is the positive definite solution of Algebraic Riccati Equation for the filter

$$AP + PA^T + Q - PC^T R^{-1} CP = 0 \tag{25}$$

(see *Kalman Filters*). The positive definite solution of ARE is computed using the eigenvectors corresponding to the stable eigenvalues (Potter 1966) of Hamiltonian matrix H. This is computed if the system is controllable. Let the eigenvalues of H with negative real parts be $\lambda_i, i = 1, 2, \cdots, n$, and the corresponding eigenvectors be $[x_i^T, y_i^T]^T, i = 1, 2, \cdots, n$. Then the positive definite solution of Algebraic Riccati Equation for LQ by Potter is

$$P = [y_1, y_2, \cdots, y_n][x_1, x_2, \cdots, x_n]^{-1} \tag{26}$$

The closed loop system by the optimal control is stable. This is proved by using Lyapunov equation, and it is found that the system is stable with the gain margin from 0.5 to infinity (Safanov Athans 1971). The characteristics of Algebraic Riccati Equation are described well by Willems (1971). Assignment of the poles of the systems in the shifted left plane has been shown in the textbook of Anderson and Moore (1985).

The feedback law to assign poles in the shifted disk for a continuous-time system (1988) can be constructed by using the modified discrete-time Riccati equation. When the criterion considered is defined over the finite time, the control law may not stabilize the closed loop system (see *Optimal Linear Quadratic Contro*l). The choice of the criterion function and the study of the stability for the LQ optimal control defined over the finite time were studied by Kwon and Pearson (1978). Riccati equation corresponding to discrete-time problems is also studied. The optimal control for the quadratic criterion

$$J = \sum_0^\infty (x_k^T Q_D x_k + 2x_k^T S_D u_k + u_k^T R_D u_k) \tag{27}$$

for the discrete time system is given by the state feedback

$$u_k = -(R_D + \Gamma^T P_D \Gamma)^{-1} (\Gamma^T P_D \Phi + S_D^T) x_k \tag{28}$$

where P_D is the positive definite solution of the following Discrete Riccati Algebraic Equation.

$$P_D = \Phi^T P_D \Phi + Q_D - (\Phi^T P_D \Gamma + S_D)(R_D + \Gamma^T P_D \Gamma)^{-1} (\Gamma^T P_D \Phi + S_D^T) \tag{29}$$

The numerical computation of the positive definite solution of Discrete Riccati Equation for the discrete-time system when the system matrix is nonsingular was given by Potter (1968) and Pappas, Laub and Sandell presented one for the singular case (1980).

The details including the historical background of the computation of the discrete-time Riccati equation are given by Chen and Francis (1995). Hitz and Anderson (1972), and Kono and Furuta (1986) have shown that there exists the same solution for the continuous-time and the discrete-time Riccati equations for the bilinearly transformed discrete-time system.

A discrete-time controller for a continuous-time system was considered by Kuo which considered the inter-sampling behavior, with the quadratic cost fucntion. The approach is extended to the case of delay in the control action (Kondo and Furuta 1985).

A closed form solution of the discrete-time Riccati equation is given by Furuta and Wongsaisuwan (1993) which makes it possible to design the LQ optimal control from the impulse response data (Furuta and Wongsaisuwan 1995) .

The Riccati equation has also been used to design continuous control system to place the poles in a specified region like a disk (Kim and Furuta 1988).

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