SAMPLE PATH ANALYSIS OF DISCRETE EVENT DYMANIC SYSTEMS (DEDS)

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Contents

Summary

Many modern engineering and social systems can be modeled as discrete event dynamic systems (DEDS). Sample path analysis explores the information provided by a single sample path of a DEDS to estimate the performance sensitivity or to implement performance optimization. The central piece of the sample path based approach is Perturbation Analysis (PA), which provides efficient algorithms for estimating the performance sensitivity for many DEDS with a queueing-type of structure. The basic concepts, principles and fundamental theory of PA are reviewed.

Markov potential theory provides another sample path approach for systems with a general structure. Other approaches include the likelihood ratio method and the weak derivatives. The research for sample path based optimization has two main directions: the approach that combines stochastic approximation with PA for continuous variables, and the approach based on Markov potentials (or online Markov decision processes) for discrete policies. Sample path techniques can be applied *online* to real systems since the

performance improves without interfering with the operation of the system.

1. Introduction

Modern civilization has certainly created many human-made systems that evolve dynamically in time and are extremely complex. Examples abound, ranging from world-wide communication networks, large automated manufacturing plants, to traffic system in the air, land, and sea, to military C^3I and logistic systems. The performance of such systems is difficult to analyze and evaluate. The only general method of last resort is *simulation*. Standard simulation methodology involves getting statistical estimates of means and confidence intervals, both of which are time consuming to compute and their accuracy is subject to the fundamental limitation of the order of $1/(N)^{1/2}$ where N denotes the length of the simulation experiments. Consequently, efforts have been directed in recent years to extract out as much information as possible from a single sample path. The topic of "Sample Path Analysis of Discrete Event Dynamic Systems (DEDS)" is devoted to this goal.

The sample path based approach explores the information provided by a single sample path of a discrete event system to estimate the performance sensitivity or to implement performance optimization. The sample path can be obtained either by simulation or online observation of a real engineering system. Sample path analysis should be distinguished from the simulation approach in the sense that the former can be implemented online for a real system, while the latter may allow non-practical schemes to be used and hence may not be implementable in real life.

Currently, the works for sample path based sensitivity analysis belong to three major areas: (i) perturbation analysis (PA), (ii) Markov potential theory, and (iii) the Likelihood ratio method and others. PA employs the dynamics of the system and is based on the following thought: what would be the sample path (i.e., all the event times) if a system parameter (or parameters) changes by a small amount while the probability nature of the system keeps unchanged? The likelihood ratio method considers the question from the opposite view: what is the probability that the current sample path would occur for the same system except that a parameter (or parameters) changes by a small amount?

With the Markov potential theory, the effect of a system jumping from one state to another state on system performance can be measured by using the difference of the potentials at these two states. Therefore, similar to the PA approach, the performance sensitivity can be obtained by the average changes in potentials due to parameter changes.

Clearly, the gradients obtained by the sample path based approach can be used in standard optimization methods, e.g., the stochastic approximation approaches (in particular, the Robbins-Monro algorithm), to obtain the optimal performance. If the parameters are discrete (i.e., with a discrete policy space), on-line Markov decision process methods can be developed by using potentials.

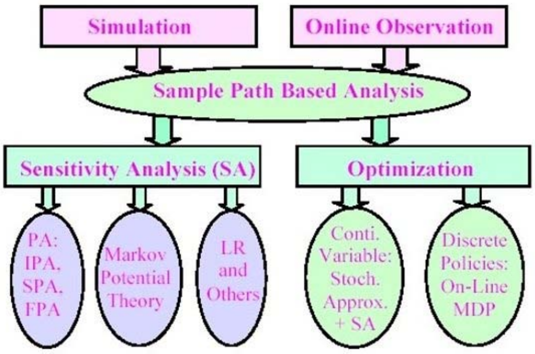

Figure 1. Relations Among Different Areas in Sample Path-Based Analysis

Figure 1 illustrates the relations among different areas in sample path based analysis.

Finally, a related simulation and sample path based stochastic optimization scheme called *Ordinal Optimization*, should be mentioned. The major goal of ordinal optimization is to minimize the number of sample path evaluations needed for optimization.

2. Perturbation Analysis

2.1 Formulation of Infinitesimal Perturbation Analysis

The following terminology is used for discussion.

θ: Design parameter vector of the system, each parameter is defined on a subset of $R = [0, \infty]$.

ξ: Random vector representing all the random aspects of the discrete event system (e.g., a sequence of [0,1] uniformly distributed random variables used in simulation).

$x(t;\theta,\xi)$: A sample path of the system which depends on θ and the realization of the random vector ξ.

$L_M\,[x(t;\theta,\xi)]$: Sample performance, the performance of the system measured on the sample path $x(t;\theta,\xi)$ from time 0 to the Mth event time.

A mathematical description of a sampled path $x(t;\theta,\xi)$ is called a Generalized Semi-Markov Process (GSMP). A formal definition will be postponed to Section 2.4 where it is needed.

The performance of interest is the average value

$$J_M(\theta) \equiv E[L_M(x(t;\theta,\xi)],$$

which is often estimated by

$$J_M(\theta) \approx \tfrac{1}{N}\sum_{i=1}^{N} L_M(x(t;\theta,\xi_i)\,.$$

The subscript M is omitted when there is no confusion. When M is large, $L_M(x(t;\theta,\xi))$ is close to the steady state performance. The confidence interval of the above estimation cannot be improved upon faster than the fundamental limit mentioned above, i.e., for one order of increase in accuracy of the estimate, two orders of increase in simulation length are required. This often times is an unacceptable computational burden.

One of the goals of modern sample path analysis of DEDS is to answer the question:

Can we infer information about x(t; $\theta+\Delta\theta,\xi$) from that of x(t; θ,ξ) alone?

More specifically,

Can we estimate dJ/dθ not from the usual formula

$$\frac{dJ}{d\theta} = \lim_{\Delta\theta -> 0} \frac{J(\theta+\Delta\theta) - J(\theta)}{\Delta\theta},$$

which is numerically fraught with difficulties, but from x(t; θ,ξ) alone?

Taking these questions on their face value, one can think of three “objections”, two immediately, and one after some thought. They are

Philosophical Objection: There is no free lunch. If the answer to the above questions were in the affirmative, then all our problems would disappear.

Intuitive Objection: If the simulation is carried out long enough, x(t; $+\Delta\theta,\xi$) will sooner or later differ from *x(t; .),* no matter how small $\Delta\theta$ is. After that, the two trajectories will diverge and may become quite different from each other.

And finally the

Mathematical Objection: The following question arises naturally: If the answer is “yes” and the derivative $\dfrac{dL(x(t;\theta,\xi))}{d\theta}$ is obtained from *x(t; .),* then is this estimate an unbiased one for $\dfrac{dJ}{d\theta}$? Or equivalently,

$$\frac{dJ}{d\theta} \equiv \frac{dE[L(x(t;\theta,\xi))]}{d\theta} = ? = E[\frac{dL(x(t;\theta,\xi))}{d\theta}].$$

In other words, can we interchange expectation with differentiation? (see Section 2.3

Unbiasedness and Consistency.) The right-hand side can be estimated by

$$\frac{1}{N}\sum_{i=1}^{N}\frac{dL(x(t;\theta,\xi_i))}{d\theta}.$$

These questions will be addressed in Section 2.3. For the moment, the following metaphor may offer an intuitive explanation and may help to dispel some doubts. Consider the problem of transmitting a series of motion pictures over a channel. The simple-minded way is to transmit fully each frame in succession. However, since successive images have a great deal of information in common, e.g., the unchanging background scene, a lot of bandwidth can be saved by transmitting only the first frame in full and the differences only for succeeding frames. By the same token, a great amount of computation involved in simulating *x(t;* $\ +\Delta\theta,\xi)$ has already been done in simulating *x(t;* $\ .\ $*)***.** To concurrently generate *x(t;* $\ +\Delta\theta,\xi)$ from *x(t;* $\ .\ $*),* only a small amount of additional computation needs to be done especially when $\Delta\theta$ is small. And when $\Delta\theta \to 0$, calculus can be used to great advantage. This is the basis of *Perturbation Analysis*, which is the heart of the modern sample path analysis of DEDS.

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Biographical Sketches

Yu-Chi (Larry) Ho received his S.B. and S.M. degrees in Electrical Engineering from M.I.T. and his Ph.D. in Applied Mathematics from Harvard University. Except for three years of full time industrial work he has been on the Harvard Faculty where he is the T. Jefferson Coolidge Chair in Applied Mathematics and the Gordon McKay Professor of Systems Engineering. He has published over 140 articles and three books. He is on the editorial boards of several international journals and is the editor-in-chief of the international Journal on Discrete Event Dynamic Systems. He is the recipient of various fellowships and awards including the Guggenheim (1970) and the IEEE Field Award for Control Engineering and Science (1989), the Chiang Technology Achievement Prize (1993), the Bellman Control Heritage Award (1999) of the American Automatic Control Council, and the ASME Rufus Oldenburger Award (1999). He is a Life fellow of IEEE, and was elected a member of the U.S. National Academy of Engineering in 1987 and a foreign member of the Chinese Academy of Sciences and the Chinese Academy of Engineering in 2000. His current research interests lie at the intersection of Control System Theory, Operations Research, and Computational Intelligence.

Xi-Ren Cao received the M.S. and Ph.D. degrees from Harvard University, in 1981 and 1984, respectively, where he was a research fellow from 1984 to 1986. He then worked as a principal and consultant engineer/engineering manager at Digital Equipment Corporation, U.S.A, until October 1993. Then, he joined the Hong Kong University of Science and Technology (HKUST), where he is a professor and the director of the Center for Networking. He held visiting positions at Harvard University, University of Massachusetts at Amherst, AT&T Labs, University of Maryland at College Park, Shanghai Jiaotong University, Nankei University, and University of Science and Technology of China, and Tsinghua University.

Dr. Cao owns two patents in data communications and published two books*: Realization Probabilities - the Dynamics of Queuing Systems*, Springer Verlag, 1994, and *Perturbation Analysis of Discrete-Event Dynamic Systems*, Kluwer Academic Publishers, 1991 (co-authored with Y. C. Ho). He received the Outstanding Transactions Paper Award from the IEEE Control System Society in 1987 and the Outstanding Publication Award from the Institution of Management Science in 1990. He is a Fellow of IEEE, Board of Governors of IEEE Control Systems Society, Associate Editor at Large of IEEE Transactions of Automatic Control, and he is/was associate editor of a number of international journals and chairman of a few technical committees of international professional societies. His current research areas include discrete event systems theory, communication systems, signal processing, stochastic processes, and optimization techniques.