ROBOT KINEMATICS AND DYNAMICS

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Summary

This chapter reviews current practical methodologies for kinematics and dynamics of robot manipulators. A kinematic model is a representation of the motion of the robot

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manipulator that does not consider mass and moments of inertia. A dynamic model is a representation of the relationship between the joint torques and the dynamical motion of the robot manipulator. Both models are used widely in the simulation of robot motion, analysis of robot manipulator structures, and design of control algorithms. In the kinematics, a degrees-of-freedom and mobility of manipulators, homogenous transformation, forward kinematics problem, inverse kinematics problem, differential kinematics, statics, singular configurations, and manipulability measure are described.

In the dynamics, a forward dynamics problem, inverse dynamics problem, Cartesian space dynamic model, and reduced dynamic model of robots restricted to a specified environment, analysis of minimum set of dynamic parameters, and dynamic parameter identification are described. Symbolic modeling of the kinematics and dynamics can be done using computer algebra systems. These topics are considered fundamental to the study and use of robots. The origins of the industrial robot date back to the early 1960s. The basic technologies of the kinematics and dynamics of robot manipulators have been established by 1980s. Applications of robots are now seeing widespread use in the manufacturing industry, space, agriculture, medicine, home, and so on.

1. Introduction

Robots have become an intelligent source of aid to humans, and are used in many fields of industry and everyday life. In the last four decades, a great deal of research and development has been carried out to equip robots with a force sensor, vision, and computerized control, to improve their intelligence and ability to interact with the environment.

The concept of the robot as a mechanical substitute for a human performing various kinds of labor was introduced by the Czech playwright Karel Capek in the play *Rossum's Universal Robots*, which appeared 1921. According to the widely accepted definition of the Robot Institute of America, a robot is *a re-programmable multifunctional manipulator designed to move materials, parts, tools or specialized devices through variable programmable motions for the performance of a variety of tasks*.

This definition, dating back to 1980, reflects the current status of robotics technology. Many other definitions of robot have been proposed; for example, that a robot is *apparently human automation, is intelligent and obedient, but is an impersonal machine*. Two common characteristic properties can be deduced from the variety of definitions; that is, versatility and adaptability. In the case of a robot, versatility is the structural potential for performing various tasks. Adaptability is the potential of the robot to achieve its objective by itself, and to interact with its environment. These are typical characteristics of almost all robots.

Kinematics and dynamics of robot manipulators are fundamental to robot technologies. The kinematics is the science of motion that does not consider mass and moments of inertia. It refers to all of the geometrical and time-based properties of the motion. The dynamics is the science of motion that represents the relationship between the joint torques and the robot motion. Both models are used widely in the simulation of motion, analysis of robot manipulator structures, and design of control algorithms. Currently,

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most of these analyses can be executed automatically using a computer algebra system equipped with a robot symbolic modeling library.

2. Kinematics

Kinematics is the science of motion. In the kinematics, the position and orientation, velocity, and acceleration of the robot manipulator are studied from the perspective of spatial geometry. To analyze the geometry, a link frame based on Denavit-Hartenberg description is attached to each link of the robot manipulator. The mobility of the manipulator, models of forward and inverse kinematics, and statics are represented.

2.1. Mechanical Structure of Robot Manipulator

Robot manipulators consist of rigid links that are connected with joints that allow relative motion of the neighboring links. The joints have one or more degrees of freedom (DOF), where the number of freedoms is equal to the number of independent parameters that would have to be specified in order to locate all parts of the mechanism.

In most of the manipulator, the joints are revolute or pivot joints with 1 DOF, or prismatic joints with 1 DOF, as shown in Figure 1, in which conventional representations of joints are also shown. A joint being moved actively by the actuator is called an active joint, and a joint not being actuated is called a passive joint.

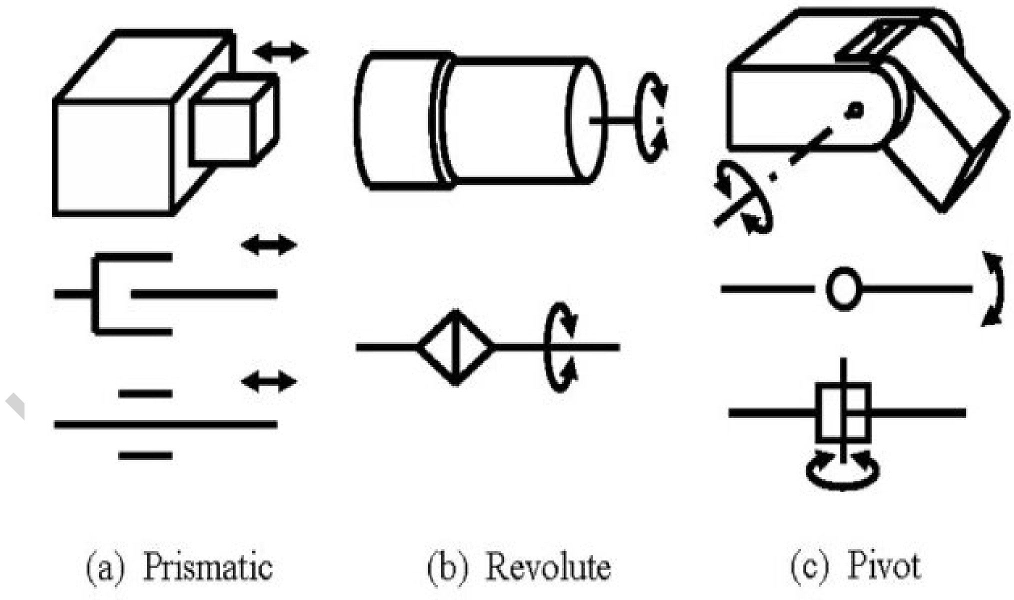

Figure1: 1 DOF joints and conventional representations

The free end of the chained links is called the end-effector. Figure 2 shows two types of link mechanisms; one is an open link mechanism, in which all links are connected serially to the end-effector, and the other is a closed-link mechanism, in which some links form a closed loop. In general, the open link mechanisms have a wider workspace than that of the closed-link mechanism, but the stiffness is lower.

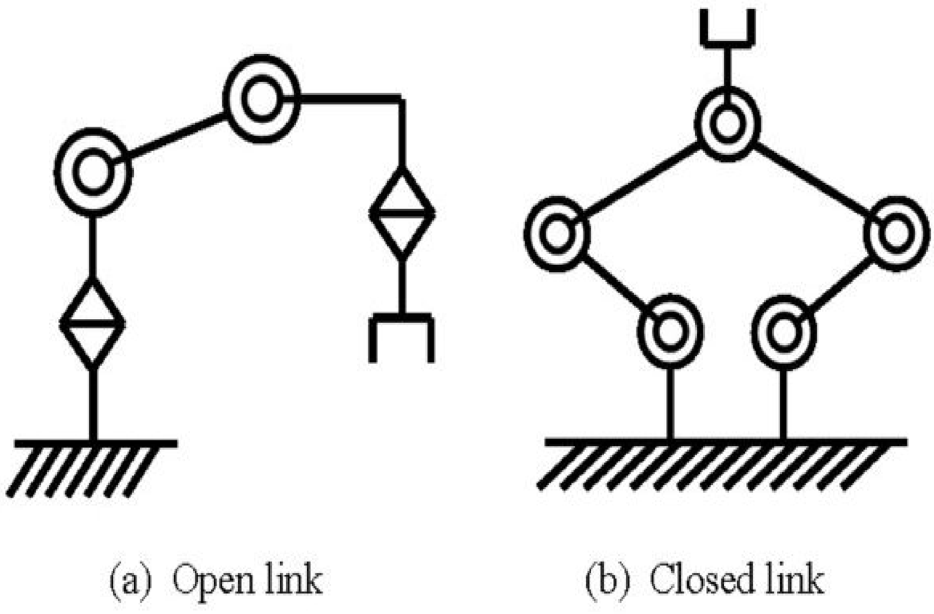

Figure 2: Link mechanisms

To move the end-effector to an arbitrary position and orientation in 3-dimensional space, a 3 DOF for position along the x, y, and z axes, and a 3 DOF for orientation around x, y, and z axes--a total of 6 DOF--is needed.

For this reason, most manipulators have 6 DOF, though they may use less than 6 DOF for work in a limited space and need more than 6 DOF to avoid obstacles in the workspace.

Given that the arm and wrist of a human has 7 DOF, the human elbow can move up and down without movement of the shoulder, and the grasped object thereby avoids obstacles.

There are many robot types, depending on the configuration of the joints. Figure 3 shows the four basic configurations of robot manipulators. A Cartesian robot has 3 prismatic joints. The mechanical stiffness is very high, but the workspace for the setting space is relatively small.

This type is suitable for a high precision positioning system. A cylindrical robot has one revolute joint and two prismatic joints. The workspace for the setting space is larger than that of the Cartesian robot.

This type is convenient for a task in which the robot handles objects arrayed in a circle. A polar robot has two revolute joints and one prismatic joint. This type is used for tasks similar to those of the cylindrical robot.

An articulated robot has 3 revolute joints. Comparatively, the workspace for the set space is largest, but it is not easy to create high stiffness.

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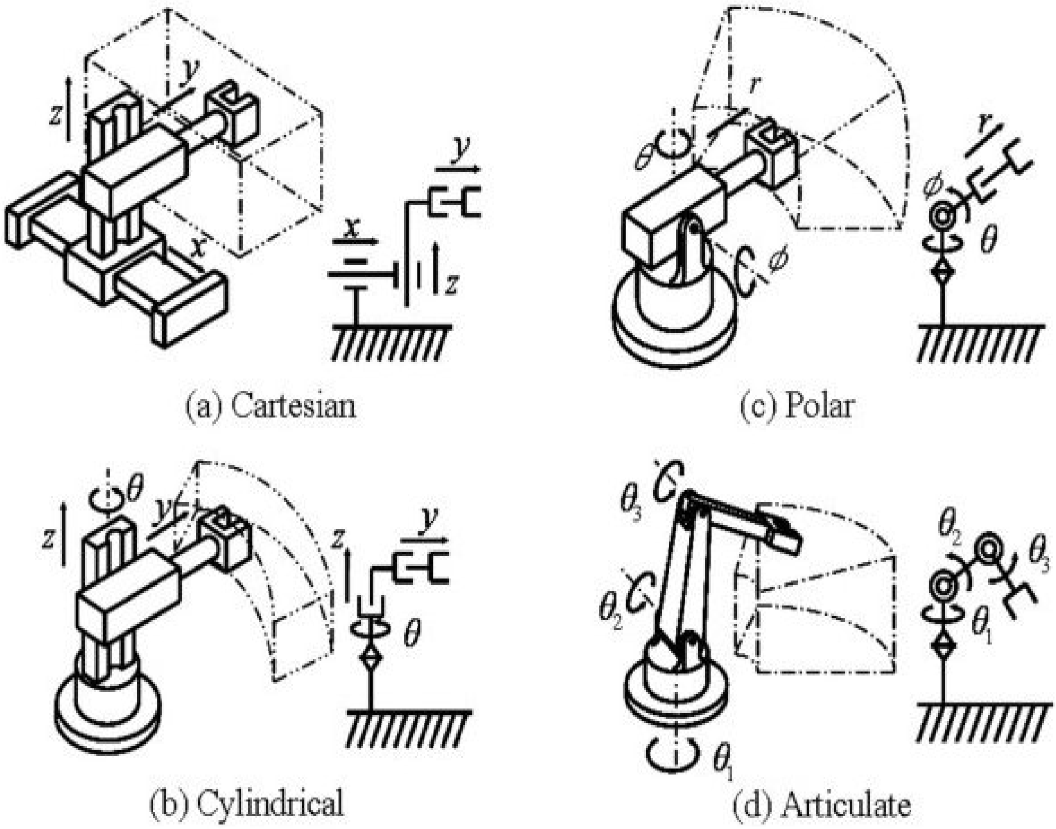

Figure 3: Robot configurations

The robot motion is characterized by the degree of freedom and the mobility. In 3-dimensional space, the maximum number of the degrees of freedom of an object is six. The mobility is the theoretical number of independent parameters by which to specify the positions and orientations of all links of the mechanism. For a spatial mechanism, the mobility M is given by

$$M = 6L - \sum_{i=1}^{N}(6 - l_i) = 6(L - N) + \sum_{i=1}^{N} l_i \qquad (1)$$

where L is the number moving links, N is the total number of joints, and l_i is the degree of freedom of joint i. For a general robot, this mechanism is called a non-redundant mechanism if $D = M$, and if $D < M$, where D is a number of DOF, the robot is called a redundant robot. For example, let us consider a parallel mechanism called the Stewart platform, shown in Figure 4.

This mechanism has 6 legs, each of which has one prismatic active joint in its center, and two spherical passive joints, one at each end. Each leg is allocated parallel to the others and jointed to the base and plate. So, the DOF of the plate is 6. On the other hand, the mobility is 12, because $L = 2\times 6 + 1$, $N = 3\times 6 = 18$, and $\sum l_i = 3\times 12 + 1\times 6 = 42$. Hence, this is a redundant mechanism, because the rotational angles around the prismatic joint axis are free even if the position and orientation of the plate is specified.

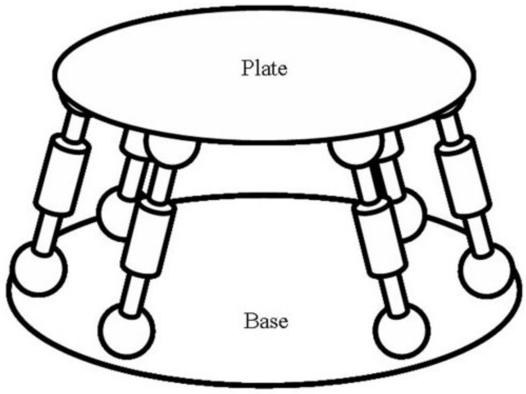

Figure 4: A general form of the Stewart Platform

2.2. Orientation of a Rigid Body

The motion of robot manipulators are analyzed using a reference frame, which includes a base frame that is attached to the nonmoving base of manipulator, an end-effector frame that is attached to the end-effector of manipulator, sensor frames that are attached, for example, to the force sensor and vision sensor, an object frame that is attached to the body of the object, and a work frame that is attached to the work cell, as shown in Figure 5, in which Frame *i* is described by Σ_i. The frame is a 3-dimensional orthogonal coordinate system. The geometric relation between links is described by the coordinate transformation. Hereafter, three unit vectors of Frame *A* are given by $(\mathbf{x}_A, \mathbf{y}_A, \mathbf{z}_A)$, a symbol ${}^A\mathbf{p}$ is defined as a vector with respect to Frame *A*, that is, the left superscript is a referenced coordinate system, and a vector $\mathbf{a}$ is given by $\mathbf{a} = (a_x, a_y, a_z)^{\mathrm{T}}$.

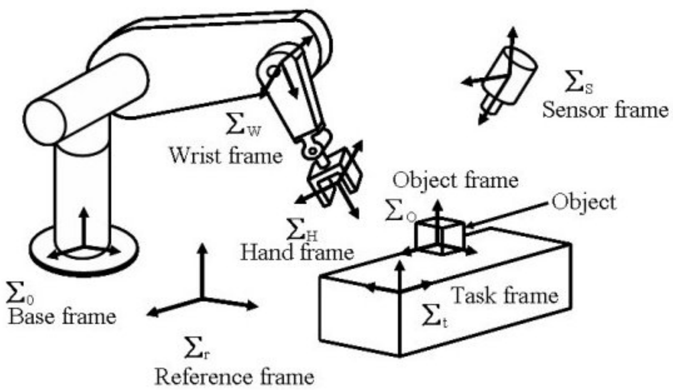

Figure 5: Frames

Consider a Frame *A*, the origin of which coincides with a Frame 0 as shown in Figure 6. A point *P* in space can be represented either as ${}^0\mathbf{p} = [{}^0p_x, {}^0p_y, {}^0p_z]^{\mathrm{T}}$ with respect to Frame 0, or as ${}^A\mathbf{p} = [{}^Ap_x, {}^Ap_y, {}^Ap_z]^{\mathrm{T}}$ with respect to Frame *A*. Subsequently, a relation between these two representations is given by

$$ {}^{0}\mathbf{p} = {}^{0}\mathbf{R}_{A}\,{}^{A}\mathbf{p} \qquad (2) $$

where ${}^{0}\mathbf{R}_{A} \in R^{3\times 3}$ is called a rotation matrix, which represents the transformation matrix of the vector coordinates in Frame A into the coordinates of the same vector in Frame 0, and is given by

$$ {}^{0}\mathbf{R}_{A} = \begin{bmatrix} \mathbf{x}_0{}^{\mathrm{T}}\mathbf{x}_A & \mathbf{x}_0{}^{\mathrm{T}}\mathbf{y}_A & \mathbf{x}_0{}^{\mathrm{T}}\mathbf{z}_A \\ \mathbf{y}_0{}^{\mathrm{T}}\mathbf{x}_A & \mathbf{y}_0{}^{\mathrm{T}}\mathbf{y}_A & \mathbf{y}_0{}^{\mathrm{T}}\mathbf{z}_A \\ \mathbf{z}_0{}^{\mathrm{T}}\mathbf{x}_A & \mathbf{z}_0{}^{\mathrm{T}}\mathbf{y}_A & \mathbf{z}_0{}^{\mathrm{T}}\mathbf{z}_A \end{bmatrix}. \qquad (3) $$

This is an orthogonal matrix; that is, $({}^{0}\mathbf{R}_{A})^{-1} = ({}^{0}\mathbf{R}_{A})^{\mathrm{T}} = {}^{A}\mathbf{R}_{0}$, and ${}^{0}\mathbf{R}_{0} = \mathbf{E}_3$ where $\mathbf{E}_3 \in R^{3\times 3}$ is a unit matrix.

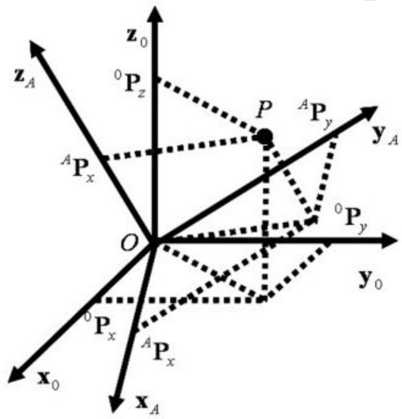

Figure 6: Representation of a point P in two different coordinate frames.

The position and orientation of an object is called the pose of the object. The object pose is described by the pose of the object frame that is attached to the object. The rotation matrix is one representation of the orientation, but it is redundant because a number of its elements equal 9 and the number of independent parameters of orientation is 3. As a description of the orientation in terms of 3 independent parameters, Z-Y-Z Euler angles and Roll-Pitch-Yaw angles are often used.

Z-Y-Z Euler angles

A rotation matrix described by the Z-Y-Z Euler angles is obtained by the following elementary rotations, as shown in Figure 7.

- Rotate the base frame by angle ϕ about axis $\mathbf{z}_0$; this rotation is described by ${}^{0}\mathbf{R}_{0'}$.
- Rotate the current frame by the angle θ about axis $\mathbf{y}_0'$; this rotation is described by ${}^{0'}\mathbf{R}_{0''}$.
- Rotate the current frame by the angle φ about axis $\mathbf{z}_0''$; this rotation is described

by ${}^{0''}\mathbf{R}_A$.

The resulting frame orientation is obtained by the composition of rotation with respect to the current frames, then the rotation matrix is given by

$$
\begin{aligned}
{}^{0}\mathbf{R}_A &= {}^{0}\mathbf{R}_{0'}\,{}^{0'}\mathbf{R}_{0''}\,{}^{0''}\mathbf{R}_A \\
&= \begin{bmatrix} C\phi C\theta C\varphi - S\phi S\varphi & -C\phi C\theta S\varphi - S\phi C\varphi & C\phi S\theta \\ S\phi C\theta C\varphi + C\phi S\varphi & -S\phi C\theta S\varphi + C\phi C\varphi & S\phi S\theta \\ -S\theta C\varphi & S\theta S\varphi & C\theta \end{bmatrix}
\end{aligned} \tag{4}
$$

where abbreviations such as $C\phi = \cos\phi$ and $S\phi = \sin\phi$ are used, and hereafter the short-hand notation is adopted. This equation shows that the rotation matrix ${}^{0}\mathbf{R}_A$ is uniquely computed for a given set of Euler angles. For a given rotation matrix ${}^{0}\mathbf{R}_A = \{R_{ij}\}$, where R_{ij} is the (*i*,*j*) element of ${}^{0}\mathbf{R}_A$, if $\sin\theta \neq 0$, two sets of Euler angles are obtained by

$$
\begin{aligned}
\theta &= A\tan 2(\pm\sqrt{R_{13}{}^2 + R_{23}{}^2},\ R_{33}) \\
\phi &= A\tan 2(\pm R_{23}, \pm R_{13}) \\
\varphi &= A\tan 2(\pm R_{32}, \mp R_{31})
\end{aligned} \tag{5}
$$

where $A\tan 2(y, x)$ is the arctangent function of two arguments. If $\sin\theta = 0$, it is possible to determine only the sum or difference of φ and ϕ, because axis $\mathbf{z}_0$ are parallel to axis $\mathbf{z}_0''$, and these axes make equivalent contributions to the rotation. These configurations characterize the representation singularities of the Euler angles. One approach to avoid the representation singularities is the unit quaternion, in which four parameters are used.

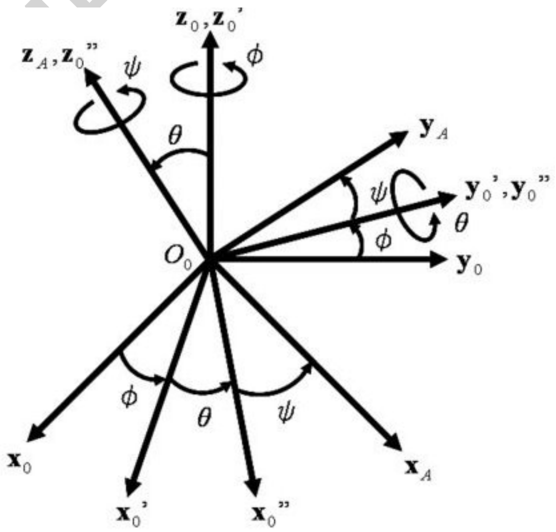

Figure 7: Representation of **Z-Y-Z** Euler angles.

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Roll-Pitch-Yaw angles

Another set Euler angles, the Z-Y-X Euler angles, are called the Roll-Pitch-Yaw angles, which originated to denote aircraft motions, as shown in Figure 8. The rotational angle about axis **x** is the yaw, the rotational angle about axis **y** is the pitch, and the rotational angle about axis **z** is the roll. The resulting rotation matrix obtained by the composition of Z-Y-X rotations is given by

$$ {}^{0}\mathbf{R}_{A} = \begin{bmatrix} C\phi C\theta & C\phi S\theta S\varphi - S\phi C\varphi & C\phi S\theta C\varphi + S\phi S\varphi \\ S\phi C\theta & S\phi S\theta S\varphi + C\phi C\varphi & S\phi S\theta C\varphi - C\phi S\varphi \\ -S\theta & C\theta S\varphi & C\theta C\varphi \end{bmatrix}. \quad (6) $$

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Biographical Sketch

Haruhisa Kawasaki was received the Master of Engineering Degree and the Doctor of Engineering degree both from Nagoya University in 1974 and 1986, respectively. He was a researcher at NTT's Laboratories from 1974 to 1990. He was Professor of Kanazawa Institute of Technology from 1990 to 1994. He is Professor of Faculty of Engineering of Gifu University since 1994. He was the chairman of executive committee of the International Conference on Virtual Systems and Multimedia in 1995 and 1996 (VSMM'95 and VSMM'96). He served as the Director of Virtual System Laboratory of Gifu University from 1997 to 1998. He was a gust professor at University of Surrey from July 1998 to January 1999. He is the editor of Journal of Robotics and Mechatronics since 1989. He is mainly engaging in the research fields of robot control, humanoid robot hand system, symbolic robot analysis system, robot teaching in a virtual realty environment and intelligent mechatronics. He is a membership of the Institute of Electrical and Electronic Engineers (IEEE), the Japan Society of Mechanical Engineers (JSME), the Robotics Society of Japan (RSJ), the Society of Instrument and Control Engineers (SICE) and the Virtual Reality Society of Japan (VRSJ).

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